LIST OF THE PLATES IN VOL. II.,

TO ILLUSTRATE THE FOLLOWING MEMOIRS.

SCIENTIFIC MEMOIRS,

SELECTED FROM

THE TRANSACTIONS OF

FOREIGN ACADEMIES OF SCIENCE

AND LEARNED SOCIETIES,

AND FROM

FOREIGN JOURNALS.

EDITED BY

RICHARD TAYLOR, F.S.A.,

FELLOW OF THE LINNÆAN, GEOLOGICAL, ASTRONOMICAL, ASIATIC, STATISTICAL, AND GEOGRAPHICAL SOCIETIES OF LONDON;
HONORARY MEMBER OF THE NATURAL HISTORY SOCIETY OF MOSCOW.
UNDER SECRETARY OF THE LINNÆAN SOCIETY.

VOL. II.

LONDON:

PRINTED BY RICHARD AND JOHN E. TAYLOR,
RED LION COURT, FLEET STREET.

SOLD BY LONGMAN, ORME, BROWN, GREEN, AND LONGMANS; CADELL; RIDGWAY AND SONS; SHERWOOD, GILBERT, AND PIPER; SIMPKIN AND MARSHALL; B. FELLOWES; S. HIGHLEY; WHITTAKER AND CO.; AND J. B. BAILLIERE, LONDON: —AND BY A. AND C. BLACK, AND THOMAS CLARK, EDINBURGH; SMITH AND SON, GLASGOW:—MILLIKEN AND SON, AND HODGES AND M'ARTHUR, DUBLIN: —DOBSON, PHILADELPHIA:—AND GOODHUGH, NEW YORK.

1841.

Fig. 1.

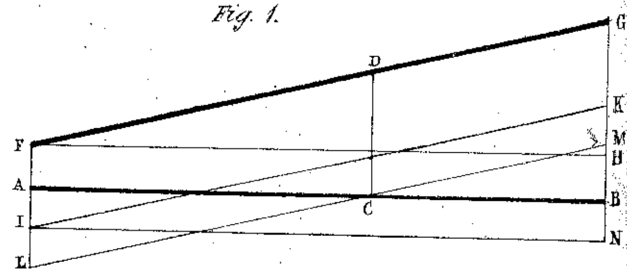


Fig. 2.

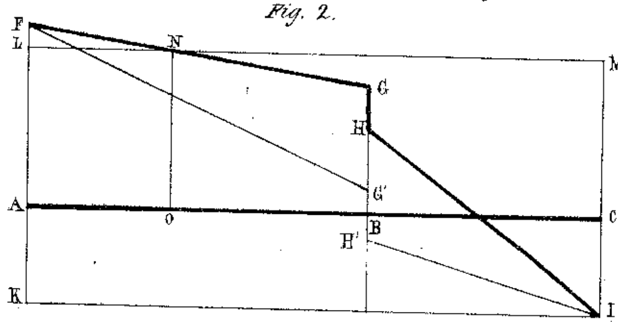
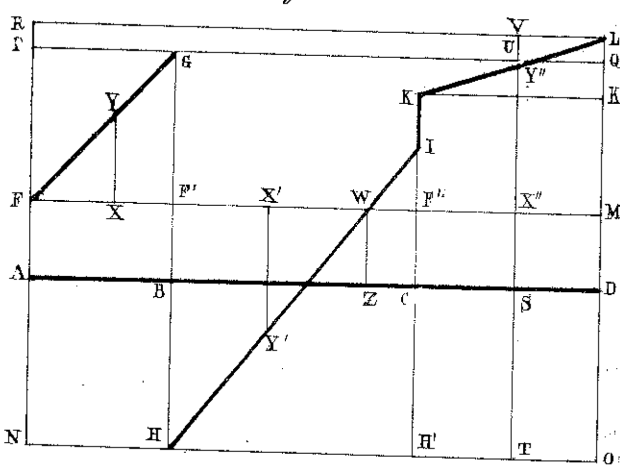


Fig. 3.



J. Basire. lith.

Ohm on the Galvanic Circuit.

Article XIII.

The Galvanic Circuit investigated Mathematically. By Dr. G. S. Ohm*.

Preface.

I HEREWITH present to the public a theory of galvanic electricity, as a special part of electrical science in general, and shall successively, as time, inclination, and means permit, arrange more such portions together into a whole, if this first essay shall in some degree repay the sacrifices it has cost me. The circumstances in which I have hitherto been placed, have not been adapted either to encourage me in the pursuit of novelties, or to enable me to become acquainted with works relating to the same department of literature throughout its whole extent. I have therefore chosen for my first attempt a portion in which I have the least to apprehend competition. May the well-disposed reader receive the performance with the same love for the object as that with which it is sent forth.

The Author.

Berlin, May 1st, 1827.

Introduction.

The design of this Memoir is to deduce strictly from a few principles, obtained chiefly by experiment, the rationale of those electrical phænomena which are produced by the mutual contact of two or more bodies, and which have been termed Galvanic:—its aim is attained if by means of it the variety of facts be presented as unity to the mind. To begin with the most simple investigations, I have confined myself at the outset to those cases where the excited electricity propagates itself only in one dimension. They form, as it were, the scaffold to a greater structure, and contain precisely that portion, the more accurate knowledge of which may be gained from the elements of natural philosophy, and which, also, on account of its greater accessibility, may be given in a more strict form. To answer

* "*Die Galvanische Kette mathematisch bearbeitet von Dr. G. S. Ohm: Berlin*, 1827." Translated from the German by Mr. William Francis, Student in Philosophy in the University of Berlin.

this especial purpose, and at the same time as an introduction to the subject itself, I give, as a forerunner of the compressed mathematical investigation, a more free, but not on that account less connected, general view of the process and its results.

Three laws, of which the first expresses the mode of distribution of the electricity within one and the same body, the second the mode of dispersion of the electricity in the surrounding atmosphere, and the third the mode of appearance of the electricity at the place of contact of two heterogeneous bodies, form the basis of the entire Memoir, and at the same time contain everything that does not lay claim to being completely established. The two latter are purely experimental laws; but the first, from its nature, is, at least partly, theoretical.

With regard to this first law, I have started from the supposition that the communication of the electricity from one particle takes place directly only to the one next to it, so that no immediate transition from that particle to any other situate at a greater distance occurs. The magnitude of the transition between two adjacent particles, under otherwise exactly similar circumstances, I have assumed as being proportional to the difference of the electric forces existing in the two particles; just as, in the theory of heat, the transition of caloric between two particles is regarded as proportional to the difference of their temperatures. It will thus be seen that I have deviated from the hitherto usual mode of considering molecular actions introduced by Laplace; and I trust that the path I have struck into will recommend itself by its generality, simplicity, and clearness, as well as by the light which it throws upon the character of former methods.

With respect to the dispersion of electricity in the atmosphere, I have retained the law deduced from experiments by Coulomb, according to which, the loss of electricity, in a body surrounded by air, in a given time, is in proportion to the force of the electricity, and to a coefficient dependent on the nature of the atmosphere. A simple comparison of the circumstances under which Coulomb performed his experiments, with those at present known respecting the propagation of electricity, showed, however, that in galvanic phænomena the influence of the atmosphere may almost always be disregarded. In Coulomb's experiments, for instance, the electricity driven to the surface of the

body was engaged in its entire expanse in the process of dispersion in the atmosphere; while in the galvanic circuit the electricity almost constantly passes through the interior of the bodies, and consequently only the smallest portion can enter into mutual action with the air; so that, in this case, the dispersion can comparatively be but very inconsiderable. This consequence, deduced from the nature of the circumstances, is confirmed by experiment; in it lies the reason why the second law seldom comes into consideration.

The mode in which electricity makes its appearance at the place of contact of two different bodies, or the electrical tension of these bodies, I have thus expressed: when dissimilar bodies touch one another, they constantly maintain at the point of contact the same difference between their electroscopic forces.

With the help of these three fundamental positions, the conditions to which the propagation of electricity in bodies of any kind and form is subjected may be stated. The form and treatment of the differential equations thus obtained are so similar to those given for the propagation of heat by Fourier and Poisson, that even if there existed no other reasons, we might with perfect justice draw the conclusion that there exists an intimate connexion between both natural phænomena; and this relation of identity increases, the further we pursue it. These researches belong to the most difficult in mathematics, and on that account can only gradually obtain general admission; it is therefore a fortunate chance, that in a not unimportant part of the propagation of electricity, in consequence of its peculiar nature, those difficulties almost entirely disappear. To place this portion before the public is the object of the present memoir, and therefore so many only of the complex cases have been admitted as seemed requisite to render the transition apparent.

The nature and form commonly given to galvanic apparatus favours the propagation of the electricity only in one dimension; and the velocity of its diffusion combined with the constantly acting source of galvanic electricity is the cause of the galvanic phænomena assuming, for the most part, a character which does not vary with time. These two conditions, to which most frequently galvanic phænomena are subjected, viz. change of the electric state in a single dimension, and its independency of time, are however precisely the reasons why the investigation is brought to a degree of simplicity which is not surpassed in any branch of natural philosophy, and is altogether adapted to secure incontrovertibly to mathematics the possession of a new field of physics, from which it had hitherto remained almost totally excluded.

The chemical changes which so frequently occur in some, generally fluid, portions of a galvanic circuit, greatly deprive the result of its natural simplicity, and conceal, to a considerable extent, by the complications they produce, the peculiar progression of the phænomenon; they are the cause of an unexampled change of the phænomenon, which gives rise to so many apparent exceptions to the rule, frequently even to contradictions, in so far as the sense of this word is itself not in contradiction to nature. I have distinctly separated the consideration of such galvanic circuits in which no portion undergoes a chemical change, from those whose activity is disturbed by chemical action, and have devoted a separate part to the latter in the Appendix. This total separation of two parts forming a whole, and, as might appear, the less dignified position of the latter, will find in the following circumstance a sufficient explanation. A theory, which lays claim to the name of an enduring and fruitful one, must have all its consequences in accordance with observation and experiment. This, it seems to me, is sufficiently established with respect to the first of the parts above-mentioned, partly by the previous experiments of others, and partly by some performed by myself, which first made me acquainted with the theory here developed, and subsequently rendered me entirely devoted to it. Such is not the case with regard to the second part. A more accurate experimental verification is in this case almost entirely wanting, to undertake which I need both the requisite time and means; and therefore I have merely placed it in a corner, from which, if worth the trouble, it may be drawn hereafter, and may then also be further matured under better nursing.

By means of the first and third fundamental positions we obtain a distinct insight into the primary galvanic phænomenon in the following way. Imagine, for instance, a ring everywhere of equal thickness and homogeneous, having, at any one place, in its whole thickness, one and the same electrical tension, *i. e.* inequality in the electrical state of two surfaces situated close to each other, from which causes, when they have come into action, and the equilibrium is consequently disturbed, the electricity will, in its endeavour to re-establish itself, if its mobility be solely confined to the extent of the ring, flow off on both sides. If this tension were merely momentary, the equilibrium would very soon be re-established; but if the tension is permanent, the equilibrium can never be restored; but the electricity, by virtue of its expansive force, which is not sensibly restrained, produces in a space of time, the duration of which almost always escapes our senses, a state which comes nearest to that of equilibrium, and consists in this; that by the constant transmission of the electricity, a perceptible change in the electric condition of the parts of the body through which the current passes is nowhere produced. The peculiarity of this state, also occurring frequently in the transmission of light and heat, has its foundation in this; that each particle of the body situated in the circle of action receives in each moment just so much of the transmitted electricity from the one side as it gives off to the other, and therefore constantly retains the same quantity. Now since by reason of the first fundamental position the electrical transition only takes place directly from the one particle to the other, and is, under otherwise similar circumstances, determined according to its energy by the electrical difference of the two particles, this state must evidently indicate itself on the ring, uniformly excited in its entire thickness, and similarly constituted in all its parts, by a constant change of the electric condition, originating from the point of excitation, proceeding uniformly through the whole ring, and finally again returning to the place of excitation; whilst at this place itself, a sudden spring in the electric condition, constituting the tension, is, as was previously stated, constantly perceptible. In this simple separation or division of the electricity lies the key to the most varied phænomena.

The mode of separation of the electricity has been completely determined by the preceding observation; but the absolute force of the electricity at the various parts of the ring still remains uncertain. This property may be best conceived, by imagining the ring, without its nature being altered, opened at the point of excitation and extended in a straight line, and representing the force of the electricity at each point by the length of a perpendicular line erected upon it; that directed upwards may represent a positive electrical, but that downwards a negative electrical, state of the part. The line A B (Plate XXIV., fig. 1) may accordingly represent the ring extended in a straight line, and the lines A F and B G perpendicular to A B may indicate by their lengths the force of the positive electricities situated at the extremities A and B. If now the straight line F G be drawn from F to G, also F H parallel to A B, the position of F G will give the mode of separation of the electricity, and the quantities B G − A F or G H the tension occurring at the extremities of the ring; and the force of the electricity at any other place C, may easily be expressed by the length of C D drawn through C perpendicularly to A B. But, from the nature of the galvanic excitation, merely the quantity of the tension or the length of the line G H, therefore the difference of the lines A F and B G, is determined, but not at all the absolute magnitudes of the lines A F and B G; consequently the mode of separation may be represented quite as well by any other line parallel to the former, *e. g.* by I K, for which the tension still constantly retains the same value expressed by K N, because the ordinates situated at present below A B assume a relation opposed to their former one. Which of the infinitely numerous lines parallel to F G would express the actual state of the ring cannot be stated in general, but must in each case be separately determined from the circumstances which occur. Moreover, it is easily conceived that, as the position of the line sought is given, it would be completely determined for one single part of the ring by the determination of any one of its points, or, in other words, by the knowledge of the electric force. If, for instance, the ring lost all its electricity by abduction at the place C, the line L M drawn through C parallel to F G would in this case express with perfect certainty the electrical state of the ring. This variability in the separation of the electricity is the source of the changeableness of the phænomenon peculiar to the galvanic circuit. I may further add, that it is evidently quite indifferent whether the position of the line F G with respect to that of A B be fixed; or whether the position of the line F G remain constantly the same, and the position of A B with respect to it be altered. The latter course is by far the more simple where the separation of the electricity assumes a more complex form.

The conclusions just arrived at, which hold for a ring homogeneous throughout its whole extent, may easily be extended to a ring composed of any number of heterogeneous parts, if each part be of itself homogeneous and of the same thickness. I may here take as an example of this extension a ring composed of two heterogeneous parts. Let this ring be imagined as before open at one of its places of excitation and stretched out to form the right line A B C (fig. 2), so that A B and B C indicate the two heterogeneous parts of the ring. The perpendiculars A F, B G, will represent by their lengths the electrical forces present at the extremities of the part A B; on the other hand, B H and C I, those present at the extremities of the part B C; accordingly A F + C I or F K will represent the tension at the opened place of excitation, and G H the tension occurring at B at the point of contact. Now if we only bear in mind the permanent state of the circuit, the straight lines F G and H I will, from the reasons above mentioned, indicate by their position the mode of separation of the electricity in the ring; but whether the line A C will keep its place, or must be advanced further up or down, remains uncertain, and can only be found out in each distinct case by other separate considerations. If, for instance, the point O of the circuit is touched abductively, and thus deprived of all electricity, O N would disappear; and therefore the line L M drawn through N parallel with A C would in this case give the position of A C required. It is hence evident, how sometimes this, sometimes another, position of the line A C in the figure F G H I, representing the separation of the electricity, may be the one suited to the circumstances; and herein we recognise the source of the variability of galvanic phænomena already mentioned.

It is, however, essentially requisite, in order to be able to judge thoroughly of the present case, to attend to a circumstance the mention of which has hitherto been purposely avoided, that the various considerations might be separated as distinctly as possible. The distances F K and G H are indeed given by the tensions existing at the two places of excitation, but the figure F G H I is not yet wholly determined by this alone. For instance, the points G and H might move down towards G′ and H′, so that G′ H′ would equal G H, giving rise to the figure F G′ H′ I, which would indicate quite a different mode of separation of the electricity, although the individual tensions in it still retain their former magnitude.

Consequently if that which has been stated with respect to the circuit of two members is to acquire a sense no longer subject to any arbitrary explanation, this uncertainty must be removed. The first fundamental law effects this in the following way:— For since the state of the ring alone, independent of the time, is regarded, each section must, as has already been stated, receive in every moment the same quantity of electricity from one side as it gives off to the other. This condition occasions upon such portions of the ring as have perfectly the same constitution at their various points, the constant and uniform change in the separation which is represented in the first figure by the straight line F G, and in the second by the straight lines F G and H I. But when the geometrical or the physical nature of the ring changes in passing from one of its component parts to another, the reason of this constancy and uniformity no longer obtains; consequently the manner in which the several straight lines are combined into a complete figure must first be deduced from other considerations. To facilitate the object, I will separately consider the geometrical and physical difference of the single parts, each independently.

Let us first suppose that every section of the part B C is m times smaller than in the part A B, while both parts are composed of the same substance; the electric state of the ring, which is independent of time, and which requires that everywhere throughout the entire ring just as much electricity be received on one side as is given off from the other, can evidently only exist under the condition that the electric transition from one particle to the other in the same time within the portion B C is m times greater than in the portion A B; because it is only in this manner that the action in both parts can maintain equilibrium. But in order to produce this m times greater transition of the electricity from element to element, the electrical difference of element to element within the portion B C must, according to the first fundamental position, be m times greater in the portion A B; or when this determination is transferred to the figure, the line H I must sink m times more on equal portions, or have an m times greater "dip" than the line F G. By the expression "dip" (*Gefälle*), is to be understood the difference of such ordinates which belong to two places distant one unit of length from each other. From this consideration results the following rule: *The dips of the lines* F G *and* H I *in the portions* A B *and* B C, *composed of like substance, will be inversely to each other as the areas of the sections of these parts.* By this the figure F G H I is now fully determined.

When the parts A B and B C of the ring have the same section but are composed of different substances, the transition of the electricity will then no longer be dependent solely on the progressive change of electricity in each part from element to element, but at the same time also on the peculiar nature of each substance. This difference in the distribution of the electricity, caused solely by the material nature of the bodies, whether it have its origin in the peculiar structure or in any other peculiar state of the bodies to electricity, establishes a distinction in the electrical conductibility of the various bodies; and even the present case may afford some information respecting the actual existence of such a distinction and give rise to its more accurate determination. In fact, since the ring composed of the two parts A B and B C differs from the homogeneous one only in this respect, that the two parts are formed of two different substances, a difference in the dip of the two lines F G and H I will make known a difference in the conductibility of the two substances, and one may serve to determine the other. In this way we arrive at the following position, supplying the place of a definition: *In a ring consisting of two parts* A B *and* B C, *of like sections but formed of different substances, the dips of the lines* F G *and* H I *are inversely as the conducting powers of the two parts.* If we have once ascertained the conducting powers of the various substances, they may be employed to determine the dips of the lines F G and H I in every case that may occur. By this, then, the figure F G H I is entirely determined. The determination of the conductibility from the separation of the electricity is rendered very difficult from the weak intensity of galvanic electricity, and from the imperfection of the requisite apparatus; subsequently we shall obtain a more easy means of effecting this purpose.

From these two particular cases we may now ascend in the usual way to the general one, where the two prismatic parts of the ring neither possess the same section nor are constituted of the same substance. *In this case the dips of the two parts must be in the inverse ratio of the products of the sections and powers of conduction.* We are hereby enabled to determine completely the figure F G H I in every case, and also to distinguish perfectly the mode of electrical separation in the ring. All the peculiarities, hitherto considered separately, of the ring composed of two heterogeneous parts, may be summed up in the following manner: *In a galvanic circuit consisting of two heterogeneous prismatic parts, there takes place in regard to its electrical state a sudden transition from the one part to the other at each point of excitation, forming the tension there occurring, and from one extremity of each point to the other a gradual and uniform transition; and the dips of these two transitions are inversely proportional to the products of the conductibilities and sections of each part.*

Proceeding in this manner, we are able without much difficulty to inquire into the electrical state of a ring composed of three or more heterogeneous parts, and to arrive at the following general law: *In a galvanic circuit consisting of any indefinite number of prismatic parts, there takes place in regard to its electrical state at each place of excitation a sudden transition, from one part to the other, forming the tension there prevailing, and within each part a gradual and uniform transition from the one extremity to the other; and the dips of the various transitions are inversely proportional to the products of the conductibilities and sections of each part.* From this law may easily be deduced the entire figure of the separation for each particular case, as I will now show by an example.

Let A B C D (fig. 3) be a ring composed of three heterogeneous parts, open at one of its places of excitation, and extended in a straight line. The straight lines F G, H I, K L represent by their position the mode of separation of the electricity in each individual part of the ring, and the lines A F, B G, B H, C I, C K, and D E drawn through A, B, C and D perpendicular to A D such quantities that G H, K I and L M or D L − A F show by their length the magnitude of the tensions occurring at the individual places of excitation. From the known magnitude of these tensions, and from the given nature of the single parts A B, B C, and C D, the figure of the electrical separation has to be entirely determined.

If we draw straight lines parallel to A D, through the points F, H and K, meeting the line drawn through B, C and D perpendicular to A D, in the points F', H', K', then according to what has already been demonstrated, the lines GF', IH' and LK' are directly proportional to the lengths of the parts A B, B C and C D, and inversely proportional to the products of the conductibility and section of the same part; consequently the relations of the lines GF', IH' and LK' to each other are given. Further, that $GF' + IH' + LK' = GH - KI + (DL - AF = LM)$ is also known, as the tensions represented by G H, K I and D L − A F are given. From the given relations of the lines GF', IH', LK' and their known sum, these lines may now be found individually; the figure F G H I K L is evidently then entirely determined. But the position of this figure with respect to the line A D remains from its very nature still undecided.

If we recollect, that proceeding in the same direction A D, the tensions represented by G H and D L − A F or L M indicate a sudden sinking of the electric force at the respective places of excitation, that represented by I K on the contrary a sudden rise of the force; and that tensions of the first kind are regarded and treated as positive quantities, while tensions of the latter kind are considered as negative quantities, we find the above example lead us to the following generally valid rule: *If we divide the sum of all the tensions of the ring composed of several parts into the same number of portions which are directly proportional to the lengths of the parts and inversely proportional to the products of their conductibilities and their sections, these portions will give in succession the amount of gradation which must be assigned to the straight lines belonging to the single parts and representing the separation of the electricity; at the same time the positive sum of all the tensions indicates a general rise, on the contrary the negative sum of all the tensions a general depression of those lines.*

I will now proceed to the determination of the electric force at any given position in every galvanic circuit, and here again I shall lay down as basis fig. 3. For this purpose let a, a', a'' indicate the tensions existing at B, C, and between A and D, so that in this case also a and a'' represent additive, a' on the contrary a subtractive line, and λ, λ', λ'' any lines which are directly as the lengths of the parts A B, B C, and C D, and inversely as the products of the conductibilities and sections of the same parts; further, let

$$a + a' + a'' = A$$

and

$$\lambda + \lambda' + \lambda'' = L$$

then according to the law just ascertained

$G F'$ is a fourth proportional to L, A and λ

$I H'$ a fourth proportional to L, A and λ'

$L K'$ a fourth proportional to L, A and λ''.

Draw the line $F M$ through F parallel to $A D$, regard this line as the axis of the abscissæ, and erect at any given points X, X', X'' the ordinates $X Y$, $X' Y'$, $X'' Y''$, we obtain their respective values, thus:

In the first place we have, since $A B = F F''$

$$A B : G F' = F X : X Y,$$

whence follows:

$$X Y = \frac{F X \, . \, G F'}{A B},$$

or if we substitute for $G F'$ its value $\frac{A \, . \, \lambda}{L}$

$$X Y = \frac{A}{L} \cdot \frac{F X \, . \, \lambda}{A B}.$$

If now x represent a line such that

$$A B : F X = \lambda : x,$$

then

$$X Y = \frac{A}{L} \, . \, x.$$

Secondly, since $B C$ and $F' X'$ are equal to the lines drawn through I and Y' to $G H$ parallel to $A D$

$$B C : I H' = F' X' : F' H - X' Y',$$

whence

$$- X' Y' = \frac{I H' \, . \, F' X'}{B C} - F' H$$

or, since $F' H = G H - G F'$

$$- X' Y' = \frac{I H' \, . \, F' X'}{B C} + G F' - a.$$

If now for $I H'$ and $G F'$ we substitute their values $\frac{A \, . \, \lambda'}{L}$ and $\frac{A \, . \, \lambda}{L}$, we obtain

$$- X' Y' = \frac{A}{L} \left(\lambda + \frac{F' X' \, . \, \lambda'}{B C} \right) - a;$$

and if by x' we represent a line such that

$$B C : F' X' = \lambda' : x',$$

then

$$- X' Y' = \frac{A}{L} (\lambda + x') - a.$$

Thirdly, since $C D = K K'$ and $F'' X''$ is equal to the part of $K K'$ which extends from K to the line $X'' Y''$, we have

$$C D : L K' = F'' X'' : X'' Y'' - K F'',$$

whence

$$X'' Y'' = \frac{L K' \, . \, F'' X''}{C D} + K F'',$$

or, since $K F'' = K I + I H' - F' H$ and $F' H = G H - G F'$,

$$X'' Y'' = \frac{L K' \, . \, F'' X''}{C D} + I H' + G F' - (a + a').$$

If now for $L K'$, $I H'$, $G F'$ we substitute their values

$\frac{A \, . \, \lambda''}{L}$, $\frac{A \, . \, \lambda'}{L}$, $\frac{A \, . \, \lambda}{L}$, we obtain

$$X'' Y'' = \frac{A}{L} \left(\lambda + \lambda' + \frac{F'' X'' \, . \, \lambda''}{C D} \right) - (a + a');$$

and if by x'' we represent a line such that

$$C D : F'' X'' = \lambda'' : x''$$

we have

$$X'' Y'' = \frac{A}{L} (\lambda + \lambda' + x'') - (a + a').$$

These values of the ordinates, belonging to the three distinct parts of the circuit and different in form from each other, may be reduced as follows to a common expression. For if F is taken as the origin of the abscissæ, $F X$ will be the abscissa corresponding to the ordinate $X Y$ which belongs to the homogeneous part $A B$ of the ring, and x will represent the length corresponding to this abscissa in the reduced proportion of $A B : \lambda$. In like manner $F X'$ is the abscissa corresponding to the ordinate $X' Y'$ which is composed of the parts $F F'$ and $F' X'$ belonging to the homogeneous portions of the ring, and λ, x' are the lengths reduced in the proportions of $A B : \lambda$ and $B C : \lambda'$ corresponding to these parts. Lastly $F X''$ is the abscissa corresponding to the ordinate $X'' Y''$, which is composed of the parts $F F'$, $F' F''$, $F'' X''$ belonging to the homogeneous portions of the ring, and λ, λ', x'' are the lengths reduced in the proportions of $A B : \lambda$, $B C : \lambda'$, $C D : \lambda''$. If in consequence of this consideration we call the values x, $\lambda + x'$, $\lambda + \lambda' + x''$

reduced abscissæ and represent them generally by y, we obtain

$$X\,Y = \frac{A}{L} \cdot y$$

$$-X'\,Y' = \frac{A}{L} \cdot y - a$$

$$X''\,Y'' = \frac{A}{L} \cdot y - (a + a'),$$

and it is evident that L is the same in reference to the whole length A D or F M as y is to the lengths F X, F X′, F X″, on account of which L is termed the entire reduced length of the circuit. Further, if we consider that for the abscissa corresponding to the ordinate X Y the tension has experienced no abrupt change, but that for the abscissa corresponding to the ordinate X′ Y′ the tension has experienced the abrupt changes a, a'; and if we represent generally by O the sum of all the abrupt changes of the tensions for the abscissa corresponding to the ordinate y, then all the values found the various ordinates are contained in the following expression:

$$\frac{A}{L} \cdot y - O.,$$

But these ordinates express, when an arbitrary constant, corresponding to the length A F, is added to them, the electric forces existing at the various parts of the ring. If therefore we represent the electric force at any place generally by u we obtain the following equation for its determination:

$$u = \frac{A}{L}\, y - O + c,$$

in which c represents an arbitrary constant. This equation is generally true, and may be thus expressed in words: *The force of the electricity at any place of a galvanic circuit composed of several parts, is ascertained by finding the fourth proportional to the reduced length of the entire circuit, the reduced length of the part belonging to the abscissa, and the sum of all the tensions, and by increasing or diminishing the difference between this quantity and the sum of all the abrupt changes of tension for the given abscissa by an arbitrary quantity which is constant for all parts of the circuit.*

When the determination of the electric force at each place of the circuit has been effected, it only remains to determine the magnitude of the electric current. Now in a galvanic circuit of

the kind hitherto mentioned, the quantity of electricity passing through a section of it in a given time is everywhere the same, because at all places and in each moment the same quantity in the section leaves it on the one side as enters it from the other, but in different circuits this quantity may be very different: therefore, in order to compare the actions of several galvanic circuits *inter se*, it is requisite to have an accurate determination of this quantity, by which the magnitude of the current in the circuit is measured. This determination may be deduced from figure 3 in the following manner. It has already been shown that the force of the electric transition in each instant from one element to the adjacent one is given by the electric difference between the two existing at that time, and by a magnitude dependent upon the kind and form of the particles of the body, viz. the conductibility of the body. But the electrical difference of the elements in the part B C, for instance, reduced to a constant unit of distance, will be expressed by the dip of the line H I or by the quotient $\frac{I\,H'}{B\,C}$; if, therefore, we now indicate by $\varkappa$ the magnitude of the conductibility of the part B C,

$$\frac{\varkappa \,.\, I\,H'}{B\,C}$$

will express the force of the transition from element to element, or the *intensity* of the current in the part B C; consequently if ω represent the magnitude of the section in the part B C, the quantity of electricity passing in each instant from one section to the adjacent one, or the *magnitude* of the current, will be expressed by

$$\frac{\varkappa \,.\, \omega \,.\, I\,H'}{B\,C};$$

or if S represent this magnitude of the current, we have

$$S = \frac{\varkappa \,.\, \omega \,.\, I\,H'}{B\,C},$$

and if we substitute for I H′ its value $\frac{A \,.\, \lambda'}{L}$

$$S = \frac{A}{L} \cdot \frac{\varkappa \,.\, \omega \,.\, \lambda'}{B\,C}.$$

Hitherto the letters λ, λ', λ'' have represented lines which are proportional to the quotients formed of the lengths of the parts A B, B C, C D, and the products of their conductibilities and their sections. If we restrict for the present this determination, which leaves the absolute magnitude of the lines λ, λ', λ'' uncertain, so that the magnitudes λ, λ', λ'' shall not be merely proportional to the said quotients, but shall be likewise equal to them, and henceforth vary this limitation in accordance with the meaning of the expression "reduced lengths," the first of the two preceding equations becomes

$$S = \frac{I\,H'}{\lambda'},$$

which gives the following generally: *The magnitude of the current in any homogeneous portion of the circuit is equal to the quotient of the difference between the electrical forces present at the extremities of this portion divided by its reduced length.*

magnitude

This expression for the ~~forces~~ of the current will be continued to be employed subsequently. The second of the former equations passes, by the adopted change, into

$$S = \frac{A}{L},$$

which is generally true, and already reveals the equality of the force of the current at all parts of the circuit; in words it may

GRösse

be thus expressed: *The force of the current in a galvanic circuit is directly as the sum of all the tensions, and inversely as the entire reduced length of the circuit,* bearing in mind that at present by reduced length is understood the sum of all the quotients obtained by dividing the actual lengths corresponding to the homogeneous parts by the product of the corresponding conductibilities and sections.

From the equation determining the force of the current in a galvanic circuit in conjunction with the one previously found, by which the electric force at each place of the circuit is given, may be deduced with ease and certainty all the phænomena belonging to the galvanic circuit. The former I had already some time ago derived from manifoldly varied experiments* with an apparatus which allows of an accuracy and certainty of measurement not suspected in this department; the latter expresses all the observations pertaining to it, which already exist in great number, with the greatest fidelity, which also continues where the equation leads to results no longer comprised in the circle of previously published experiments. Both proceed uninterruptedly hand in hand with nature, as I now hope to

* Schweigger's *Jahrbuch*, 1826, part 2.

S = current ; A = volt diff.

L = "reduced length" = ℓL/A = resistance

demonstrate by a short statement of their consequences; at the same time I consider it necessary to observe, that both equations refer to all possible galvanic circuits whose state is permanent, consequently they comprise the voltaic combination as a particular case, so that the theory of the pile needs no separate comment. In order to be distinct, I shall constantly, instead of employing the equation $u = \frac{A}{L} y - O + c$, only take the third figure, and therefore will merely remark here, once for all, that all the consequences drawn from it hold generally.

In the next place, the circumstance that the separation of the electricity, diffusing itself over the galvanic circuit, maintains at the different places a permanent and unchangeable gradation, although the force of the electricity is variable at one and the same place, deserves a closer inspection. This is the reason of that magic mutability of the phænomena which admits of our predetermining at pleasure the action of a given place of the galvanic circuit on the electrometer, and enables us to produce it instantly. To explain this peculiarity I will return to figure 3. Since the figure of separation F G H I K L, is always wholly determined from the nature of any circuit; but its position with respect to the circuit A D, as was seen, is fixed by no inherent cause, but can assume any change produced by a movement common to all its points in the direction of the ordinates, the electrical condition of each point of the circuit expressed by the mutual position of the two lines, may be varied constantly, and at will, by external influences. When, for example, A D is at any time the position representing the actual state of the circuit, so that, therefore, the ordinate S Y″ expresses by its length the force of the electricity at the place of the circuit to which that ordinate belongs, then the electrical force corresponding to the point A, at the same time will be represented by the line A F. If now the point A be touched abductively, and thus be entirely deprived of all its force, the line A D will be brought into the position F M, and the force previously existing in the point S will be expressed by the length X″ Y″; this force, therefore, has suddenly undergone a change, corresponding to the length S X″. The same change would have occurred if the circuit had been touched abductively at the point

SL = A equiv. to IR = V

Z, because the ordinate Z W is equal to that of A F. If the circuit were touched at the place where the two parts A B and B C join, but so that the contact was made within the part B C, we should have to imagine A D advanced to N O; the electrical force at the point S would in this case be increased to the force indicated by T Y$''$. But if the contact took place, still at the same point, viz. where the parts A B and B C touch each other, but within the part A B, the line A D would be moved to P Q, and the force belonging to the point S would sink to the negative force expressed by U Y$''$. If, lastly, the pile had been touched abductively at the point D, we should have prescribed for the line A D the position R L, and the electrical force at the point S would have assumed the negative force indicated by V Y$''$. The law of these changes is obvious, and may be expressed generally thus: *each place of a galvanic circuit undergoes mediately, in regard to its outwardly acting electrical force, the same change which is produced immediately at any other place of the circuit by external influences.*

Since each place of a galvanic circuit undergoes, of itself, the same change to which a single place was compelled, the change in the quantity of electricity, extending over the whole circuit, is proportional, on the one hand, to the sum of all the places, *i. e.* to the space over which the electricity is diffused in the circuit, and moreover, to the change in the electric force produced at one of these places. From this simple law result the following distinct phænomena. If we call r the space over which the electricity is diffused in the galvanic circuit, and imagine this circuit touched at any one place by a non-conducting body, and designate by $u_{,}$ the electric force at this place before contact, by u that after contact, the change produced in the force at this place is $u_1 - u$; consequently the change of the whole quantity of electricity in the circuit is $(u_1 - u)\,r$. If, now, we suppose that the electricity in the touched body is diffused over the space R, and is at all places of equal strength, and, at the same time, that at the place of contact itself the circuit and the body possess the same electric force, viz. u, it it is evident $u\,\mathrm{R}$ will be the quantity of electricity imparted to the body, and

$$(u_{,} - u)\,r = u\,\mathrm{R},$$

whence we obtain

$$u = \frac{u_{,}\,r}{r + \mathrm{R}}.$$

The intensity of the electricity received by the body will, therefore, be the more nearly equal to that which the circuit possessed at the place of contact before being touched, the smaller R *is with respect to r; it will amount to the half when* R $= r$, *and become weaker, as* R *becomes greater in comparison with r.* Since these changes are merely dependent on the relative magnitude of the spaces r and R, and not at all on the qualitative nature of the circuit, they are merely determined by the dimensions of the circuit, nay, even by foreign masses brought into conducting connexion with the circuit. If we connect this fact with the theory of the condensor, we arrive at an explanation of all the relations of the galvanic circuit to the condensor, noticed by Jäger*, which is perfectly surprising. I content myself with regard to this point to refer to the memoir itself, to give room here for the insertion of some new peculiarities of the galvanic circuit.

The mode of separation of the electricity, within a homogeneous part of the circuit, is determined by the magnitudes of the dips of the lines F G, H I, K L, (fig. 3,) and there again by the magnitudes of the ratios $\frac{\mathrm{G\,F'}}{\mathrm{A\,B}}$, $\frac{\mathrm{I\,H'}}{\mathrm{B\,C}}$, $\frac{\mathrm{L\,K'}}{\mathrm{C\,D}}$. But, as was already shown,

$$\mathrm{G\,F'} = \frac{\mathrm{A}}{\mathrm{L}} \cdot \lambda, \quad \mathrm{I\,H'} = \frac{\mathrm{A}}{\mathrm{L}} \cdot \lambda', \quad \mathrm{L\,K'} = \frac{\mathrm{A}}{\mathrm{L}} \cdot \lambda'';$$

hence it may be seen, without much trouble, that the magnitude of the dip of the line corresponding to any part of the circuit, and representing the separation of the electricity, is obtained by multiplying the value $\frac{\mathrm{A}}{\mathrm{L}}$ by the ratio of the reduced to the actual length of the same part. If, therefore, (λ) represent the reduced length of any homogeneous part of the circuit and (l) its actual length, the magnitude of the dip of the straight line belonging to this part, and representing the separation of the electricity, is

$$\frac{\mathrm{A}}{\mathrm{L}}' \cdot \frac{(\lambda)}{(l)},$$

which expression, if we designate by $(\varkappa)$ the conductibility,

* Gilbert's *Annalen*, vol. xiii.

and by (ω) the section of the same part, may also be written thus:

$$\frac{A}{L} \cdot \frac{(\lambda)}{(\varkappa)\,(\omega)}.$$

This expression leads to a more detailed knowledge of the separation of the electricity in a galvanic circuit. For since A and L designate values which remain identical for each part of the same circuit, it is evident *that the dips in the separate homogeneous parts of a circuit are to one another inversely as the products of the conductibility, and the section of the part.* If consequently a part of the circuit surpasses all others from the circumstance, that the product of its conductibility and its section is far smaller than in the others, it will be the most adapted to reveal, by the magnitude of its dip, the differences of the electric force at its various points. If its actual length is, at the same time, not inferior to those of the other parts, its reduced length will far surpass those of the other parts; and it is easily conceived that such a relation between the various parts can be brought about, that even its reduced length may remain far greater than the sum of the reduced lengths of all the other parts. But in this case the reduced length of this one part is nearly equal to the reduced lengths of the entire circuit, so that we may substitute, without committing any great error, $\frac{(l)}{(\varkappa)\ (\omega)}$ for L, if (l) represent the actual length of the said part, $(\varkappa)$ its conductibility, and (ω) its section; but then the dip of this part changes nearly into

$$\frac{A}{(l)},$$

whence it follows *that the difference of the electrical forces at the extremities of this part is nearly equal to the sum of all the tensions existing in the circuit.* All the tensions seem, as it were, to tend towards this one part, causing the electrical separation to appear in it with otherwise unusual energy, when all the tensions, or, at least, the greater part in number and magnitude, are of the same kind. In this way the scarcely perceptible gradation in the separation of the electricity, in a closed circuit, may be rendered distinctly evident, which, otherwise, would not be the case without a condensor, on account of the weak intensity of galvanic forces. This remarkable pro-

perty of galvanic circuits, representing, as it were, their entire nature, had already been noticed long ago in various bad conducting bodies, and its origin sought for in their peculiar constitution*; I have, however, enumerated in a letter to the editor of the *Annalen der Physik*†, the conditions under which this property of the galvanic circuit may be observed, even in the best conductors, the metals; and the necessary precautions, founded on experience, by which the success of the experiment is assured, described in it, are in perfect accordance with the present considerations.

The expression $\frac{A}{L} \cdot \frac{(\lambda)}{(l)}$, denoting the dip of any portion of the circuit, vanishes when L is indefinitely great, while A and $\frac{(\lambda)}{(l)}$ retain finite values. Consequently, if L assumes an indefinitely great value, while A remains finite, the dip of the straight lines representing the separation of the electricity, in all such parts of the circuit, whose reduced length has a finite ratio to the actual length, vanishes, or what comes to the same thing, the electricity is of equal force at all places of each such part. Now, since L represents the sum of the reduced lengths of all the parts of the circuit, and these reduced lengths evidently can only assume positive values, L becomes indefinite as soon as one of the reduced lengths assumes an infinite value. Further, since the reduced length of any part represents the quotient obtained by dividing the actual length by the product of the conductibility and the section of the same part, it becomes infinite when the conductibility of this part vanishes, *i. e.* when this part is a non-conductor of electricity. *When, therefore, a part of the circuit is a non-conductor, the electricity expands uniformly over each of the other parts, and its change from one part to the other is equal to the whole tension there situated.* This separation of the electricity, relative to the open circuit, is far more simple than that in the closed circuit, which has hitherto formed the object of our consideration, as is geometrically represented by the lines F G, H I, K L, (fig. 3) taking a position parallel to A D. It distinctly demonstrates *that the difference between the electrical forces, occurring at any two*

* Gilbert's *Annalen*, vol. viii. pp. 205, 207, and 456. Vol. x. p. 11.
† Jahrgang, 1826. Part v. p. 117.

places of the circuit, is equal to the sum of all the tensions situated between these two places, and consequently increases or decreases exactly in the same proportion as this sum. When, therefore, one of these places is touched abductively, the sum of all the tensions, situated between the two, makes its appearance at the other place, at the same time the direction of the tensions must always be determined by an advance from the latter place. All the experiments on the open pile, with the help of the electroscope, instituted at such length by Ritter, Erman, and Jäger, and described in Gilbert's *Annalen**, are expressed in this last law.

All the electroscopic actions of a galvanic circuit of the kind, described at the outset, have been above stated; I therefore pass at present to the consideration of the current originating in the circuit, the nature of which, as explained above, is expressed at every place of the circuit by the equ[...]

$$S = \frac{A}{L}.$$

Both the form of this equation, as well as the mode by which we arrive at it, show directly *that the magnitude of the current in such a galvanic circuit remains the same at all places of the circuit, and is solely dependent on the mode of separation of the electricity, so that it does not vary, even though the electric force at any place of the circuit be changed by abductive contact, or in any other way.* This equality of the current at all places of the circuit has been proved by the experiments of Becquerel†, and its independency of the electric force at any determinate place of the circuit by those of G. Bischof‡. An abduction or adduction does not alter the current of the galvanic circuit so long as they only act immediately on a single place of the circuit; but if two different places were acted upon contemporaneously, a second current would be formed, which would necessarily, according to circumstances, more or less change the first.

The equation

$$S = \frac{A}{L}$$

shows that the current of a galvanic circuit is subjected to a

* Vol. viii., xii., and xiii.

† *Bulletin universel. Physique.* Mai, 1825.

‡ Kastner's *Archiv*, vol. iv. Part 1.

change, by each variation originating either in the magnitude of a tension or in the reduced length of a part, which latter is itself again determined, both by the actual length of the part, as well as by its conductibility and by its section. This variety of change may be limited, by supposing only one of the enumerated elements to be variable, and all the remainder constant. We thus arrive at distinct forms of the general equation corresponding to each particular instance of the general capability of change of a circuit. To render the meaning of this phrase evident by an example, I will suppose that in the circuit only the actual length of a single part is subjected to a continual change; but that all the other values denoting the magnitude of the current remain constantly the same, and, consequently, also in its equation. If we designate by x this variable length, and the conductibility corresponding to the same part by $\varkappa$, its section by ω, and the sum of the reduced lengths of all the others by Λ, so that $L = \Lambda + \frac{x}{\varkappa \,.\, \omega}$, then the general expression for the current changes into the following:

$$S = \frac{A}{\Lambda + \frac{x}{\varkappa \,.\, \omega}};$$

or if we multiply both the numerator and denominator by $\varkappa\,\omega$, and substitute a for $\varkappa\,\omega\,A$, and b for $\varkappa\,\omega\,\Lambda$, into the following:

$$S = \frac{a}{b + x},$$

where a and b represent two constant magnitudes, and x the variable length of a portion of the circuit fully determined with respect to its substance and its section. This form of the general equation, in which all the invariable elements have been reduced to the smallest number of constants, is that which I had practically deduced from experiments to which the theory here developed owes its origin*. The law which it expresses relative to the length of conductors, differs essentially from that which Davy formerly, and Becquerel more recently, were led to by experiments; it also differs very considerably from that advanced by Barlow, as well as from that which I had previously drawn from other experiments, although the two latter come much nearer to the truth. The first, in fact,

* See Schweigger's *Jahrbuch*, 1826. Part 2.

is nothing more than a formula of interpolation, which is valid only for a relatively very short variable part of the entire circuit, and, nevertheless, is still applicable in very different possible modes of conduction, which is already evident, from its merely admitting the variable portion of the circuit, and leaving out of consideration all the other part; but all partake in common of this evil, that they have admitted a foreign source of variability, produced by the chemical change of the fluid portion of the circuit, of which I shall speak more fully hereafter. I have already treated, in other places, more at length of the relations of the various forms of the law to one another.

From the numerous separate peculiarities of the galvanic circuit resulting from the general equation

$$S = \frac{A}{L},$$

I will here merely mention a few. It is immediately evident that a change in the arrangement of the parts has no influence on the magnitude of the current if the sum of the tensions be not affected by it. Nor is the magnitude of the current altered, when the sum of the tensions, and the entire reduced length of the circuit, change in the same proportion; consequently a circuit, the sum of whose tensions is very small in comparison to that of another circuit, may still produce a current, which, in energy, may be equal to that in the other circuit, when merely that which it loses in force of tensions is replaced by a shortening of its reduced length. *In this circumstance is the source of the peculiar difference between thermo- and hydro-circuits.* In the former only metals occur as parts of the circuit; in the latter, besides the metals, aqueous fluids, whose power of conduction, in comparison to that of the metals, is exceedingly small; on which account the reduced lengths of the fluid surpass, beyond all proportion, those of the metallic parts, with in all respects equal dimensions, and even remain considerably greater when diminished by shortening their actual lengths, and increasing their sections, so long, at least, as this diminution is not carried too far. And thence it is that the reduced length of the thermo-circuit is, in general, far smaller than that of the hydro-circuit, whence we may infer a tension smaller in the same proportion in the former, although the magnitude of the current,

in the thermo-circuit, cedes in nothing to that in the hydro-circuit. *The great difference between a thermo- and hydro-circuit, both of which produce a current of the same energy, is evident when the same change is made on both, as will be shown in the following consideration.* Let the reduced length of a thermo-circuit be L, and the sum of its tensions A, the reduced length of an hydro-circuit m L, and the sum of its tensions m A, then the magnitude of the current in the former is expressed by $\frac{A}{L}$, in the latter by $\frac{m\,A}{m\,L}$, and is consequently the same in both circuits. But this equality of the current no longer exists if the same new part λ of the reduced length be introduced into both, for then the magnitude of the current is in the first

$$\frac{A}{L + \lambda},$$

in the second

$$\frac{m\,A}{m\,L + \lambda}.$$

If we connect with this determination an evaluation, even if merely superficial, of the quantities m, L, and λ, we shall readily be convinced that in cases where the simple hydro-circuit can still produce in the part λ actions of heat or chemical decomposition, the simple thermo-circuit may not possess the hundredth, and in some cases not the thousandth part of the requisite force, whence the absence of similar effects in it is easily to be understood. We are also able to understand why a diminution of the reduced lengths of the thermo-circuit (by increasing, for instance, the section of the metals constituting it) cannot give rise to the production of those effects, although the magnitude of its current may be increased by this means to a higher degree than in the hydro-circuit producing such effects. This difference in the conductibility of metallic bodies and aqueous fluids, is the cause of a peculiarity noticed with respect to hydro-circuits, which it is here, perhaps, the proper place to mention. Under the usual circumstances, the reduced length of the fluid portion is so large, in comparison to that of the metallic portion, that the latter may be overlooked, and the former alone taken instead of the reduced length of the entire circuit; but then the magnitude of the current in circuits which have the same tension is in the inverse ratio to the reduced length of the fluid portion. Consequently, if merely such circuits are compared in which the fluid parts have the same actual lengths and the same conductibilities, *then the magnitude of the current in these circuits is in direct ratio to the section of the fluid portion.* However, it must not be overlooked, that a more complex definition must take the place of this simple one when the reduced length of the metallic portion can no longer be regarded as evanescent towards that of the fluid, which case occurs whenever the metallic portion is very long and thin, or the fluid portion is a good conductor, and with unusually large terminal surfaces.

From the equation

$$S = \frac{A}{L}$$

we can easily perceive that, when a portion is taken from the galvanic circuit, and is replaced by another, and after this change the sum of the tensions as well as the energy of the current still remains perfectly the same, these two parts have the same reduced length, *consequently their actual lengths are as the products of their conductibilities and sections. The actual lengths of such parts are therefore, when they have like sections, as their conductibilities, and when they have like conductibilities as their sections.* By the first of these two relations we are enabled to determine the conductibilities of various bodies in a far more advantageous manner than by the previously mentioned process, and it has already been employed by Becquerel and myself for several metals*. The second relation may serve to demonstrate experimentally the independence of the effect on the form of the section, as has previously been done by Davy, and recently by myself†.

In the voltaic pile, the sum of the tensions, and the reduced length of the simple circuit, is repeated as frequently as the number of elements of which it consists expresses. If, therefore, we designate by A the sum of all the tensions in the simple circuit, by L its reduced length, and by n the number of elements in the pile, the magnitude of the current in the closed pile is evidently

* *Bulletin universel. Physique,* Mai 1825, and Schweigger's *Jahrbuch,* 1826. Part 2.

† Gilbert's *Annalen,* nn. Folge. Vol. xi. p. 253, and Schweigger's *Jahrbuch,* 1827.

$$\frac{nA}{nL},$$

while in the simple closed circuit it is

$$\frac{A}{L}.$$

If we now introduce into the simple circuit, as well as into the pile, one and the same new part Λ of the reduced length, upon which the current is to act, the magnitude of the current thus altered in the simple circuit will be

$$\frac{A}{L+\Lambda},$$

and in the voltaic pile

$$\frac{nA}{nL+\Lambda}, \quad \text{or} \quad \frac{A}{L+\frac{\Lambda}{n}}.$$

It is hence evident *that the current is constantly greater in a voltaic pile than in the simple circuit, but it is merely imperceptibly greater so long as Λ is very small in comparison with L; on the contrary, this increase approximates the nearer to n times, the greater Λ becomes to nL, and consequently the more so in comparison with* L. Besides this mode of increasing the magnitude of the galvanic current, there is a second one, which consists in shortening the reduced lengths of the simple circuit, which may be effected by increasing its section, or placing several simple circuits by the side of each other, and connecting them in such a way that together they only form one single simple circuit. If we now retain the same signs, so that

$$\frac{A}{L+\Lambda}$$

again denotes the magnitude of the current in one element, then, in the above-mentioned combination of n elements into a single circuit, the magnitude of the current is evidently

$$\frac{A}{\frac{L}{n}+\Lambda}, \quad \text{or} \quad \frac{nA}{L+n\Lambda},$$

which indicates a slight increase in the action of the new combination when Λ is very great in comparison with L; *on the contrary, a very powerful one when Λ is very small in comparison with $\frac{L}{n}$, and consequently the more so in comparison with* L. It hence follows that the one combination is most active in those cases where the other ceases to be so, and *vice versâ*. If therefore we are in possession of a certain number of simple circuits intended to act upon the portion whose reduced length is Λ, much depends on the way in which they are placed, in order to produce the greatest effect of current; whether all be side by side, or all in succession, or whether part be placed by the side of each other, and part in series. It may be mathematically shown that it is most advantageous to form them into a voltaic combination, of so many equal parts, that the square of this number be equal to the quotient $\frac{\Lambda}{L}$. When $\frac{\Lambda}{L}$ is equal to, or smaller than Λ, they had best be arranged by the side of each other, and in succession when $\frac{\Lambda}{L}$ is eq[illegible], or larger than the square of the number of all the elements. *We see in this determination the reason why in most cases a simple circuit, or at least a voltaic combination of only a few simple circuits, is sufficient to produce the greatest effect.* If we bear in mind, that since the quantity of the current is the same at all places of the circuit, its intensity at the various places must be in inverse proportion to the magnitude of the section there situated, and if we grant that the magnetic and chemical effects, as well as the phænomena of light and heat in the circuit, are direct indications of the electrical current, and that their energy is determined by that of the current itself, then a detailed analysis of the current, here indicated merely in outline, will lead to the perfect explanation of the numerous and partially enigmatical anomalies observed in the galvanic circuit, in as far as we are justified in considering the physical nature of the circuit as invariable*. Those great differences which are frequently met with in the statements of various observers, and which are not consequences of the dimensions of their different apparatus, have undoubtedly their origin in the double capability of change of the hydro-circuits, and will therefore cease when this circumstance is taken into consideration on a repetition of the experiments.

The remarkable variability in the circle of action of one and the same multiplier in various circuits, and of different multipliers in the same circuit, is completely explained by the

* See Schweigger's *Jahrbuch*, 1826, Part 2, where I have given a somewhat more detailed explanation of the separate points.

preceding consideration. For if we denote by A the sum of the tensions, and by L the reduced length of any galvanic circuit,

$$\frac{A}{L}$$

expresses the magnitude of its current. If we now imagine a multiplier of n similar convolutions each of the reduced length λ,

$$\frac{A}{L + n\lambda}$$

indicates the magnitude of the current when the multiplier is brought into the circuit as an integral part. Moreover, if we grant, for the sake of simplicity, that each of the n convolutions exerts the same action on the magnetic needle, the action of the multiplier on the magnetic needle is evidently

$$\frac{nA}{L + n\lambda},$$

when the action of an exactly similar coil of the circuit, without the multiplier on the needle, is taken as

$$\frac{A}{L}.$$

Hence it follows directly that the action on the magnetic needle is augmented or weakened by the multiplier, according as nL *is greater or smaller than* $L + n\lambda$, *i. e., according as n times the reduced length of the circuit without the multiplier is greater or smaller than the reduced length of the circuit with the multiplier.* Further, a mere glance at the expression by which the action of the multiplier on the needle has been determined, will show that the greatest or smallest action occurs as soon as L may be neglected with reference to $n\lambda$, and is expressed by

$$\frac{A}{\lambda}.$$

If we compare this extreme action of the multiplier with that which a perfectly similarly constructed convolution of the circuit without the multiplier produces, we perceive that both are in the same ratio to one another as the reduced lengths L and λ, which relation may serve to determine one of the values when the others are known. *The expression found for the extreme action of the multiplier shows that it is proportional to the tension of the circuit, and independent of its reduced length*; consequently the extreme action of the same multiplier may serve not merely to determine the tensions in various circuits, but it also indicates that the extreme action may be also augmented to the same degree as the sum of the tensions is increased, which may be effected by forming a voltaic combination with several simple circuits. If we represent the actual length of a coil of the multiplier by l, its conductibility by $\varkappa$, and its section by ω, so that $\lambda = \frac{l}{\varkappa . \omega}$, the expression for the extreme action of the multiplier is converted into

$$\varkappa . \omega . \frac{A}{l},$$

from which it will be seen *that the extreme action of two multipliers of different metals, constructed of wire of the same thickness, are in the same ratio to each oth the conductibilities of these metals, and that the extreme actions of two multipliers formed of wire of the same metal, are in the same proportion to each other as the sections of the wires.* All these various peculiarities of the multiplier I have shown to be founded on experience, partly on experiments performed by other persons, and partly on those by myself*. The most recent experiments made on this subject on thermo-circuits, have, in a different, and, in a certain sense, opposite manner, already afforded the conclusion deduced above from an equation of the reduced lengths, that the sum of the tensions in a thermo-circuit is far weaker than in the ordinary hydro-circuits; and a preliminary comparison has convinced me, that, with respect to the heating effects, if they are to be predicted with certainty, a voltaic combination of some hundred well-chosen simple thermo-circuits is requisite, and for chemical effects of some energy a far greater apparatus. Experiments, which place this prediction beyond doubt, will afford a new and not unimportant confirmation to the theory here propounded.

The previous considerations are also sufficient to indicate the process which is carried on when the galvanic circuit is divided at any place into two or more branches. For this purpose I call attention to the circumstance, that at the time we found the equation $S = \frac{A}{L}$, we also obtained the rule that the magnitude of the current in any homogeneous part of the galvanic

* Schweigger's *Jahrbuch*, 1826. Part 2; and 1827.

circuit is given by the quotient of the difference between the electrical forces existing at the extremities of the portion and its reduced length. It is true, this rule was only advanced above for the case in which the circuit nowhere divides into several branches; but a very simple consideration, analogous to the one then made, derived from the equality of the abducted and adducted quantity of electricity in all sections of each prismatic part, is sufficient to prove that the same rule holds good for every single branch in case of a division of the circuit. Let us suppose that the circuit be divided, for instance, into three branches, whose reduced lengths are λ, λ', λ''; and, moreover, that at each of these places the undivided circuit and the single branches possess equal electrical force, and consequently no tension occurs there, and designate by α the difference between the electrical forces at these two places; then, according to the above rule, the magnitude of the current in each of the three branches is

$$\frac{\alpha}{\lambda}, \frac{\alpha}{\lambda'}, \frac{\alpha}{\lambda''};$$

whence it directly follows *that the currents in the three branches are inversely as their reduced lengths*; so that each separate one may be found when the sum of all three together is known. But the sum of all three is evidently equal to the magnitude of the current at any other place of the non-divided portion of the circuit, for otherwise the permanent state of the circuit, which is still constantly supposed, would not be maintained. If we connect with this the conclusion resulting from the above considerations, namely, that the magnitude of the current, and the nature of each homogeneous part of the circuit, give the dip of the corresponding straight line, representing the separation of the electricity, we are certain that the figure of the separation belonging to the non-divided portion of the circuit must remain the same so long as the current in it retains the same magnitude, and *vice versâ*; whence it follows that the variability of the current in the non-divided portion necessarily supposes that the difference between the electrical forces at the extremities of this portion is constant. If we now imagine, instead of the separate branches, a single conductor of the reduced length Λ brought into the circuit which does not at all alter the magnitude of its current and its tensions, then, according to what has just been stated, the difference between the electrical forces at its extremities must still always remain α, and consequently be

$$\frac{\alpha}{\Lambda} = \frac{\alpha}{\lambda} + \frac{\alpha}{\lambda'} + \frac{\alpha}{\lambda''},$$

or

$$\frac{1}{\Lambda} = \frac{1}{\lambda} + \frac{1}{\lambda'} + \frac{1}{\lambda''},$$

which equation serves to determine the value of Λ. But if this value is known, and we call A the sum of all the tensions in the circuit, and L the reduced length of the non-divided portion of the circuit, *we obtain, as is known, for the magnitude of the current in the last-mentioned circuit*

$$\frac{A}{L + \Lambda},$$

which is equal to the sum of the curre[illegible]curring in the separate branches. Now since it has already been proved that the currents in the separate branches are in inverse proportion to one another as the reduced lengths of these branches, we obtain for the magnitude of the current in the branch whose reduced length is λ,

$$\frac{A}{L + \Lambda} \cdot \frac{\Lambda}{\lambda};$$

in the branch whose reduced length is λ',

$$\frac{A}{L + \Lambda} \cdot \frac{\Lambda}{\lambda'};$$

and in the branch whose reduced length is λ'',

$$\frac{A}{L + \Lambda} \cdot \frac{\Lambda}{\lambda''}.$$

This remote, and hitherto but slightly noticed peculiarity of the galvanic circuit, I have also found to be perfectly confirmed by experiment*.

I herewith conclude the consideration of such galvanic circuits which have already attained the permanent state, and which neither suffer modifications by the influence of the surrounding atmosphere, nor by a gradual change in their chemical composition. But from this point the simplicity of the subject decreases more and more, so that the previous elementary treatment soon entirely disappears. With respect to those

* Schweigger's *Jahrbuch*, 1827.

circuits on which the atmosphere exercises some influence, and whose condition varies with time, without this change originating in a progressive chemical transformation of the circuit, and is thus distinguished from all the others by the magnitude of its current being different at different places,—I have been content, with respect to each of these, always to treat of only the most simple case, as they but rarely occur in nature, and in general appear to be of less interest. I have adopted this plan the more willingly, as I intend to return to this subject at some future time. But with regard to that modification of galvanic circuits which is produced by a chemical change in the circuit, proceeding first from the current, and then again reacting on it, I have devoted separate attention to it in the Appendix. The course adopted is founded on a vast number of experiments performed on this subject, the communication of which, however, I omit, because they appear to be capable of being far more accurately determined than I was able to do at that time, from failing to attend to several elements in operation; nevertheless, I consider it proper to mention the circumstance in this place, in order that the careful manner with which I advance in the inquiry, and which I consider to be due to truth, may not operate more than is just against its reception.

I have sought for the source of the chemical changes caused by the current, in the above-described peculiar mode of separation of the electricity of the circuit; and, I can scarcely doubt, have at least found the main cause. It is immediately evident that each disk belonging to a section of a galvanic circuit, which obeys the electric attractions and repulsions and does not oppose their movement, must in the closed circuit be propelled always towards one side only, as these attractions and repulsions, in consequence of the continually varying electric force, are different at the two sides; and it is mathematically demonstrable *that the force with which it is driven to the one side, is in the ratio compounded of the magnitude of the electric current and of the electric force in the disk.* It is true, however, that merely a change of position in space would be immediately produced by this. But if this disk be regarded as a compound body, the constituent parts of which, according to electro-chemical views, are distinguished by a difference in their electrical relation to one another, it thence directly follows that this one-sided pressure on the various constituent parts would in most cases act with unequal force, and sometimes even in contrary direction, and must thus excite a tendency in them to separate from one another. From this consideration results a distinct activity of the circuit, tending to produce a chemical change in its parts, which I have termed its *decomposing force*, and I have endeavoured to determine its magnitude for each particular case. This determination is independent of the mode, in which the electricity may be conceived to be asssociated with the atoms.* Granting, which seems to be most natural, that the electricity is diffused proportionately to the mass over the space which these bodies then occupy, a complete analysis will show *that the decomposing force of the circuit is in direct proportion to the energy of the current, and, moreover, that it depends on a coefficient, to be derived from the nature the constituent parts and their chemical equivalents.* From the nature of this decomposing force of the circuit, which is of equal energy at all places of an homogeneous portion, it directly follows, that when it is capable of overcoming, under all circumstances, the reciprocal connexion of the constituent parts, the separation and abduction of the constituents to both sides of the circuit are limited solely by mechanical obstacles; but if the connexion of the constituent parts *inter se*, either immediately at the commencement everywhere, or in the course of the action anywhere, overcome the decomposing force of the circuit, then from that time no further movement of the elements can take place. This general description of the decomposing force is in accordance with the experiments of Davy and others.

There is a peculiar state which seems to be produced in most cases of the separation of the two elements of a chemically compound liquid, which is especially worthy of attention, and which is caused in the following manner. When the decomposition is confined solely to a limited portion of the circuit, and the constituent parts of the one kind are propelled towards the one side of this part, and the constituent parts of the other kind to its opposite side, then, for this very reason, a natural limit is prescribed to the action; for the constituent

* I shall shortly have occasion to speak of the peculiar import of this remark, when I shall attempt to reduce the actions of the parts of a galvanic circuit on one another, as discovered by Ampère, to the usual electrical attractions and repulsions.

part preponderating on the one side of any disk, within the portion in the act of decomposition, will, by force of its innate repulsive power, constantly oppose the movement of a similar constituent to the same side, so that the decomposing force of the circuit has not merely to overcome the constant connexion of the two constituents *inter se*, but also this reaction of each constituent on itself. It is hence evident that a cessation in the chemical change must occur, if at any time there arises an equilibrium between the two forces. This state, founded on a peculiar chemical and permanent separation of the constituents of the portion of the circuit in the act of decomposition, is the very one from which I started, and whose nature I have endeavoured to determine as accurately as possible in the Appendix. Even the mere description of the mode of origin of this highly remarkable phænomenon shows that at the extremities of the divided portion no natural equilibrium can occur, on which account the two constituents must be retained at these two places by a mechanical force, unless they pass over to the next parts of the circuit, or, where the other circumstances allow, separate entirely from the circuit. Who would not recognise in this plain statement all the chief circumstances hitherto observed of the external phænomenon in chemical decompositions by the circuit?

If the current, and, at the same time, the decomposing force, be suddenly interrupted, the separated constituents gradually return to their natural equilibrium; but tend to re-assume immediately the relinquished state, if the current is re-established. During this process, both the conductibility, and the mode of excitation between the elements of the portion in the act of decomposition, obviously vary with their chemical nature; but this necessarily produces a constant change in the electrical separation, and in the magnitudes of the current in the galvanic circuit dependent thereon, which only finds its natural limits in the permanent state of the electrical separation. For the accurate determination of this last stage of the electric current it is requisite to be acquainted with the law which governs the conductibility and force of excitation of the variable mixtures, formed of two different liquids. Experiment has hitherto afforded insufficient data for this purpose, I have therefore given the preference to a theoretical supposition, which will supply its place until the true law is discovered.

With the help of this law, which is not altogether imaginary, I now arrive at the equations which make known for each case all the individual circumstances constituting the permanent state of the chemical separation in the galvanic circuit; I have, however, neglected the further use of them, as the present state of our experimental knowledge in this respect did not appear to me to repay the requisite trouble. Nevertheless, in order to compare in their general features the results of this examination with what has hitherto been supplied by experiments, I have fully carried out one particular case, and have found that the formula represents very satisfactorily the kind of wave of the force, as I have above described it*.

Having thus given a slight outline of the contents of this Memoir, I will now proceed to the fundamental investigation of the individual points.

* Schweigger's *Jahrbuch*, 1826. Part 2.

[To be continued.]

SCIENTIFIC MEMOIRS.

VOL. II.—PART VIII.

ARTICLE XIII. continued.

The Galvanic Circuit investigated Mathematically. By Dr. G. S. OHM*.

THE GALVANIC CIRCUIT.

A. *General observations on the diffusion of electricity.*

1. A PROPERTY of bodies, called into activity under certain circumstances, and which we call *electricity*, manifests itself in space, by the bodies which possess it, and which on that account are termed *electric*, either attracting or repelling one another.

In order to investigate the changes which occur in the electric condition of a body A in a perfectly definite manner, this body is each time brought, under similar circumstances, into contact with a second moveable body of invariable electrical condition, called the *Electroscope*, and the force with which the electroscope is repelled or attracted by the body is determined. This force is termed the *electroscopic* force of the body A; and to distinguish whether it is attractive or repulsive we place before the expression for its measure the sign + in the one case, and − in the other.

The same body A may also serve to determine the electroscopic force in various parts of the same body. For this purpose we take the body A of very small dimensions, so that when we bring it into contact with the part to be tested of any third body, it may from its smallness be regarded as a sub-

* "*Die Galvanische Kette mathematisch bearbeitet von Dr. G. S. Ohm: Berlin*, 1827."

stitute for this part; then its electroscopic force, measured in the way described, will, when it happens to be different at the various places, make known the relative difference with regard to electricity between these places.

The intention of the preceding explanations is to give a simple and determinate signification to the expression "electroscopic force"; it does not come within the limits of our plan to take notice either of the greater or less practicability of this process, nor to compare *inter se* the various possible modes of proceeding for the determination of the electroscopic force.

2. We perceive that the electroscopic force moves from one place to another, and from one body to another, so that it does not merely vary at different places at the same time, but also at a single place at different times. In order to determine in what manner the electroscopic force is dependent upon the time when it is perceived, and on the place where it is elicited, we must set out from the fundamental laws to which the exchange of electroscopic force occurring between the elements of a body is subject.

These fundamental laws are of two kinds, either borrowed from experiment, or, where this is wanting, assumed hypothetically. The admissibility of the former is beyond all doubt, and the justness of the latter is distinctly evident from the coincidence of the results deduced from calculation with those which actually occur; for since the phænomenon with all its modifications is expressed in the most determinate manner by calculation, it follows, since no new uncertainties arise and increase the earlier ones during the process, that an equally perfect observation of nature must in a decisive manner either confirm or refute its statements. This in fact is the chief merit of mathematical analysis, that it calls forth, by its never-vacillating expressions, a generality of ideas, which continually excites to renewed experiments, and thus leads to a more profound knowledge of nature. Every theory of a class of natural phænomena founded upon facts, which will not admit of analytical investigation in the form of its exposition, is imperfect; and no reliance is to be placed upon a theory developed in ever so strict a form, which is not confirmed to a sufficient extent by observation. So long therefore as not even one portion of the effects of a natural force has been observed with the greatest accuracy in all its gradations, the calculation employed in its investigation only treads

on uncertain ground, as there is no touchstone for its hypotheses, and in fact it would be far better to wait a more fit time; but when it goes to work with the proper authority, it enriches, at least in an indirect manner, the field it occupies with new natural phænomena, as universal experience shows. I have thought it necessary to premise these general remarks, as they not only serve to throw more light on what follows, but also because they explain the reason why the galvanic phænomena have not long since been mathematically treated with greater success, although, as we shall subsequently find, the requisite course has been already earlier pursued in another, apparently less prepared, branch of Physics.

After these reflections we will now proceed to the establishment of the fundamental laws themselves.

3. When two electrical elements, E and E′, of equal magnitude, of like form and similarly placed with respect to each other, but unequally powerful, are situated at the proper distance from each other, they exhibit a mutual tendency to attain electric equilibrium, which is apparent in both constantly and uninterruptedly approaching nearer to the mean of their electric state, until they have actually attained it. That is to say, both elements reciprocally change their electric state so long as a difference continues to exist between their electroscopic forces; but this change ceases as soon as they have both attained the same electroscopic force. Consequently this change of the electric difference of the elements is so dependent that the one disappears at the same time with the other. We now suppose that the change, effected in an extremely short instant of time in both elements, is proportional to the difference of their cotemporaneous electroscopic force and the magnitude of the instant of time; and without yet attending to any material distinctions of the electricity, it is always to be understood that the forces designated by + and − are to be treated exactly as opposite magnitudes. That the change is effected accurately according to the difference of the forces, is a mathematical supposition, the most natural because it is the most simple; all the rest is given by experiment. The motion of electricity is effected in most bodies so rapidly that we are seldom able to determine its changes at the various places, and on that account we are not in a condition to discover by observation the law according to which they act. The galvanic phænomena, in which such

changes occur in a constant form, are therefore of the highest importance for testing this assumption: for if the conclusions drawn from the supposition are thoroughly confirmed by those phænomena, it is admissible, and may then be applied without any further consideration to all analogous researches, at least within the same limits of force.

We have assumed, in accordance with the observations hitherto made, that when by any two exteriorly like constituted elements, whether they be of the same or of different matter, a mutual change in their electrical state is produced, the one loses just so much force as the other gains. Should it hereafter be shown by experiments that bodies exhibit a relation similar to that which in the theory of heat is termed the capacity of bodies, the law we have established will have to undergo a slight alteration, which we shall point out in the proper place.

4. When the two elements E and E′ are not of equal magnitude, it is still allowed to regard them as sums of equal parts. Granting that an element E consist of m perfectly equal parts, and the other E′ of m' exactly similar parts, then, if we imagine the elements E and E′ exceedingly small in comparison with their mutual distance, so that the distances from each part of the one to each part of the other element are equal, the sum of the actions of all the m' parts of the element E′ on a part of E will be m' times that which a single part exerts, and the sum of all the actions of the element E′ on all the m parts of E will be $m\,m'$ times that which a part of E′ exerts on a part of E. It is hence evident, that in order to ascertain the mutual actions of dissimilar elements on each other, they must be taken as proportional not merely to the difference of their electroscopic forces and their duration, but also to the product of their relative magnitudes. We shall in future term the sum of the electroscopic actions, referred to the magnitude of the elements—by which therefore we have to understand the force multiplied by the magnitude of the space over which it is diffused, in the case where the same force prevails at all places in this space—the *quantity of electricity*, without intending to determine anything thereby with respect to the material nature of electricity. The same observation is applicable to all figurative expressions introduced, without which, perhaps for good reasons, our language could not exist.

In cases where the elements cannot be regarded as evanescent

in comparison with their relative distances, a function, to be determined separately for each given case from their dimensions and their mean distance, must be substituted for the product of the magnitudes of the two elements, and which we will designate where it is employed by F.

5. Hitherto we have taken no notice of the influence of the mutual distance of the elements between which an equalization of their electric state takes place, because as yet we have only considered such elements as always retained the same relative distance. But now the question arises, whether this exchange is directly effected only between adjacent elements, or if it extends to others more distant, and how on the one or the other supposition is its magnitude modified by the distance? Following the example of Laplace, it is customary in cases where molecular actions at the least distance come into play, to employ a particular mode of representation, according to which a direct mutual action between two elements separated by others, still occurs at finite distances, which action, however, decreases so rapidly, that even at any perceptible distance, be it ever so minute, it has to be considered as perfectly evanescent. Laplace was led to this hypothesis, because the supposition that the direct action only extended to the next element produced equations, the individual members of which were not of the same dimension relatively to the differentials of the variable quantities*,—a non-uniformity which is opposed to the spirit of the differential calculus. This apparent unavoidable

* Poisson, in his Mémoire sur la Distribution de la Chaleur, *Journ. de l'Ecole Polytechn.* cah. xix. expresses himself on this subject thus:—

"If a bar be divided, by sections perpendicular to the axis, into an infinite number of infinitely small elements, and if we consider the mutual action of three consecutive elements, that is to say, the quantity of heat that the intermediate element at each instant communicates to and abstracts from the two others, in proportion to the positive or negative excess of its temperature over that of each of them, we may thence easily determine the augmentation of temperature of this element during an infinitely small instant; assuming therefore this quantity equal to the differential of its temperature taken with respect to the time, the equation of the propagation of heat according to the length of the bar is formed; but on examining the question more attentively, it is seen without difficulty that this equation would be founded on the comparison of two infinitely small non-homogeneous quantities, or of different orders, which would be contrary to the first principles of the differential calculus. This difficulty can only be made to disappear by supposing, as M. Laplace first remarked, (Mémoires de la 1re classe de l'Institut, année 1809,) that the action of each element of the bar extends itself beyond the contact, and that it exerts itself on all the elements contained within a finite space, as small as we please."

disproportion between the members of a differential equation, belonging nevertheless necessarily to one another, is too remarkable not to attract the attention of those to whom such inquiries are of any value; an attempt therefore to add something to the explanation of this ænigma will be the more proper in this place, as we gain the advantage of rendering thereby the subsequent considerations more simple and concise. We shall merely take as an instance the propagation of electricity, and it will not be difficult to transfer the obtained results to any other similar subject, as we shall subsequently have occasion to demonstrate in another example.

6. Above all, it is requisite that the term goodness of conduction be accurately defined. But we express the energy of conduction between two places by a magnitude which under otherwise similar circumstances, is proportional to the quantity carried over in a certain time from one place to the other multiplied by the distance of the two places from each other. If the two places are extended, then we have to understand by their distance the straight line connecting the centres of the dimensions of the two places. If we transfer this idea to two electric elements, E and E′, and call s the mutual distance of their centres, ϱ the quantity of electricity, which under accurately determined and invariable circumstances is carried over from one element to the other, and $\varkappa$ the conductibility between them,

$$\varkappa = \varrho \, . \, s.$$

We will now endeavour to determine more precisely the quantity of electricity denoted by ϱ. According to § 4 the quantity of electricity, which is transferred in an exceedingly short time from one element to the other, is, the distance being invariable, in general proportional to the difference between the electroscopic forces, the duration, and the size of each of the two elements. If therefore we designate the electroscopic forces of the two elements E and E′ by u and u', and the space they occupy by m and m', we obtain for the quantity of electricity carried over from E′ to E in the element of time $d\,t$ the following expression:

$$\alpha \, m \, m' \, (u' - u) \, d\,t,$$

where α represents a coefficient depending in some way on the distance s. This quantity changes every moment if $u' - u$ is variable; but if we suppose that the forces u' and u remain constant at all times, it merely depends on the magnitude of the instant of time $d\,t$, we can consequently extend it to the unity of time; if we place the present constant difference of the forces $u' - u$ equal to the unity of force, it then becomes

$$\alpha \, m \, m'.$$

This quantity of electricity is for the two elements E and E′ whose position is invariable, constant under the same circumstances, on which account it may be employed in the determination of the power of conduction just mentioned. For if we understand by ϱ the quantity of electricity transferred from E′ to E in the unity of time, with a constant difference of the electroscopic forces equal to the unity of force, we have

$$\varrho = \alpha \, m \, m',$$

and then

$$\varkappa = \alpha \, m \, m' \, s.$$

If we take from this last equation the value of $\alpha \, m \, m'$ and substitute it in the expression

$$\alpha \, m \, m' \, (u' - u) \, d\,t,$$

we obtain for the variable quantity of electricity which passes over in the instant of time $d\,t$ from E′ to E, the following:

$$\frac{\varkappa \, (u' - u) \, d\,t}{s}, \qquad (\delta)$$

which expression is not accompanied by the above-mentioned disproportion between the members of the differential equation, as will soon be perceived.

7. The course hitherto pursued was based upon the supposition that the action exerted by one element on the other is proportional to the product of the space occupied by the two elements, an assumption which, as was already observed in § 4, can no longer be allowed in cases where it is a question of the mutual action of elements situated indefinitely near each other, because it either establishes a relation between the magnitudes of the elements and their mutual distances, or prescribes to these elements a certain form. The previously found expression (δ) for the variable quantity of electricity passing from one element to the other, possesses therefore no slight advantage in being entirely independent of this supposition; for whatever may have to be placed in any determinate case instead of the product $m \, m'$, the expression (δ) constantly remains the

same, this peculiarity being solely referrible to the power of conduction $\varkappa$. If, for instance, F designate, as was stated in § 4, the function, corresponding to such a case, of the dimensions and of the mean distance of both elements, the expression

$$\alpha\ m\ m'\ (u' - u)\ d\ t$$

not merely changes apparently into

$$\mathrm{F}\ (u' - u)\ d\ t,$$

but also the equation

$$\varkappa = \alpha\ m\ m'\ s$$

into the other,

$$\varkappa = \mathrm{F} \,.\, s, \qquad (\odot)$$

so that if we take the value of F from this equation and place it in the above expression, we always obtain

$$\frac{\varkappa\ (u' - u)\ d\ t}{s}.$$

Moreover, the circumstance of the expression (♂) still remaining valid for corpuscles, whose dimensions are no longer indefinitely small, is of some importance when the same electroscopic force only exists merely at all points of each such part. It is hence evident how intimately our considerations are allied to the spirit of the differential calculus; for uniformity in all points with reference to the property which enters into the calculation is precisely the distinctive characteristic required by the differential calculus from that which it is to receive as an element.

If we institute a more profound comparison between the process originating with Laplace and that here advanced, we shall arrive at some interesting points of comparison. If for instance we consider that for infinitely small masses at infinitely short distances all particular relations must necessarily have the same weight as for finite masses at finite distances, it is not directly evident how the method of the immortal Laplace—to whom we are indebted for so many valuable explanations respecting the nature of molecular actions,—according to which the elements must be constantly treated as if they were placed at finite distances from each other, could nevertheless still afford correct results; but we shall find on closer examination that it acts in fact otherwise than it expresses. Indeed, since Laplace, when determining the changes of an element by all surrounding it, makes the higher powers of the distance disappear compared with the lower, he therewith assumes, quite in the spirit of the differential calculus, the difference of action itself to be infinitely small, but terms it finite, and treats it also as such; whence it is immediately apparent that he in fact treats that which is infinitely small at an infinitely short distance as finite. Disregarding however the great certainty and distinctness which accompany our manner of representation, there might still be something more to say, and perhaps with some justice, against Laplace's mode of treatment in favour of ours, in this respect, that the former takes not the least account of the possible nature of the *given* elements of bodies, but merely has to do with *imaginary* elements of space, by which the physical nature of the bodies is almost entirely lost sight of. We may, to render our assertion intelligible by an example, undoubtedly imagine bodies in nature which consist only of homogeneous elements, but whose position to each other, taken in one direction, might be different than when in another direction; such bodies, as our mode of representation immediately shows, might conduct the electricity in one direction in a different manner than in another, notwithstanding that they might appear uniform and equally dense. In such a case, did it occur, we should have to take refuge, according to Laplace, in considerations which have remained entirely foreign to the general process. On the other hand, the mode in which bodies conduct affords us the means by which we are enabled to judge of their internal structure, which, from our almost total ignorance on the subject, cannot be immediately shown. Lastly, we may add, that this, our hitherto developed view of molecular actions, unites in itself the two advanced by Laplace and by Fourier in his theory of heat, and reconciles them with each other.

8. We need now no longer hesitate about allowing the electrical action of an element not to extend beyond the adjacent surrounding elements, so that the action entirely disappears at every finite distance, however small. The extremely limited circle of action with the almost infinite velocity with which electricity passes through many bodies might indeed appear suspicious; but we did not overlook on its admission, that our comparison in such cases is only effected by an imaginary relative standard, which is deceitful, and does therefore not justify us to vary a law so simple and independent until the conclusions drawn from it are in contradiction to nature, which in our subject, however, does not seem to be the case.

The sphere of action thus fixed by us, although it is infinitely small, precisely the same circumference as that introduced by Laplace, and called finite, where he lets the higher powers of the distance vanish compared with the lower, the reason of which may be found already in what has been stated above. The supposition of a finite distance of action in our sense would correspond to the case where Laplace still retains higher powers of distance together with the lower.

9. The bodies on which we observe electric phænomena are in most cases surrounded by the atmosphere; it is therefore requisite, in order to investigate profoundly the entire process, not to disregard the changes which may be produced by the adjacent air. According to the experiments left us by Coulomb on the diffusion of electricity in the surrounding atmosphere, the loss in force thus occasioned is (during a very short constant time), at least when the intensities are not very considerable, on the one hand proportional to the energy of the electricity, and on the other is dependent on a coefficient varying according to the cotemporaneous nature of the air, but otherwise invariable for the same air. The knowledge of this enables us to bring the influence of the atmosphere on galvanic phænomena into calculation wherever it might be requisite. It must however not be overlooked here, that Coulomb's experiments were made on electricity which had entered into equilibrium and was no longer in the process of excitation, with respect to which both observation and calculation have convinced us that it is confined to the surface of bodies, or merely penetrates to a very slight depth into their interior; for from thence may be drawn the conclusion, of some importance with respect to our subject, that all the electricity present in those experiments may have been directly concerned in the transference to the atmosphere. If we now connect with this observation the law just announced, according to which two elements, situated at any finite distance from each other, no longer exert any direct action on each other, we are justified in concluding, that where the electricity is uniformly diffused throughout the entire mass of a finite body, or at least so that proportionately but a small quantity resides in the vicinity of the surface, which case does not in general occur when it has entered into motion, the loss which is occasioned by the circumambient air can be but extremely small in comparison to that which takes place when the entire force is situated immediately at the surface, which invariably happens when it has entered into equilibrium; and thence, therefore, it happens that the atmosphere exerts no perceptible influence on galvanic phænomena in the closed circuit when this is composed of good conductors, so that the changes produced by the presence of the atmosphere in phænomena of contact-electricity may be neglected in such cases. This conclusion, moreover, receives new support from the circumstance, that in the same cases the contact-electricity only remains during an exceedingly short time in the conductors, and even on that account would only give up a very slight portion to the air, even if it were in immediate contact with it.

Although, from what has been stated, it is placed beyond all doubt that the action of the atmosphere has no perceptible influence on the magnitude of effect of the usual galvanic circuits, it by no means is intended to admit the reverse of the conclusion, viz. that the galvanic conductor exerts no perceptible influence on the electric state of the atmosphere; for mathematical investigation teaches us that the electroscopic action of a body on another has no direct connexion with the quantity of electricity which is carried over from the one to the other.

10. We arrive at last at that position founded on experiment, and which is of the highest importance for the whole of natural philosophy, since it forms the basis of all the phænomena to which we apply the name of galvanic: it may be expressed thus: Different bodies, which touch each other, constantly preserve at the place of contact the same difference between their electroscopic forces by virtue of a contrariety proceeding from their nature, which we are accustomed to designate by the expression *electric tension*, or *difference of bodies*. Thus enounced, the position stands, without losing any of its simplicity, in all the generality which belongs to it; for we are nearly always referred to it by every single phænomenon. Moreover, the above expression is adopted in all its generality, either expressly or tacitly, by all philosophers in the explanation of the electroscopic phænomena of the voltaic pile. According to our previously developed ideas respecting the mode in which elements act on one another, we must seek for the source of this phænomenon in the elements directly in contact, and consequently we must allow the abrupt transition to take place from one body to the other in an infinitely small extent of space.

11. This being established, we will now proceed to the subject, and in the first place consider the motion of the electricity in a homogeneous cylindric or prismatic body, in which all points throughout the whole extent of each section, perpendicular to its axis, possess contemporaneously equal electroscopic force, so that the motion of the electricity can only take place in the direction of its axis. If we imagine this body divided by a number of such sections into disks of infinitely small thickness, and so that in the whole circumference of each disk the electroscopic force does not vary sensibly for each pair of such disks, the expression ♂ given in § 6 can be applied to determine the quantity of electricity passing from one disk to the other; but by the limitation of the distance of action to only infinitely small distances mentioned in the preceding paragraph, its nature is so modified that it disappea - as soon as the divisor ceases to be infinitely small.

If we now choose one of the infinite number of sections invariably for the origin of the abscissæ, and imagine anywhere a second, whose distance from the first we denote by x, then $d\,x$ represents the thickness of the disk there situated, which we will designate by M. If we conceive this thickness of the disk to be of like magnitude at all places, and term u the electroscopic force present at the time t in the disk M, whose abscissa is x, so that therefore u in general will be a function of t and x; if we further suppose u' and $u_{\prime}$ to be the values of u when $x + d\,x$ and $x - d\,x$ are substituted respectively for x, then u' and $u_{\prime}$ evidently express the electroscopic forces of the disks situated next the two sides of the disk M, of which we will denote the one belonging to the abscissa $x + d\,x$ by M′, and that belonging to the abscissa $x + d\,x$ by $M_{\prime}$; and it is clearly evident that the distance of the centre of each of the disks M′ and $M_{\prime}$ from the centre of the disk M is $d\,x$. Consequently, by virtue of the expression (♂) given in § 6, if $\varkappa$ represents the conducting power of the disk M′ to M,

$$\frac{\varkappa\,(u' - u)\,d\,t}{d\,x}$$

is the quantity of electricity which is transferred during the interval of time $d\,t$ from the disk M′ to the disk M, or from the latter to the former, according as $u' - u$ is positive or negative. In the same manner, when we admit the same power of conduction between $M_{\prime}$ and M,

$$\frac{\varkappa\,(u_{\prime} - u)\,d\,t}{d\,x}$$

is the quantity of electricity passing over from $M_{\prime}$ to M when the expression is positive, and from M to $M_{\prime}$ when it is negative. The total change of the quantity of electricity which the disk M undergoes from the motion of the electricity in the interior of the body in the particle of time $d\,t$, is consequently

$$\frac{\varkappa\,(u' + u_{\prime} - 2\,u)\,d\,t}{d\,x},$$

and an increase in the quantity of electricity is denoted when this value is positive, and when negative a diminution of the same.

But according to Taylor's theorem

$$u' = u + \frac{d\,u}{d\,x} \cdot d\,x + \frac{d^2 u}{d\,x^2} \cdot \frac{d\,x^2}{2} + \ldots\ldots,$$

and in the same way

$$u_{\prime} = u - \frac{d\,u}{d\,x} \cdot d\,x + \frac{d^2 u}{d\,x^2} \cdot \frac{d\,x^2}{2} - \ldots\ldots;$$

consequently

$$u' + u_{\prime} = 2\,u + \frac{d^2 u}{d\,x^2}\,d\,x^2.$$

According to this the expression just found for the total change of the quantity of electricity present in the disk M is converted during the time $d\,t$ into

$$\varkappa \cdot \frac{d^2 u}{d\,x^2}\,d\,x\;d\,t,$$

where $\varkappa$ represents the power of conduction which prevails from one disk to the adjacent one, which we suppose to be invariable throughout the length of the homogeneous body. It must here be observed, that this value $\varkappa$ is, on account of the infinitely small distance of action, proportional to the section of the cylindric or prismatic body; if therefore we denote the magnitude of this section by ω, and separate this factor from the value $\varkappa$, always calling the remaining portion $\varkappa$, the former expression changes into the following:

$$\varkappa\,\omega\,\frac{d^2 u}{d\,x^2}\,d\,x\;d\,t,$$

in which $\varkappa$ now represents the conductibility of the body independent of the magnitude of the section, which we will term the *absolute* conductibility of the body in opposition to the former, which may be called the *relative*. Henceforward wherever the word conductibility occurs without any closer definition, the absolute conductibility is always to be understood.

Hitherto we have left out of consideration the change which the disk suffers from the adjacent atmosphere. This influence may easily be determined. If, for instance, we designate by c the circumference of the disk belonging to the abscissa x, then $c\,d\,x$ is the portion of its surface which is exposed to the air; consequently, according to the experiments of Coulomb, mentioned in § 9,

$$b\,c\,u\,d\,x\,d\,t$$

is the change of the quantity of electricity which is occasioned in the disk M by the passing off of the electricity into the atmosphere during the moment of time $d\,t$, where b represents a coefficient dependent on the cotemporaneous nature of the atmosphere, but constant for the same atmosphere. It expresses a decrease when u is positive, and an increase when u is negative. But in accordance with our original supposition, this action cannot occasion an inequality of the electroscopic force in the same section of the body; or at least, this inequality must be so slight that no perceptible alteration is produced in the other quantities; a circumstance which may nearly always be supposed in the galvanic circuit.

Accordingly, the entire change which the quantity of electricity in the disk M undergoes in the moment of time $d\,t$ is

$$\varkappa\,\omega\,\frac{d^2u}{dx^2}\,d\,x\,.\,dt - b\,c\,u\,d\,x\,d\,t,$$

in which the portion is comprised which arises from the motion of the electricity in the interior of the body as well as that which is caused by the circumambient atmosphere.

But the entire change of the electroscopic force u in the disk M effected in the moment of time dt is

$$\frac{du}{dt}\,dt,$$

consequently the total change in the quantity of electricity in the disk M during the time dt is

$$\omega\,\frac{du}{dt}\,dx\;dt,$$

where, however, it is supposed that under all circumstances similar changes in the electroscopic force correspond to similar changes in the quantity of electricity. If observation showed that different bodies of the same surface underwent a different change in their electroscopic force by the same quantity of electricity, then there would still remain to be added a coefficient γ corresponding to this property of the various bodies. Experience has not yet decided respecting this supposition borrowed from the relation of heat to bodies.

If we assume the two expressions just found for the entire change in the quantity of electricity in the disk M during the moment of time $d\,t$ to be equal, and divide all the members of the equation by $\omega\;dx\;dt$, we obtain

$$\gamma\,\frac{d\,u}{d\,t} = \varkappa\,\frac{d^2\,u}{d\,x^2} - \frac{b\,c}{\omega}\,u, \qquad (a)$$

from which the electroscopic force u has to be determined as a function of x and t.

12. We have in the preceding paragraph found for the change in the quantity of electricity occurring between the disks M′ and M during the time $d\,t$

$$\frac{\varkappa\,(u' - u)\;d\,t}{d\,x},$$

and have seen that the direction of the passage is opposed to the course of the abscissæ when the expression is positive; on the contrary, it proceeds in the direction of the abscissæ when it is negative. In the same way the magnitude of the transition between the disks $M_{,}$ and M, when we retain the same relation to its direction, is

$$\frac{\varkappa\,(u_{,} - u)\;d\,t}{d\,x}.$$

If we substitute in these two expressions for $u_{,}$ and u' the transformations given in the same paragraph, and at the same time $\varkappa\,\omega$ for $\varkappa$, *i. e.* the absolute power of conduction for the relative, we obtain in both cases

$$\varkappa\,\omega\,\frac{d\,u}{d\,x}\;d\,t,$$

whence it results that the same quantity of electricity which enters from the one side into the disk M during the element of time $d\,t$, is again in the same time expelled from it towards the other side. If we imagine this transmission of the electricity, occurring at the time t in the disk belonging to the abscissa x, of invariable energy reduced to the unity of time, call it the *electric current*, and designate the magnitude of this current by S, then

$$S = \varkappa\,\omega\,\frac{du}{dx}; \qquad (b)$$

and in this equation positive values for S show that the current takes place opposed to the direction of the abscissæ; negative, that it occurs in the direction of the abscissæ.

13. In the two preceding paragraphs we have constantly had in view a homogeneous prismatic body, and have inquired into the diffusion of the electricity in such a body, on the supposition that throughout the whole extent of each section, perpendicular to its length or axis, the same electroscopic force exists at any time whatsoever. We will now take into consideration the case where two prismatic bodies A and B, of the same kind, but formed of different substances, are adjacent, and touch each other in a common surface. If we establish for both A and B the same origin of abscissæ, and designate the electroscopic force of A by u, that of B by u', then both u and u' are determined by the equation (a) in paragraph 11, if $\varkappa$ only retain the value each time corresponding to the peculiar substance of each body: but u represents a function of t and x, which holds only so long as the abscissa x corresponds to points in the body A; u on the other hand denotes a function of t and x, which holds only when the abscissa x corresponds to the body B. But there are still some other conditions at this common surface, which we will now explain. If we denote for this purpose the separate values of u and u', which they first assume at the common surface, by enclosing the general ones between crotchets, we find according to the law advanced in § 10 the following equation between these separate values:

$$(u) - (u') = a,$$

where a represents a constant magnitude otherwise dependent on the nature of the two bodies. Besides this condition, which relates to the electroscopic force, there is still a second, which has reference to the electric current. It consists in this, that the electric current in the common surface must in the first place possess equal magnitude and like direction in both bodies, or, if we retain the common factor ω,

$$\varkappa\,\omega\left(\frac{d\,u}{d\,x}\right) = \varkappa'\,\omega\left(\frac{d\,u'}{d\,x}\right),$$

where $\varkappa$ represents the actual power of conduction of the body A, $\varkappa'$ that of the body B, and $\left(\frac{d\,u}{d\,x}\right)$, $\left(\frac{d\,u'}{d\,x}\right)$ the particular values of $\frac{d\,u}{d\,x}$, $\frac{d\,u'}{d\,x}$ immediately belonging to them at the common surface, and in which it was assumed that the origin of the abscissæ was not taken on this common surface. The necessity of this last equation may easily be conceived; for were it otherwise, the two currents would not be of equal energy in the common surface, but there would be more conveyed from the one body to this surface than would be abstracted from it by the other; and if this difference were a finite portion of the entire current, the electroscopic force would increase at that very place, and indeed, considering the surprising fertility of the electric current, would arrive in the shortest time to an exceedingly high degree, as observation has long since demonstrated. Nor can a smaller quantity of electricity be imparted from the one body to the common surface than it is deprived of by the other, as this circumstance would be evinced by an infinitely high degree of negative electricity.

It is not absolutely requisite for the validity of the preceding determinations, that the two bodies in contact have the same base. The section in the one prismatic body may be different in size and form to that in the other, if this does not render the electroscopic force sensibly different at the various points of the same section, which, considering the great energy with which the electricity tends to equilibrium, will not be the case when the bodies are good conductors, whose length far surpasses their other dimensions. In this case everything remains as before, only that the section of the body B must everywhere be distinguished from that of A; consequently the second conditional equation for the place where the two bodies are in contact changes into the following:—

$$\varkappa\,\omega\left(\frac{d\,u}{d\,x}\right) = \varkappa'\,\omega'\left(\frac{d\,u'}{d\,x}\right),$$

where ω still represents the section of A, but ω' that of the body B, which at present differs from the former.

There may even exist in the prolongation of the body A two prismatic bodies, B and C, separated from each other, which are both situated immediately on the one surface of A. If in this case $\varkappa'\,\omega'\,u'$ signifies for the body B, and $\varkappa''\,\omega''$, u'' for the body

C what $\varkappa \omega u$ does for A, we obtain instead of the one conditional equation the two following:—

$$(u) - (u') = a,$$
$$(u) - (u'') = a',$$

where a represents the electric tension between the bodies A and B, and a' that between A and C. In the same manner we now obtain instead of the second conditional equation the following:—

$$\varkappa \omega \left(\frac{d u}{d x}\right) = \varkappa' \omega' \left(\frac{d u'}{d x}\right) + \varkappa'' \omega'' \left(\frac{d u''}{d x}\right).$$

It is immediately apparent how these equations must change when a greater number of bodies are combined. We shall not enter further into these complications, as what has been stated suffices to throw sufficient light upon the changes which have in such a case to be performed on the equations.

14. To avoid misconception, I will, at the close of these general observations, once more accurately define the circle of application within which our formulæ have universal validity. Our whole inquiry is confined to the case where all the parts of the same section possess equal electroscopic force, and the magnitude of the section varies only from one body to the other. The nature of the subject, however, frequently gives rise to circumstances which render one or the other of these conditions superfluous, or at least diminishes their importance. Since the knowledge of such circumstances is not without use, I will here illustrate the most prominent by an example.

A circuit of copper, zinc, and an aqueous fluid, will wholly come under the above formula when the copper and zinc are prismatic and of equal section; when, further, the fluid is likewise prismatic and of the same or of smaller section, and its terminal surfaces everywhere in contact with the metals. Nay, when only these last conditions are fulfilled with respect to the fluid, the metals may possess equal sections or not, and touch one another with their full sections, or only at some points, and even their form may deviate considerably from the prismatic form, and nevertheless the circuit must constantly obey the laws deduced from our formulæ; for the motion of the electricity produced with such ease in the metals, is obstructed to such a considerable extent by the non-conductive nature of the fluid, that it gains sufficient time to diffuse itself thoroughly with equal energy over the metals, and thus re-establishes in the fluid the conditions upon which our calculation is founded. But it is a very different matter when the prismatic fluid is only touched in disproportionately small portions of its surfaces by the metals, as the electricity arriving there can only advance slowly and with considerable loss of energy to the untouched surfaces of the fluid, whence currents of various kinds and directions result. The existence of such currents has been sufficiently demonstrated by Pohl's manifoldly varied experiments, and nothing more now stands in the way of their determination by analysis, after the additions which it has received from the successful investigations respecting the theory of heat, than the complexities of the expressions. Since their determination exceeds the limits of this small work, which has for its object to investigate the current only in one dimension, we will defer them to a more fit occasion.

We will now proceed to the application of the formulæ advanced, and divide, for the sake of a more easy and general survey, the whole into two sections, of which the one will treat of the electroscopic phænomena, and the other of the phænomena of the electric current.

B. *Electroscopic Phænomena.*

15. In our preceding general determinations we have constantly confined our attention to prismatic bodies, whose axes, upon which the abscissæ have been taken, formed a straight line. But all these considerations still retain their entire value, if we imagine the conductor constantly curved in any way whatsoever, and take the abscissæ on the present curved axis of the conductor. The above formulæ acquire their entire applicability from this observation, since galvanic circuits, from their very nature, can but seldom be extended in a straight line. Having anticipated this point, we will immediately proceed to the most simple case, where the prismatic conductor is formed in its entire length of the same material, and is curved backwards on itself, and conceive the seat of the electric tension to be where its two ends touch. Although no case in nature resembles this imaginary one, it will nevertheless be of great service in the treatment of the other cases which do really occur in nature.

The electroscopic force, at any place of such a prismatic body, may be deduced from the differential equation (a) found in § 11. For this purpose we have only to integrate it, and to determine, in accordance with the other conditions of the problem, the arbitrary functions or constants entering into the integral. This matter is, however, generally very much facilitated, with respect to our subject, by omitting one or even two members, according to the nature of the subject, from the equation (a). Thus nearly all galvanic actions are such that the phænomena are permanent and invariable immediately at their origin. In this case, therefore, the electroscopic force is independent of time, consequently the equation (a) passes into

$$o = \varkappa \frac{d^2 u}{d x^2} - \frac{b c}{\omega} u.$$

Moreover, the surrounding atmosphere has (as we have already noticed in § 9.) in most cases no influence on the electric nature of the galvanic circuit; then $b = o$, by which the last equation is converted into

$$o = \frac{d^2 u}{d x^2}.$$

But the integral of this last equation is

$$u = f x + c, \qquad (c)$$

where f and c represent any constants remaining to be determined. The equation (c) consequently expresses the law of electrical diffusion, in a homogeneous prismatic conductor, in all cases where the abduction by the air is insensible, and the action no longer varies with time. As these circumstances in reality most frequently accompany the galvanic circuit, we shall on that account dwell longest upon them.

We are enabled to determine one of the constants by the tension occurring at the extremities of the conductor, which has to be regarded as invariable and given in each case. If, for instance, we imagine the origin of the abscissæ anywhere in the axis of the body, and designate the abscissa belonging to one of its ends by x_1, then the electroscopic force there situated is, according to the equation (c),

$$f x_1 + c;$$

in the same way we obtain for the electroscopic force of the other extremity, when we represent its abscissa by x_2,

$$f x_2 + c.$$

If we now call the given tension or difference of the electroscopic force a, we have

$$a = \pm f (x_1 - x_2).$$

But $x_1 - x_2$ evidently represents the entire, positive or negative, length of the prismatic conductor; if we designate this by l, we obtain accordingly

$$a = \pm f l,$$

whence the constant f may be determined. If we now introduce the value of the constant thus found into the equation (c), it is converted into

$$u = \pm \frac{a}{l} x + c,$$

so that only the constant c remains to be determined. We may consider the ambiguity of the sign $\pm$ to be owing to the tension a, by ascribing to it a positive value when the extremity of the conductor, belonging to the greater abscissa, possesses the greatest electroscopic force, and when the contrary a negative. Under this supposition is then generally

$$u = \frac{a}{l} x + c. \qquad (d)$$

The constant c remains in general wholly undetermined, which admits of our allowing the diffusion of the electricity in the conductor to vary arbitrarily, by external influences, in such manner that it occupies the entire conductor everywhere uniformly.

Among the various considerations respecting this constant, there is one of especial importance to the galvanic circuit, I mean that which supposes the circuit to be connected at some one place with a perfect conductor, so that the electroscopic force has to be regarded as constantly destroyed at this place. If we call the abscissa belonging to this place λ, then according to the equation (d)

$$o = \frac{a}{l} \lambda + c.$$

By determining from this the constant c, and placing its value in the same equation (d), we obtain

$$u = \frac{a}{l} (x - \lambda),$$

from which the electroscopic force of a galvanic circuit of the length l, and of the tension a, which is touched at any given place whose abscissa is λ, may be found for every other place.

If any constant and perfect adduction, from outwards to the galvanic circuit, were to be given instead of the permanent abduction outwards, so that the electroscopic force pertaining to the abscissa λ were compelled to assume constantly a given energy, which we will designate by α, we should obtain for the determination of the constant c the equation

$$\alpha = \frac{a}{l}\lambda + c,$$

and for the determination of the electroscopic force of the circuit at any other place the following:

$$u = \frac{a}{l}(x - \lambda) + \alpha.$$

We have seen how the constant c may be dete[illegible]ined when the electroscopic force is indicated at any place of th[illegible] circuit by external circumstances; but now the question arises, what value are we to ascribe to the constant when the circuit is left entirely to itself, and this value can consequently no longer be deduced from outward circumstances? The answer to this question is found in the consideration, that each time both electricities proceed contemporaneously, and in like quantity from a previously indifferent state. It may, therefore, be asserted, that a simple circuit of the present kind, which is formed in a perfectly neutral and isolated condition, would assume on each side of the place of contact an equal but opposite electric condition, whence it is self-evident that their centre would be indifferent. For the same reason, however, it is also apparent that when the circuit at the moment of its origin is compelled by any circumstance to deviate from this, its normal state, it would certainly assume the abnormal one until again caused to change.

The properties of a simple galvanic circuit, such as we have hitherto considered them to be, accordingly consist essentially in the following, as is directly evident from the equation (d):

a. The electroscopic force of such a circuit varies throughout the whole length of the conductor continually, and on like extents constantly to the same amount; but where the two extremities are in contact, it changes suddenly, and, indeed, from one extremity to the other, to the extent of an entire tension.

b. When any place of the circuit is disposed by any circumstance to change its electric state, all the other places of the circuit change theirs at the same time, and to the same amount.

16. We will now imagine a galvanic circuit, composed of two parts, P and P$'$, at whose two points of contact a different electric tension occurs, which case comprises in it the thermal circuit. If we call u the electroscopic force of the part P, and u' that of the part P$'$, then, according to the preceding paragraph, as here, the case there noticed is repeated twice, in consequence of the equation (c),

$$u = fx + c$$

for the part P, and

$$u' = f'x + c'$$

for the part P$'$, where f, c, f', c' are any constant magnitudes to be deduced from the peculiar circumstances of our problem, and each equation is only valid so long as the abscissæ refer to that part to which the equations belong. If we now place the origin of the abscissæ at one of the places of contact of the part P, and suppose the direction of the abscissæ in this part to proceed inwards; moreover, designate by l the length of the part P, and by l' that of P$'$; and, lastly, represent by u'_2 and u_1 the values of u and u' at the place of contact where $x = o$, and by u_2 and u'_1 the values of u and u' at the place of contact where $x = l$, we then obtain

$$u'_2 = f'\ (l + l') + c' \qquad u_1 = c$$
$$u_2 = fl + c \qquad u'_1 = f'l + c'.$$

If we now designate by a the tension which occurs at the place of contact where $x = o$, and by a' that which occurs at the place of contact where $x = l$; and if we once for all assume, for the sake of uniformity, that the tension at each individual place of contact always expresses the value which is obtained when we deduct the electroscopic force of one extremity from the electroscopic force of that extremity belonging to the place in question, upon which the abscissa falls before the abrupt change takes place—(it is not difficult to perceive that this general rule contains that advanced in the preceding paragraph, and which, in fact, expresses nothing more than that the tensions of such places of contact, by the springing over of which, in the direction of the abscissæ, we arrive from the greater to the smaller electroscopic force, are to be regarded as positive, in the contrary case as negative, where, however, it must not be overlooked that every positive force has to be taken as greater than every negative, and the negative as greater than the actually smaller), we obtain

$$a = f' \, (l + l) + c' - c,$$

and

$$a' = f\,l - f'\,l + c - c',$$

whence directly results

$$a + a' = f\,l + f'\,l'.$$

But now at each of the places of contact when $\varkappa$ and ω represent the power of conduction and the section of the part P, and $\varkappa'$ and ω' the same for P$'$, in accordance with the considerations developed in § 13, there arises the conditional equation

$$\varkappa\,\omega \left(\frac{d\,u}{d\,x}\right) = \varkappa'\omega' . \left(\frac{d\,u'}{d\,x}\right),$$

where $\left(\frac{d\,u}{d\,x}\right)$ and $\left(\frac{d\,u'}{d\,x}\right)$ represent the values of $\frac{d\,u}{d\,x}$ and $\frac{d\,u'}{d\,x}$ at the place of contact. From the equations at the commencement of this paragraph for the determination of the electroscopic force in each single part of the circuit, we, however, obtain the value of x to be allowed to each,

$$\frac{d\,u}{d\,x} = f \text{ and } \frac{d\,u'}{d\,x} = f',$$

which converts the conditional equation in question into

$$\varkappa\,\omega f = \varkappa'\,\omega' f'.$$

From this, and the equation $a + a' = f\,l + f'\,l'$ just deduced from the tensions, we now find the values of f and f' thus:

$$f = \frac{(a+a')\,\varkappa'\,\omega'}{\varkappa'\,\omega'\,l + \varkappa\,\omega\,l'},$$

$$f' = \frac{(a+a')\,\varkappa\,\omega}{\varkappa'\,\omega'\,l + \varkappa\,\omega\,l'},$$

and with the help of these values we find

$$c' = c - a' + \frac{(a+a')\,(\varkappa'\,\omega'\,l - \varkappa\,\omega\,l)}{\varkappa'\,\omega'\,l + \varkappa\,\omega\,l'}.$$

Hence the electroscopic force of the circuit in the part P is expressed by the equation

$$u = \frac{(a+a')\,\varkappa'\,\omega'\,x}{\varkappa'\,\omega'\,l + \varkappa\,\omega\,l'} + c,$$

and that in the part P$'$ by the equation

$$u' = \frac{(a+a')\,\varkappa\,\omega\,x - \varkappa\,\omega\,l + \varkappa'\,\omega'\,l}{\varkappa'\,\omega'\,l + \varkappa\,\omega\,l'} - a + c.$$

If we substitute λ and λ' for $\frac{l}{\varkappa\,\omega}$ and $\frac{l'}{\varkappa'\,\omega'}$, the following more simple form may be given to these equations:—

$$\left.\begin{aligned} u &= \frac{a+a'}{\lambda+\lambda'} \cdot \frac{x}{\varkappa\,\omega} + c \\ u_{\prime} &= \frac{a+a'}{\lambda+\lambda'}\left(\frac{x-l}{\varkappa'\,\omega'} + \frac{l}{\varkappa\,\omega}\right) - a' + c \end{aligned}\right\} \text{(L)}.$$

From the form of these equations it will be immediately perceived, that when the conductibility, or the magnitude of the section, is the same in both parts, the expressions for u and u' undergo no other change than that the letter representing the conductibility or the section entirely disappears.

17. We will now proceed to the consideration of a galvanic circuit, composed of three distinct parts P, P$'$, and P$''$, which case comprises the hydro-circuit.

If we represent by u, u', u'' respectively the electroscopic forces of the parts P, P$'$, and P$''$, then, according to § 15, the case there mentioned being here thrice repeated, we have, in accordance with the equation (c) there found, with respect to the part P,

$$u = f\,x + c,$$

with respect to the part P$'$,

$$u' = f'\,x + c',$$

and with respect to the part P$''$,

$$u'' = f''\,x + c'',$$

where f, f', f'', c, c', c'' may represent any constant magnitudes remaining to be determined from the nature of the problem, and each equation has only so long any meaning as the abscissæ refer to that part to which the equations appertain. If we suppose the origin of the abscissæ at that extremity of the part P, which is connected with the part P$''$, and choose the direction of the abscissæ so that they lead from the part P to that of P$'$, and from thence into P$''$; if we further respectively designate by l, l', and l'' the lengths of the parts P, P$'$, P$''$; and lastly, let u''_2 and u_1 represent the values of u'' and u at the place of contact where $x = o$, and u_2 and u' the values of u and u' at the place of contact where $x = l$, and u'_2 and u''_1 the values of u' and u'' at the place of contact where $x = l + l'$, then we obtain

$$\begin{array}{ll} u''_2 = f''(l + l' + c'') + c'' & u_1 = c \\ u_2 = f\,l + c & u'_1 = f'\,l + c' \\ u'_2 = f'(l + l') + c' & u''_1 = f''(l + l') + c''. \end{array}$$

If we call a the tension which occurs at the place of contact where $x = o$, a' that at the place of contact where $x = l$, and a'' that at the place of contact where $x = l + l'$, we obtain, if we pay due attention to the general rule stated in the preceding paragraph,

$$\begin{aligned} a &= f''(l + l' + l'') + c'' - c \\ a' &= f\,l - f'\,l' + c - c' \\ a'' &= f'(l + l') - f''(l + l') + c' - c'', \end{aligned}$$

and hence

$$a + a' + a'' = f\,l + f'\,l' + f''\,l''.$$

But from the considerations developed in § 13, when $\varkappa$ and ω represent the power of conduction and the section for the part P, $\varkappa'$ and ω' the same for the part P$'$, and $\varkappa''$ and ω'' for the part P$''$, at the individual places of contact, the following conditional equations are obtained:

$$\varkappa\,\omega\left(\frac{d\,u}{d\,x}\right) = \varkappa'\,\omega'\left(\frac{d\,u'}{d\,x}\right) = \varkappa''\,\omega''\left(\frac{d\,u''}{d\,x}\right),$$

where $\left(\frac{d\,u}{d\,x}\right)$, $\left(\frac{d\,u'}{d\,x}\right)$, $\left(\frac{d\,u''}{d\,x}\right)$ represent the particular values of $\frac{d\,u}{d\,x}$, $\frac{d\,u'}{d\,x}$, $\frac{d\,u''}{d\,x}$, belonging to the places of contact. From the equations stated at the commencement of the present paragraph for the determination of the electroscopic force in the single parts of the circuit, we obtain for every admissible value of x,

$$\frac{d\,u}{d\,x} = f, \quad \frac{d\,u'}{d\,x} = f', \quad \frac{d\,u''}{d\,x} = f'',$$

by which the preceding conditional equations are converted into

$$\varkappa\,\omega\,f = \varkappa'\,\omega'\,f' = \varkappa''\,\omega''\,f''.$$

From these, and the equation between f, f', and f'' above deduced from the tensions, we now find, when λ, λ', λ'' are respectively substituted for $\frac{l}{\varkappa\,\omega}$, $\frac{l'}{\varkappa'\,\omega'}$, $\frac{l''}{\varkappa''\,\omega''}$,

$$\begin{aligned} f &= \frac{a + a' + a''}{\lambda + \lambda' + \lambda''} \cdot \frac{1}{\varkappa\,\omega}, \\ f' &= \frac{a + a' + a''}{\lambda + \lambda' + \lambda''} \cdot \frac{1}{\varkappa'\,\omega'}, \\ f'' &= \frac{a + a' + a''}{\lambda + \lambda' + \lambda''} \cdot \frac{1}{\varkappa''\,\omega''}, \end{aligned}$$

and by the aid of these values we find further,

$$\begin{aligned} c' &= \frac{a + a' + a''}{\lambda + \lambda' + \lambda''}\left(\frac{l}{\varkappa\,\omega} - \frac{l}{\varkappa'\,\omega'}\right) - a' + c, \\ c'' &= \frac{a + a' + a''}{\lambda + \lambda' + \lambda''} \cdot \left(\frac{l}{\varkappa'\,\omega'} - \frac{l + l'}{\varkappa''\,\omega''} + \frac{l}{\varkappa\,\omega}\right) - (a' + a'') + c. \end{aligned}$$

By substituting these values, we obtain for the determination of the electroscopic force of the circuit in the parts P, P$'$, P$''$ respectively, the following equations:

$$\left.\begin{aligned} u &= \frac{a + a' + a''}{\lambda + \lambda' + \lambda''} \cdot \frac{x}{\varkappa\,\omega} + c \\ u' &= \frac{a + a' + a''}{\lambda + \lambda' + \lambda''} \cdot \left(\frac{x - l}{\varkappa'\,\omega'} + \frac{l}{\varkappa\,\omega}\right) - a' + c \\ u'' &= \frac{a + a' + a''}{\lambda + \lambda' + \lambda''} \cdot \left(\frac{x - (l + l')}{\varkappa''\,\omega''} + \frac{l'}{\varkappa'\,\omega'} + \frac{l}{\varkappa\,\omega}\right) - (a' + a'') + c \end{aligned}\right\} (\mathrm{L}'),$$

and it is easy to see, that these equations, with the omission of the letter $\varkappa$ or ω (both where they are explicit, as well as in the expressions for λ, λ', λ''), are the true ones for the case $\varkappa = \varkappa'$, or $\omega = \omega' = \omega''$.

18. These few cases suffice to demonstrate the law of progression of the formulæ ascertained for the electroscopic force, and to comprise them all in a single general expression. To do this with the requisite brevity, for the sake of a more easy and general survey, we will call the quotients, formed by dividing the length of any homogeneous part of the circuit by its power of conduction and its section, *the reduced length* of this part; and when the entire circuit comes under consideration, or a portion of it, composed of several homogeneous parts, we understand by its reduced length the sum of the reduced lengths of all its parts. Having premised this, all the previously found expressions for the electroscopic force, which are given by the equa-

tions (L) and (L′), may be comprised in the following general statement, which is true when the circuit consists of any number of parts whatever.

The electroscopic force of any place of a galvanic circuit, composed of any number of parts, is found by dividing the sum of all its tensions by its reduced length, multiplying this quotient by the reduced length of the part of the circuit comprised by the abscissa, and subtracting from this product the sum of all the tensions abruptly passed over by the abscissa; lastly, by varying the value thus obtained by a constant magnitude to be determined elsewhere.

If, therefore, we designate by A the sum of all the tensions of the circuit, by L its entire reduced length, by y the reduced length of the part which the abscissa passes through, and by O the sum of all the tensions to the points to which the abscissa corresponds, lastly, by u, the electroscopic force of any place in any part of the circuit, then

$$u = \frac{\mathrm{A}}{\mathrm{L}}\, y - \mathrm{O} + c,$$

where c represents a constant, but yet undetermined magnitude.

Thus transformed, this exceedingly simple expression for the electroscopic force of any circuit will allow us hereafter to combine generality with conciseness, for which purpose we will, moreover, indicate by y the *reduced abscissa*. This form of the equation has besides the peculiar advantage that, without anything further, it is even applicable when in any part of the circuit the tensions and conductibilities constantly vary; for in this case we should merely have to take, instead of the sums, the corresponding integrals, and to define their limits according as the nature of the expression required. Since O does not change its value within the entire extent of the same homogeneous part of the circuit, and y constantly varies to the same amount on like portions of this extent, the following properties, already demonstrated less generally with respect to the simple circuit, evidently apply to every galvanic circuit, and in these is expressed the main character of galvanic circuits :—

a. The electric force of each homogeneous portion of the circuit varies throughout its entire length constantly, and on like extents always to the same amount; but where it ceases and another commences, it suddenly changes to the extent of the entire tension situated at that place.

b. If any single place of the circuit is induced by any circumstance whatsoever to change its electric condition, all the other places of the circuit change theirs at the same time, and the same amount.

The constant c is in the rule determined by ascertaining the electroscopic force at any place of the circuit. If, for instance, we designate by u' the electroscopic force at a place of the circuit, the reduced abscissa of which is y', then, in accordance with the general equation above stated,

$$u' = \frac{\mathrm{A}}{\mathrm{L}}\, y' - \mathrm{O}' + c,$$

where O′ represents the sum of the tensions abruptly passed over by the abscissa y'. If we now subtract this equation, valid for a certain place of the circuit, from the previous one belonging in the same manner to all places, we obtain

$$u - u' = \frac{\mathrm{A}}{\mathrm{L}}\,(y - y') - (\mathrm{O} - \mathrm{O}'),$$

in which nothing more remains to be determined.

If the circuit, during its production, is exposed to no external deduction or adduction, the constant c must be sought for in the circumstance that the sum of all the electricity in the circuit must be zero. This determination is founded on the fundamental position, that, from a previously indifferent state, both electricities constantly originate at the same time and in like quantity. To illustrate, by an example, the mode in which the constant c is found in such a case, we will again consider the case treated of in § 16. In the portion P of that circuit, u is generally $= \frac{\mathrm{A}}{\mathrm{L}}\, y + c$, where $y = \frac{x}{\varkappa\, \omega}$, and in the portion P′ we have constantly $u = \frac{\mathrm{A}}{\mathrm{L}}\, y - a' + c$, where $y = \frac{x - l}{\varkappa'\, \omega'} + \lambda$. Since now the magnitude of the element, in the portion P, is $\omega\, d\,x$ or $\varkappa\, \omega^2\, d\,y$, but in the portion P′ is $\omega'\, d\,x$ or $\varkappa'\, \omega'^2\, d\,y$, we obtain for the quantity of electricity contained in an element of the first portion

$$\varkappa\, \omega^2\;\, d\,y\left(\frac{\mathrm{A}}{\mathrm{L}}\, y + c\right),$$

and for the quantity contained in an element of the second portion

$$\varkappa' \omega'^2 \, dy \left(\frac{A}{L} y - a' + c\right).$$

If we now integrate the first of the two preceding expressions from $y = O$ to $y = \lambda$, we then obtain for the whole quantity of electricity contained in the part P,

$$\varkappa \omega^2 \left[\frac{A}{2L} \lambda^2 + c\lambda\right];$$

in the same manner we obtain, by integrating the second expression from $y = \lambda$ to $y = \lambda + \lambda'$, for the entire quantity of electricity contained in the portion P'

$$\varkappa' \omega'^2 \left[\frac{A}{2L} (\lambda'^2 + 2\lambda\lambda') - a'\lambda' + c\lambda'\right].$$

But the sum of the two last found quantities must, in accordance with the above-advanced fundamental position, be zero. We thus obtain the equation required for the determi ·tion of the constant c, and it only remains to be observed that .nd λ' are the reduced lengths corresponding to the portions P and P'.

We have hitherto always tacitly supposed only positive abscissæ. But it is easy to be convinced that negative abscissæ may be introduced quite as well. For let $-y$ represent such a negative reduced abscissa for any place of the circuit, then $L-y$ is the positive reduced abscissa pertaining to the same place, for which the general equation found is valid; we accordingly obtain

$$u = \frac{A}{L}(L-y) - O + c$$

or

$$u = -\frac{A}{L} y - (O - A) + c.$$

But $O - A$ evidently expresses, if regard be had to the general rule expressed in § 16, the sum of all the tensions abruptly passed over by the negative abscissa, whence it is evident that the equation still retains entire its former signification for negative abscissæ.

19. If we imagine one of the parts of which the galvanic circuit is composed to be a non-conductor of electricity, *i. e.* a body whose capacity of conduction is zero, the reduced length of the entire circuit acquires an indefinitely great value. If we now make it a rule never to let the abscissæ enter into the nonconducting part, in order that the reduced abscissa y may constantly retain a finite value, the general equation changes into the following:

$$u = -O + c,$$

which indicates that the electroscopic force in the whole extent of each other homogeneous portion of the circuit is everywhere the same, and merely changes suddenly from one part to the other to the amount of the entire tension prevailing at its place of contact.

To determine the constant c in this equation, we will suppose the electroscopic force, at any one place of the circuit, to be given. If we call this u', and the sum of the tensions there abruptly passed over by the abscissa O', we have

$$u - u' = -(O - O').$$

The difference of the electroscopic forces of any two places of an open circuit, *i. e.* a galvanic circuit interrupted by a nonconductor, is consequently equal to the sum of all the tensions situated between the two places, and the sign which has to be placed before this sum is always easily to be determined from mere inspection.

20. We will now notice another peculiarity of the galvanic circuit, which merits especial attention. To this end let us keep in view exclusively one of the homogeneous parts of the circuit, and imagine, for the sake of simplicity, the origin of the abscissæ placed in one end of it, and the abscissæ directed towards the other end. If we designate its reduced length by λ, and the reduced length of the other portion of the circuit by Λ, then

$$u = \frac{A}{\Lambda + \lambda} \cdot y + c$$

within the length λ; the following form may also be given to this equation:

$$u = \frac{\frac{A\lambda}{\Lambda + \lambda}}{\lambda} \cdot y + c;$$

the extent is consequently similarly circumstanced to a simple homogeneous circuit, at whose ends the tension $\frac{A\lambda}{\Lambda + \lambda}$ occurs. If, accordingly, A has a very sensible value, such as it can acquire in the voltaic pile, and if the ratio $\frac{\lambda}{\Lambda + \lambda}$ approaches to unity, then the tension $\frac{A\lambda}{\Lambda + \lambda}$ will likewise be still very perceptible; consequently its various gradations in the extent of the portion λ are very easily perceptible. This conclusion is of importance, because it affords the means of presenting to the senses the law of electric distribution even on compound circuits, when it is no longer possible on the simple circuit, on account of its extremely feeble force. It is, moreover, immediately evident, that, with equal tensions, this phænomenon will be indicated with greater intensity, the greater λ is in comparison with Λ.

21. A phænomenon common to all galvanic circuits is the sudden change to which its electroscopic force may incessantly, and arbitrarily, be subjected. This phænomenon has its source in the previously developed properties of such circuits. Since, as we have found, each place of a galvanic circuit undergoes the same alterations to which a single one is exposed, we have it in our power to give sometimes one, sometimes another value to the electroscopic force at any certain place. Among these changes those are the most remarkable which we are able to produce by deductive contact, *i. e.* by destroying the electroscopic force sometimes at one, and sometimes at another place of the circuit; its magnitude, however, has its natural limits in the magnitude of the tensions.

There is another class of phænomena which is immediately connected with these. If, for instance, we call r the space over which the electroscopic force is diffused in a given galvanic circuit, u the electroscopic force of the circuit at one of its points, which is immediately connected with an external body M, and u' the electroscopic force of the same circuit at the same place as it was previous to contact with the body M, $u' - u$ is evidently the alteration in the electroscopic force produced at this place; consequently, since this change likewise occurs uniformly at all the other places of the circuit, $r\,(u'-u)$ is the quantity of electricity which the change produced over the entire circuit comprises, and accordingly that which has passed over into the body M. If now we suppose that in the state of equilibrium the electroscopic force is everywhere of equal intensity at all places of the body M in which it occurs, and represent by R the space over which it is diffused in the body M, then its electroscopic force is evidently $\frac{r\,(u'-u)}{\mathrm{R}}$. But this force is in the state of equilibrium equal to the u', which the

place of the circuit, brought into contact with M, has assumed when no new tension originates at this place of contact; under this supposition therefore

$$u = \frac{r\,(u'-u)}{\mathrm{R}},$$

whence we find

$$u = \frac{r\,u'}{r + \mathrm{R}}.$$

From this equation it results that the electroscopic force in the body M will constantly be smaller than it was at the touched place before contact; and also that both will approximate the more to each other, the greater r is in comparison to R. If we regard R as a constant magnitude, the relation of the electroscopic forces u and u' to each other depends solely upon the magnitude of the space which the electricity occupies in the circuit; we can therefore bring the electroscopic force of the body M nearer to its greatest value solely by increasing the capacity of the circuit, either by a general increase of its dimensions, or by attaching anywhere to it foreign masses. Upon the nature of these masses, when they are merely conductors of electricity, and do not give rise to new tensions, none of this effect, in my opinion, depends, but solely upon their magnitudes. If the attached masses occupy an infinitely great space, which case occurs when the circuit has anywhere a complete deduction, then the electroscopic force in the body M will constantly be equal to that which the place of the circuit touched by it possesses.

To connect these effects with the action of the condenser, we have merely to bear in mind, that a condenser, whose magnitude is R, and whose number of charges is m, must be considered equal to a common conductor of the magnitude $m\,\mathrm{R}$, yet with the difference that its electroscopic force is m times that of the common conductor. If, therefore, we designate by u the electroscopic force of the condenser, which is brought into connexion with a place of the circuit whose force is u', we obtain

$$u = \frac{m\,r\,u'}{r + m\,\mathrm{R}},$$

whence it follows that the condenser will indicate m times the force of the touched place when r is very great in comparison with $m\,\mathrm{R}$; but that it will have a weakening action so soon as r is equal to, or smaller than R. Masses attached anywhere to the circuit will accordingly make the indications of the condenser approximate to its maximum in proportion as they are greater, and a circuit touched at any place will constantly produce in the condenser the maximum of increase.

The preceding determinations suppose that one plate of the condenser remains constantly touched deductively. We will now take into consideration the case where the two plates of an insulated condenser are connected with various points of a galvanic circuit. In the first place, it is evident that the two plates of the condenser will assume the same difference of free electricity which the various places of the circuit with which they are in contact require unconditionally, from the peculiar nature of galvanic actions. Consequently, if d represents the difference of the electroscopic force at the two places of the circuit, and u the free electricity of one plate of the condenser, then $u + d$ is the free electricity of the other plate, and everything will depend on finding, from the known free electricities existing in the plates of the condenser, those actually present in them. If, for this purpose, we call A the actual intensity of electricity in the plate, whose free electricity is $u + d$, then $A - u - d$ represents the portion retained in the same plate; in the same manner $B - u$ designates the portion of electricity retained in the plate, whose free electricity is u, when B represents the actual intensity of the electricity in this plate. If now we represent by n the relation between the electricity retained by one plate, and the actual electricity of the other plate, the following two equations arise:

$$A - u - d + n\,B = 0,$$
$$B - u + n\,A = 0,$$

from which the values A' and B result, as follows:

$$A = \frac{d + u\,(1-n)}{1-n^2},$$

$$B = \frac{u\,(1-n) - n\,d}{1-n^2}.$$

But from the theory of the condenser, it is well known that $1 - n = \frac{1}{m}$, if m is the number of charges of the condenser; if, therefore, we substitute $\frac{1}{m}$ for $1 - n^2$ in the expressions for A and B, and at the same time $1 - \frac{1}{2\,m}$ for n, which is permitted when m, as is usually the case, denotes a very large number, we obtain

$$A = m\,d + \tfrac{1}{2}\,u,$$
$$B = -m\,d + \tfrac{1}{2}\,u + \tfrac{1}{2}\,d.$$

Or when m is a very large number, and n not much greater than d, we may, without committing any perceptible error, place

$$A = m\,d,$$
$$B = -m\,d,$$

in which is expressed the known law, that when two different places of a voltaic pile are brought into connexion with the two plates of an insulated condenser, each plate takes the same charge as if the other plate, and the corresponding place of the pile, had been touched deductively. At the same time our considerations show that this law ceases to be true when u can no longer be regarded as evanescent towards $m\,d$. This case would occur if, for instance, two places, near the insulated upper pole of a voltaic pile, constructed of a great number of elements, came in contact with the plates of the condenser, while the inferior pole of this pile remained in deductive connexion with the earth.

The determinations hitherto given respecting the mode in which the galvanic circuit imparts its electricity to foreign bodies, and which appear to me to leave nothing more to be wished for in the explanation of this subject, might, however, give rise to researches of a very different kind, and of no slight interest. For it is placed beyond all doubt, both from theoretical considerations, as well as from experiments, that electricity in motion penetrates into the interior of bodies, and its quantity accordingly depends on the space occupied by the bodies; while, on the other hand, it is no less ascertained that static electricity accumulates at the surface of bodies, and its quantity therefore is dependent on the extent of surface. But it would hence result, that in the closed galvanic circuit, r in the above formulæ would express the volume of the circuit; in the open circuit, on the contrary, the magnitude of its surface, on which point, in my opinion, experiments might decide without great difficulty.

22. We have hitherto kept in view a circuit on which the surrounding atmosphere exercised no influence, and which has

already arrived at its permanent state, and we have treated it at a length which it merits from the abundance and importance of the phænomena connected with it. However, not to let even here the other circuits pass entirely unnoticed, we will briefly indicate the method to be pursued for the most simple case, and thus point out the path to be followed, although only at a distance.

If it is intended to take into consideration the influence of the atmosphere on the galvanic circuit, the member $\frac{b\,c}{\omega}\,u$ must be added to the member $\varkappa\,\frac{d^2 u}{d\,x^2}$ of the equation (a) in § 11, we then obtain for the circuit which has acquired a permanent state, for which $\frac{d\,u}{d\,t} = 0$, the equation

$$0 = \varkappa\,\frac{d^2 u}{d\,x^2} - \frac{b\,c}{\omega}\,u;$$

or if we put $\frac{b\,c}{\varkappa\,\omega} = \beta^2$,

$$0 = \frac{d^2 u}{d\,x^2} - \beta^2 u.$$

The integral of this equation is

$$u = c\,.\,e^{\beta x} + d\,.\,e^{-\beta x},$$

where e represents the base of the natural logarithms, and c, d any constants to be determined from the other circumstances of the problem.

If we now call $2\,l$ the length of the entire circuit, and fix the origin of the abscissæ in that place of the circuit which is equidistant from the point of excitation; if, further, we designate by a the tension existing at the point of excitation, we obtain

$$a = (c - d)\,(e^{\beta l} - e^{-\beta l}).$$

If we write the previously found equation thus,

$$u = (c - d)\,e^{\beta x} + d\,(e^{\beta x} + e^{-\beta x}),$$

and substitute for $c - d$, the value just ascertained, we have

$$u = \frac{a\,.\,e^{\beta x}}{e^{\beta l} - e^{-\beta l}} + d\,(e^{\beta x} + e^{-\beta x}).$$

If we now suppose for the determination of the other constant, that the sum of the two electroscopic forces, situated at the

point of excitation, is known, and is equal to b, which case always occurs when the electroscopic force of the circuit is given at any one of its places, we obtain

$$b = \frac{a\,(e^{\beta l} + e^{-\beta l})}{e^{\beta l} - e^{-\beta l}} + 2\,d\,(e^{\beta l} + e^{-\beta l});$$

and after substitution and proper reduction,

$$u = \frac{\frac{1}{2}\,a\,(e^{\beta x} - e^{-\beta x})}{e^{\beta l} - e^{-\beta l}} + \frac{\frac{1}{2}\,b\,(e^{\beta x} + e^{-\beta x})}{e^{\beta l} + e^{-\beta l}},$$

which for $b = 0$, *i. e.* for a circuit left entirely to itself, changes into

$$u = \frac{\frac{1}{2}\,a\,(e^{\beta x} - e^{-\beta x})}{e^{\beta l} - e^{-\beta l}}.$$

These equations, which hold for a circuit homogeneous and prismatic in its whole extent, change when $\beta = 0$ again into the above, where the influence of the atmosphere on the circuit was, under the circumstances given above, left out of consideration. Since $\beta^2 = \frac{b}{\varkappa}\,.\,\frac{c}{\omega}$, it follows that the influence of the atmosphere on the galvanic circuit must be less, the smaller the conducting power of the atmosphere is in comparison to that of the circuit, and the smaller the quotient $\frac{c}{\omega}$ is. But the quotient $\frac{c}{\omega}$ expresses the relation of the surface of a disc of the conductor surrounded by the atmosphere to the volume of the same disc, and it might therefore appear that $\frac{c}{\omega}$ must constantly be infinitely small. However, it must not be forgotten that we have not here to deal with mathematical, but with physical determinations; for, strictly taken, c does not represent a surface, but that portion of a disc of the circuit on which the atmosphere has direct influence, and ω in fact signifies nothing more than that part of a disc of the circuit which is traversed by the electricity continually passing through the circuit. In general, therefore, c is indeed incomparably smaller than ω; but where the electric current can only move forwards with the greatest difficulty, and on that account but very slowly, as is more or less the case in dry piles, the magnitude c may, in accordance with what was stated in the preceding paragraph, become very nearly equal to ω; for undoubtedly a gradual transition, modified by the cotemporaneous circumstances, must occur from that which is peculiar to the rapid current to that belonging to the state of perfect equilibrium. Here, then, is a wide field open for future researches.

23. In cases where the permanent state is not instantaneously assumed, as it usually is in dry piles, we should, in order to become acquainted with the changes of the circuit up to that period, proceed from the complete equation

$$\gamma \frac{d\,u}{d\,t} = \varkappa \frac{d^2\,u}{d\,x^2} - \frac{b\,c}{\omega} u, \qquad (*)$$

because in this case we cannot consider $\frac{d\,u}{d\,t} = 0$, and the member $\frac{b\,c}{\omega}\,u$ must either remain in it, or be removed from it, according to whether it is considered worth while to take the influence of the atmosphere on the circuit into consideration not. If we again place, as in the previous paragraph, $\beta^2 = \frac{b\,c}{\varkappa\,\omega}$, and, also $\frac{\varkappa}{\gamma} = \varkappa'$, the preceding equation changes into the following:

$$\frac{d\,u}{d\,t} = \varkappa' \left(\frac{d^2\,u}{d\,x^2} - \beta^2\,u\right),$$

and we immediately perceive, that on admitting $\beta = 0$, the action of the atmosphere is left out of the question.

In the present case u represents a function of x and t, which, however, in proportion as the time t increases, becomes gradually less dependent on t, and at last passes over into a mere function of x, which expresses the permanent state of the circuit, with the nature of which we have already become acquainted. If we designate this latter function by u', and place $u = u' + v$, then v is evidently a function of x and t, which indicates every deviation of the circuit from its permanent state, and consequently after the lapse of a certain time entirely disappears. If we now substitute $u' + v$ for u in the equation $(*)$, and bear in mind that u' is independent of t, and of such nature that

$$0 = \frac{d^2\,u'}{d\,x^2} - \beta^2\,u',$$

the equation

$$\frac{d\,v}{d\,t} = \varkappa' \left(\frac{d^2\,v}{d\,x^2} - \beta^2\,v\right) \qquad (☽)$$

then remains for the determination of the function v, which still possesses the same form as the equation $(*)$, but differs from it in this respect, that v is a function of x, and t of a different nature from u, by which its final determination is much facilitated.

The integral of the equation (☽), in the form in which it was first obtained by Laplace, is

$$v = \frac{e^{-\varkappa'\beta^2 t}}{\sqrt{\pi}} \int e^{-y^2}\, f(x + 2\,y\,\sqrt{\varkappa' t})\,d\,y, \qquad (☿)$$

where e represents the base of the natural logarithms, π the ratio of the circumference of a circle to its diameter, and f an arbitrary function to be determined from the peculiar nature of each problem, while the limits of the integration must be taken from $y = -\infty$ to $y = +\infty$. For $t = 0$ we have $v = f\,x$, because between the indicated limits $\int e^{-y^2}\,d\,y = \sqrt{\pi}$, whence it results that if we know how to find the function v in the case where $f = 0$, we should thereby likewise discover $f\,x$, consequently the arbitrary function f. Now in general $v = u - u'$; but if we reckon the time t from the moment when, by the contact at the two extremities of the circuit, the tension originates, then u, when $t = 0$ has evidently fixed values only at these extremities, at all other places of the circuit u is $= 0$; accordingly, in the whole extent of the circuit $v = -u'$ in general when $t = 0$; only at the extremities of the circuit at the same time $v = u - u'$. If, therefore, we imagine a circuit left from the first moment of contact entirely to itself, then v constantly $= 0$ at its extremities, so that therefore in the interior of the circuit $v = -u'$, when $t = 0$, and at its extremities $v = 0$. Since, in accordance with our previous inquiries, u' may be regarded as known for each place of the circuit, this likewise applies to v when $t = 0$; we know then the form of the arbitrary function $f\,x$, so long as x belongs to a point in the circuit.

However, the integral given for the determination of v requires the knowledge of the function $f\,x$ for all positive and negative values of x; we are thus compelled to give, by transformation, such as the researches respecting the diffusion of heat have made us acquainted with, such a form to the above equation that only pre-supposes the knowledge of the function $f\,x$ for the extent of the circuit. The transformation applicable to the present case gives, when $2\,l$ designates the length of the circuit, and the origin of the abscissæ is placed in its centre*,

$$v = \frac{e^{-\varkappa'\beta^2 t}}{l}\left[\Sigma\left(e^{\frac{-\varkappa' i^2\pi^2 t}{l^2}} \cdot \sin\frac{i\pi x}{l}\int \sin\frac{i\pi y}{l} f y\, d y\right)\right.$$

$$\left. + \Sigma\left(e^{\frac{-(2i-1)^2\pi^2 t}{4 l^2}} \cos\frac{(2i-1)\pi x}{2l}\int \cos\frac{(2i-1)\pi y}{2l} f y\, d y\right)\right]$$

where the sums must be taken from $i = 1$ to $i = \infty$, and the integrals from $y = -l$ to $y = +l$. If we now substitute in this equation for $f x$ its value $-u'$, whereby according to our supposition in the preceding paragraph, if a represents the tension at the place of contact,

$$u' = \frac{\frac{1}{2}\, a\, (e^{\beta x} - e^{-\beta x})}{e^{\beta l} - e^{-\beta l}},$$

and then integrate, we obtain, since between the indicated limits

$$\tfrac{1}{2}\, a \int \sin\frac{i\pi y}{l} \cdot \frac{e^{\beta y} - e^{-\beta y}}{e^{\beta l} - e^{-\beta l}} \cdot d y = -\frac{a\, i\pi l \cos i\pi}{i^2\pi^2 + \beta^2 l^2},$$

and

$$\tfrac{1}{2}\, a \int \frac{e^{\beta y} - e^{-\beta y}}{e^{\beta l} - e^{-\beta l}} \cdot \cos\frac{(2i-1)\pi y}{2l} \cdot d y = 0,$$

for the determination of v the equation

$$v = a \cdot e^{-\varkappa'\beta^2 t}\, \Sigma\left(\frac{i\pi \sin\frac{i\pi(l+x)}{l}}{i^2\pi^2 + \beta^2 l^2} \cdot e^{\frac{-\varkappa'\pi^2 i^2 t}{l^2}}\right),$$

and, lastly, since $u = u' + v$

$$u = \frac{\frac{1}{2}\, a\, (e^{\beta x} - e^{-\beta x})}{e^{\beta l} - e^{-\beta l}} + a \cdot e^{-\varkappa'\beta^2 t} \times \Sigma\left(\frac{i\pi \sin\frac{i\pi(l+x)}{l}}{i^2\pi^2 + \beta^2 l^2} \cdot e^{\frac{-\varkappa'\pi^2 i^2 t}{l^2}}\right),$$

which equation, for $\beta = 0$, *i. e.* when it is not intended to take into consideration the influence of the atmosphere, passes into

$$u = \frac{a}{2l}x + a\,\Sigma\left(\frac{1}{i\pi} \sin\frac{i\pi(l+x)}{l} \cdot e^{\frac{-\varkappa'\pi^2 i^2 t}{l^2}}\right).$$

It is easily perceived that the value of the second member to the right in the equations which have been found for the determination of u, becomes smaller and smaller as the time increases, and that it at last entirely vanishes; the permanent state of the circuit has then occurred. This moment can, as is evident from the form of the expression, be retarded by a diminished power of conduction, and in a far greater degree by an increased length of the circuit.

* See *Journal de l'Ecole Polytechnique*, cap. xix. p. 53.

This expression found for u, however, holds perfectly only so long as the circuit, which we have supposed, is not induced by any external disturbance to change its natural state. If the circuit is at any time compelled by any external cause, for instance, by deductive contact at any place, to approximate to an altered permanent state, the above method has to undergo some changes, which I intend to develope on another occasion. I will, moreover, observe, that it is in this last class of galvanic circuits, in which the peculiar phænomena of dry piles, and, in general, of circuits of unusually great length, have to be sought for; to which class likewise belong the circuits of very great length employed in the experiments of Basse, Erman, and Aldini, if the influence of their great length be not annulled by an increased goodness of conduction, or by an increased section.

C. *Phænomena of the Electric Current.*

24. According to what was advanced in paragraph 12, the magnitude of the electric current, in a prismatic body, will in general be expressed for each of its places by the equation

$$S = \omega\varkappa\frac{d u}{d x},$$

where S denotes the magnitude of the current, and u the electroscopic force at that place of the circuit whose abscissa is x, while ω represents the section of the prismatic body, and $\varkappa$ its power of conduction at the same place. To connect this equation with the general equation found in § 18 for any circuit, composed of any number of parts, we write it thus:

$$S = \varkappa\omega\frac{d u}{d y} \cdot \frac{d y}{d x},$$

and substitute for $\frac{d u}{d y}$ the value $\frac{A}{L}$ resulting from that general equation, and for $\frac{d y}{d x}$ the value $\frac{1}{\varkappa\omega}$ easily deducible from the same paragraph, both which values are valid for each place, situated between two points of excitation, we then very simply obtain

$$S = \frac{A}{L},$$

where L denotes the entire reduced length of the circuit, and A the sum of all its tensions. By means of this equation we obtain the magnitude of the electric current of a galvanic circuit, composed of any number of prismatic parts, which has acquired its permanent state, which is not affected by the surrounding atmosphere, and the single sections of which possess in all their points one and the same electroscopic force; in this category are comprised the most frequently occurring cases, on which account we shall dissect this result in the most careful manner.

Since A represents the sum of all the tensions in the circuit, and L the sum of the reduced lengths of all the individual parts, there results, in the first place, from the equation found, the following general properties relative to the electric current of the galvanic circuit.

I. The electric current is decidedly of equal magnitude at all places of a galvanic circuit, and is independent of the value of the constant c, which, as we have seen, fixes the intensity of the electroscopic force at a determined place. In the open circuit the current ceases entirely, for in this case the reduced length L acquires an infinitely great value.

II. The magnitude of the current, in a galvanic circuit, remains unchanged when the sum of all its tensions and its entire reduced length are varied, either not at all, or in the same proportion; but it increases, the reduced length remaining the same, in proportion as the sum of the tensions increases, and the sum of the tensions remaining the same, in proportion as the reduced length of the circuit diminishes. From this general law we will, moreover, particularly deduce the following.

1. A difference in the arrangement and distribution of the individual points of excitation, by a transposition of the parts of which the circuit consists, has no influence on the magnitude of the current when the sum of all the tensions remains the same. Thus, for instance, the current would remain unaltered in a circuit formed in the order copper, silver, lead, zinc, and a fluid, even when the silver and lead change places with each other; because, according to the laws of tension observed with respect to metals, this transposition would, it is true, alter the individual tensions, but not their sum.

2. The intensity of a galvanic current continues the same, although a part of the circuit be removed, and another prismatic conductor be substituted in its place, only both must have the same reduced length, and the sum of the tensions in both cases remain the same; and *vice versâ*, when the current of a circuit is not altered by the substitution of one of its parts for a foreign prismatic conductor, and we can be convinced that the sum of the tensions has remained the same, then the reduced lengths of the two exchanged conductors are equal.

3. If we imagine a galvanic circuit always constructed of a like number of parts, of the same substance, and arranged in the same order, in order that the individual tensions may be regarded as unchangeable, the current of this circuit increases, the length of its parts remaining unaltered, in the same proportion in which the sections of all its parts increase in a similar manner, and the sections remaining unaltered, in the same proportion in which the length of all its parts uniformly decrease. When the reduced length of a part of the circuit far exceeds that of the other parts, the magnitude of the current will principally depend on the dimensions of this part; and the law here enounced will assume a much more simple form, if, in the comparison, attention be solely directed to this one part.

The conclusion arrived at in II. 2. presents a convenient means for the determination of the conductibility of various bodies. If, for instance, we imagine two prismatic bodies, whose lengths are l and l', their sections respectively ω and ω', and whose powers of conduction are $\varkappa$ and $\varkappa'$, and both bodies possess the property of not altering the current of a galvanic circuit when they alternatively form a portion of it, and both leave the individual tensions of the circuit unchanged, then

$$\frac{l}{\varkappa\,\omega} = \frac{l'}{\varkappa'\,\omega'},$$

consequently

$$\varkappa : \varkappa' = \frac{l}{\omega} : \frac{l'}{\omega'};$$

the powers of conduction, therefore, of both bodies are directly proportionate to their lengths, and inversely proportionate to their sections. If it is intended to employ this relation in the determination of the powers of conduction of various bodies, and we choose for the experiments prismatic bodies of the same section, which indeed is requisite for the sake of great accuracy, their lengths will enable us to determine accurately their conductibilities.

25. In the preceding paragraph we have deduced the magnitude of the current from the general equation given in § 18,

$$u = \frac{A}{L} y - O + c,$$

and have found that it is expressed by $\frac{A}{L}$, the coefficient of y.

To ascertain the value $\frac{A}{L}$ it is in general requisite to possess an accurate knowledge of all the single parts of the circuit, and their reciprocal tensions; but our general equation indicates a means of deducing this value likewise from the nature of any single part of the circuit in the state of action, which we will not disregard, as it will be of great service to us hereafter. If, namely, we conceive in the above equation y to be increased by any magnitude Δy, and designate by ΔO the corresponding change of O, and by Δu that of u, there results from that equation

$$\Delta u = \frac{A}{L} \Delta O - \Delta O,$$

and we thence find

$$\frac{A}{L} = \frac{\Delta u + \Delta O}{\Delta y};$$

we find, therefore, the magnitude of the electric current by adding to the difference of the electroscopic forces at any two places of the circuit the sum of all the tensions situated between these two places, and dividing this sum by the reduced length of the part of the circuit which lies between these same places. If there should be no tension within this portion of the circuit, then $\Delta O = 0$, and we obtain

$$\frac{A}{L} = \frac{\Delta u}{\Delta y}.$$

26. The voltaic pile, which is a combination of several similar

simple circuits, merits peculiar attention in this place, from the numerous and varied experimental results obtained by its means.

If A represent the sum of the tensions of a closed galvanic circuit, and L its reduced length, the magnitude of its current is, as we have found,

$$\frac{A}{L}.$$

Now, if we imagine n such circuits perfectly similar to the former, but open, and constantly bring the end of each one in direct connexion with the commencement of the next following one, in such a manner that between each two circuits no new tension occurs, and all the previous tensions remain afterwards as before, then the magnitude of the current of this voltaic combination, closed in itself, is evidently

$$\frac{nA}{nL},$$

consequently equal to that in the simple circuit. This equality of the circuit, however, no longer exists when a new conductor, which we will call the interposed conductor, is inserted in both. If, namely, we designate the reduced length of this interposed conductor by Λ, then, when no new tension is produced by it, the magnitude of the current in the simple circuit will be

$$\frac{A}{L + \Lambda},$$

and in the voltaic combination, consisting of n, such elements

$$\frac{nA}{nL + \Lambda} \quad \text{or} \quad \frac{A}{L + \frac{\Lambda}{n}};$$

therefore in the latter circuit it is constantly greater than in the former, and, in fact, a gradual transition takes place from equality of action, which is evinced when Λ disappears, to where the voltaic combination exceeds n times the action of the simple circuit, which case occurs when Λ is incomparably greater than nL. If by Λ we represent the relative length of the body upon which the circuit is to act by the force of its current, then from the observations just brought forward it results that it is most advantageous to employ a powerful simple circuit when Λ is very small in comparison to L; and, on the contrary, the voltaic pile, when Λ is very great in comparison with L.

But how must, in each separate case, a given galvanic apparatus be arranged so as to produce the greatest effect? Let us suppose, in solving this problem, that we possess a certain magnitude of surface; for instance, of copper and zinc, from which we can form, according to pleasure, a single large pair of plates, or any number of smaller pairs, but in the same proportion, and, moreover, that the liquid between the two metals is constantly the same, and of the same length, which latter supposition means nothing more than that the two metals between which the liquid is confined retain, under all circumstances, the same distance from each other.

Let Λ be the reduced length of the body upon which the electric current is to act, L the reduced length of the apparatus when formed into a simple circuit, and A its tension; then, when it is altered into a voltaic combination of x elements, its present tension will be $x\,\mathrm{A}$, and the reduced length of each of its present elements $x\,\mathrm{L}$, accordingly the reduced length of all the x elements $x^2\,\mathrm{L}$, consequently the magnitude of the action of the voltaic combination of x elements is

$$\frac{x\,\mathrm{A}}{x^2\,\mathrm{L} + \Lambda}.$$

This expression acquires its greatest value $\dfrac{\mathrm{A}}{2\sqrt{\Lambda\,.\,\mathrm{L}}}$ when $x = \sqrt{\dfrac{\Lambda}{\mathrm{L}}}$. We hence see that the apparatus in form of a simple circuit is most advantageous, so long as Λ is not greater than L; on the contrary, the voltaic combination is most useful when Λ is greater than L, and indeed it is best constructed of two elements when Λ is four times greater than L, of three elements when Λ is nine times greater than L, and so forth.

27. The circumstance that the current always remains the same at all places, affords us the means of multiplying its external action, as in the case when the current influences the magnetic needle. We will, for perspicuity, suppose that, in order to test the action of the current on the magnetic needle, each time a part of the circuit be formed into a circle of a given radius, and so placed in the magnetic meridian that its centre coincides with the point of rotation of the needle. Several such distinct coils, formed of the circuit in exactly the same manner, will, taken singly, produce, on account of the equality of the current in each, equally powerful effects on the magnetic needle; if we imagine them, therefore, so arranged near one another, that though they are separated by a non-conducting layer, they are yet situated so close together that the position of each one toward the magnetic needle may be regarded as the same, they would produce a greater effect on the magnetic needle in proportion as their number increased. Such an arrangement is termed a *multiplier*.

Now, let A be the sum of the tensions of any circuit, and L its reduced length; let also Λ be the reduced length of one of the interposed conductors formed into a multiplier of n convolutions; then, if we represent the reduced length of one such convolution by λ, $\Lambda = n\,\lambda$, the action of the multiplier on the magnet needle will be proportional to the value

$$\frac{n\,\mathrm{A}}{\mathrm{L} + n\,\lambda}.$$

But the action of a similar coil of the circuit, without the multiplier, is, according to the same standard,

$$\frac{\mathrm{A}}{\mathrm{L}},$$

and we will suppose the portion of the circuit, whence the coil is taken, to be of the same nature as in the multiplier; accordingly the difference between the former and the present effect is

$$\frac{n\,\mathrm{L} - (\mathrm{L} + n\,\lambda)}{\mathrm{L} + n\,\lambda}\cdot\frac{\mathrm{A}}{\mathrm{L}},$$

which is positive or negative according as $n\,\mathrm{L}$ is greater or less than $\mathrm{L} + n\,\lambda$. Consequently the action on the magnetic needle will be augmented or diminished by the multiplier formed of n coils, according as the n times reduced length of the circuit, without interposed conductor, is greater or less than the entire reduced length of the circuit with the interposed conductor.

If $n\,\lambda$ is incomparably greater than L, the action of the multiplier on the needle will be

$$\frac{\mathrm{A}}{\lambda}.$$

To this value, which indicates the extreme limit of the action by means of the multiplier, whether it be strengthening or weakening, belong several remarkable properties, which we will briefly notice. It is constantly supposed that the multiplier is formed of so many coils that the magnitude of its action may, without committing any sensible error, be considered equal to the limit value.

Since the action of a coil of the circuit is $\frac{A}{L}$, while the action of the multiplier, in connexion with the same circuit, is $\frac{A}{\lambda}$, it is evident that the two actions are in the same ratio to each other as the reduced length λ and L; if, therefore, we are acquainted with the two actions, and with one of the two reduced lengths, the other may be found, and in the same manner one of the two actions may be deduced from the other, and the two reduced lengths.

Since the limit of the action of the multiplier is $\frac{A}{\lambda}$, it increases when λ is invariable in the same proportion as the sum of the tensions A in the circuit increases; we may, therefore, by comparing the extreme actions of the same multiplier in various circuits, arrive at the determination of their relative tensions. At the same time we perceive that the extreme action of the multiplier increases, when several simple circuits are formed into a voltaic combination, and, indeed, in direct proportion to the number of the elements. In this manner it is always in our power, in cases where the multiplier in connexion with the simple circuit produces a weakening effect, to cause it to indicate any increase of force whatever.

If we call the actual length of a coil of the multiplier l, its conductibility $\varkappa$, and its section ω, then $\lambda = \frac{l}{\varkappa\,\omega}$, and consequently the extreme action of the multiplier

$$\varkappa\,\omega \, . \, \frac{A}{l},$$

whence it results that in the same circuit the extreme actions of two multipliers of coils of equal diameter, are in the ratio to each other of the products of their conductibility and their section. These extreme actions are, therefore, in two multipliers, which differ only in being formed of two distinct metals, in proportion to the conductibility of these metals; and when the multipliers consist of similar convolutions, and of one metal, their extreme actions are proportional to their sections.

But all these determinations are based upon the supposition that the action of a portion of the circuit on the magnetic needle, under otherwise similar circumstances, is proportional to the magnitude of the current. But long since direct experiments have established the correctness of this supposition.

28. We will now proceed to the consideration of a multiple conduction existing at the same time. If, for instance, we imagine an open circuit, whose separated extremities are connected by several conductors, arranged by the side of each other, it may be asked, according to what law is the current distributed in the adjacent conductors? In answering this question, we might proceed directly from the considerations contained in § 11 to 13; but we shall more simply attain the required object from the peculiarity of galvanic circuits ascertained in § 25, in which case we will, for the sake of simplicity, suppose that none of the former tensions is destroyed by the opening of the circuit, nor a new tension produced by the conductor which is introduced.

For if λ, λ', λ'', &c. represent the reduced lengths of the conductors brought into connexion with the extremities of the open circuit, and α the difference of the electroscopic forces at the extremities of the circuit, after the conductors have been introduced, the same difference will also occur at the ends of the single adjacent conductors, since, according to the supposition we have made, no new tension is introduced by the conductor. Since now, according to § 13, the magnitude of the current in the circuit must be equal to the sum of all the currents in the adjacent conductors, we may imagine the circuit to be divided into as many parts as there are adjacent conductors; then, according to § 25, the magnitude of the current in each adjacent conductor, and in the corresponding part of the circuit, will respectively be

$$\frac{\alpha}{\lambda},\ \frac{\alpha}{\lambda'},\ \frac{\alpha}{\lambda''},\ \text{\&c.},$$

whence, in the first place, it results that the magnitude of the current in each adjacent conductor is in inverse ratio to its reduced length. If we now imagine a single conductor of such nature, that, being substituted for all the adjacent conductors in the circuit, it does not at all alter its current; then, in the first place, α, according to § 25, must retain the same value, and, if we designate by Λ the reduced length of this conductor, must moreover be

$$\frac{1}{\Lambda} = \frac{1}{\lambda} + \frac{1}{\lambda'} + \frac{1}{\lambda''} + \&c.$$

From the preceding explanations we may conclude, that when A denotes the sum of all the tensions, and L the entire reduced length of the circuit without adjacent conductors, the magnitude of the current, while the adjacent conductors are in connexion with the circuit, will be expressed in the circuit itself by

$$\frac{A}{L + \Lambda};$$

in the joint conductor, whose reduced length is λ, by

$$\frac{A}{L + \Lambda} \cdot \frac{\Lambda}{\lambda};$$

in the joint conductor, whose reduced length is λ', by

$$\frac{A}{L + \Lambda} \cdot \frac{\Lambda}{\lambda'};$$

in the joint conductor, whose reduced length is λ'', by

$$\frac{A}{L + \Lambda} \cdot \frac{\Lambda}{\lambda''};$$

and so on, where for Λ its value obtained from the equation

$$\frac{1}{\Lambda} = \frac{1}{\lambda} + \frac{1}{\lambda'} + \frac{1}{\lambda''} + \&c.$$

has to be placed.

29. That in the above the galvanic current is found to be of equal magnitude at all places of the circuit, arises from the value of $\frac{d\,u}{d\,x}$, deduced from the equation

$$u = \frac{A}{L} y - O + c,$$

being constant. This circumstance no longer happens if we start from the equations given in § 22 and 23. In all these cases $\frac{d\,u}{d\,x}$ is dependent on x, which indicates that the magnitude of the current is different at different places of the circuit. We may hence draw the conclusion, that the electric current is only of equal intensity at all places of the circuit, when the circuit has already assumed a permanent state, and the atmosphere has no sensible action upon it. This property likewise appears best adapted to enable us to find out, by experiment, whether the atmosphere exercises a perceptible influence on a galvanic circuit, or not, we will therefore enter into this case at greater length.

Since, according to § 12, the magnitude of the electric current is given by the equation

$$S = \varkappa\,\omega \cdot \frac{d\,u}{d\,x},$$

we have only in each separate case to obtain the value of $\frac{d\,u}{d\,x}$ from the equation found for the determination of the electroscopic force, and to place it in the one above. Thus, for a circuit which has assumed its permanent state, but upon which the surrounding atmosphere exercises no sensible influence, according to § 22,

$$u = \tfrac{1}{2}\,a \cdot \frac{e^{\beta x} - e^{-\beta x}}{e^{\beta l} - e^{-\beta l}} + \tfrac{1}{2}\,b\,\frac{e^{\beta x} + e^{-\beta x}}{e^{\beta l} + e^{-\beta l}},$$

where a represents the tension at the place of excitation, and b the sum of the electroscopic forces immediately adjacent on both sides of the place of excitation. We hence obtain

$$S = \varkappa\,\omega\,\beta\left(\tfrac{1}{2}\,a\,\frac{e^{\beta x} + e^{-\beta x}}{e^{\beta l} - e^{-\beta l}} + \tfrac{1}{2}\,b\,\frac{e^{\beta x} - e^{-\beta x}}{e^{\beta l} + e^{-\beta l}}\right).$$

This expression gives the magnitude of the current at each place of the circuit; but the law, according to which the alteration of the current at various places of the circuit is effected, may be rendered more easily intelligible in the following manner. If, for instance, we differentiate the equation

$$S = \varkappa\,\omega\,\frac{d\,u}{d\,x},$$

we obtain the equation

$$\frac{d\,S}{d\,x} = \varkappa\,\omega\,\frac{d^2\,u}{d\,x^2};$$

and by multiplying both together,

$$\frac{d\,S}{d\,u} = \varkappa^2\,\omega^2\,\frac{d^2\,u}{d\,x^2}.$$

If we now substitute for $\frac{d^2\,u}{d\,x^2}$ its value $\beta^2\,u$, as obtained from the equation $0 = \frac{d^2\,u}{d\,x^2} - \beta^2\,u$, we have

$$\frac{d\,S}{d\,u} = \varkappa^2\,\omega^2\,\beta^2\;u;$$

and we hence obtain by integration

$$S^2 = c^2 + \varkappa^2 \omega^2 \beta^2 u^2,$$

where c represents a constant remaining to be determined. If we designate by u' the smallest absolute value which u occupies in the circumference of the circuit, and by S' the corresponding value of S, and determine, in accordance with this, the constant c, we obtain

$$S^2 - S'^2 = \varkappa^2 \omega^2 \beta^2 (u^2 - u'^2).$$

It may easily be deduced from this equation, that the current of a circuit, which is influenced by the atmosphere, is weakest where the electroscopic force, without regard to the sign, is smallest, and that it is of the same magnitude at places with equal but opposite electroscopic forces.

APPENDIX.

ON THE CHEMICAL POWER OF THE GALVANIC CIRCUIT.

On the Source and character of the Chemical Changes in a Galvanic Circuit, and on the Nature of the Fluctuations of its Force dependent thereon.

30. In the present Memoir we have constantly supposed that those bodies, which are under the influence of the electric current, remain unchangeable; we will now, however, take into consideration the action of the current on the bodies subjected to it, and the alterations in their chemical constitution thence resulting in any possible manner, as also the changes of the current itself produced by reaction. If what we here give does by no means exhaust the subject, nevertheless our first attempt shows that we are advancing in this path towards important conclusions respecting the relation of electricity towards bodies.

To proceed on sure ground, let us return to what has been enounced in § 1 to 7, and connect our present considerations with those expressions and developments. We will suppose, therefore, two particles, and designate by s their mutual distance, by u and u' their electroscopic forces, which we admit to be of equal intensity in all points of the same particle; then, as may easily be deduced from what has been previously stated,

the repulsive force between these two elements is proportional to the time dt, to the product uu', and, moreover, to a function dependent on the position, size, and form of the two particles, which we will represent by F'; we accordingly obtain for the repulsive force between two particles the expression

$$F'\, u u'\, dt.$$

If we here proceed in the same manner as in § 6, and signify by the *moment of action* $\varkappa'$ between two places, the product of q', which expresses the force produced under perfectly determined circumstances between both, and its mean distance s', so that

$$\varkappa' = q' \,.\, s',$$

and determine q' by putting $u = u' = 1$ in the expression $F'\, u u'\, dt$, and extending the action to the unit of time, we have

$$\varkappa' = F'\, s',$$

whence it follows that

$$F' = \frac{\varkappa'}{s'}.$$

Let us now imagine, as we did in § 11, the prismatic circuit to be divided into equally large, infinitely thin discs, and call M', M, $M_{,}$ those immediately following one another, which belong to the abscissæ $x + dx$, x, $x - dx$; then, according to what has just been shown, the pressure which the disc M' exerts on the disc M is

$$F'\, u u'\, dt;$$

and if we admit that the position, size, and form of the particles remain in all discs the same, the counter pressure, which the disc $M_{,}$ exerts on the disc M, is

$$F'\, u u_{,}\, dt;$$

the difference between these two expressions, viz.

$$F'\, u\, (u' - u_{,})\, dt,$$

gives accordingly the magnitude of the force, with which the disc M tends to move along the axis of the circuit. This force acts contrary to the direction of the abscissæ when its value is positive, and in the direction of the abscissæ when it is negative.

If we substitute for $u' - u$ its value proceeding from the developments given in § 11 for u' and $u_{,}$, the expression just found changes into the following:

$$2\,F'\,u\,\frac{d\,u}{d\,x}\,d\,x\;\;d\,t,$$

and if we take, instead of the function F′ dependent on the nature of each single body, its value $\frac{\varkappa'}{s'}$, this expression, since s' is evidently here $d\,x$, changes into

$$2\,\varkappa'\,u\,\frac{d\,u}{d\,x}\,d\,t\,;$$

or if we reduce the moment of action $\varkappa'$, referring to the magnitude of the section ω, to the unit of surface, and at the same time extend the action to the unit of time, into

$$2\,\varkappa'\,\omega\,u\,\frac{d\,u}{d\,x},$$

where the present $\varkappa'$ represents the magnitude of the moment of action reduced to the unit of surface. If we write this latter expression thus:

$$2\,\frac{\varkappa'}{\varkappa}\,\varkappa\,\omega\,u\,\frac{d\,u}{d\,x},$$

in which $\varkappa$ denotes the absolute power of conduction of the circuit; and if we substitute for $\varkappa\,\omega\,\frac{d\,u}{d\,x}$, by which, according to the equation (*b*) in § 12, the magnitude of the electric current is expressed, the sign S chosen for it, and i instead of $\frac{\varkappa'}{\varkappa}$, it is changed into

$$2\,i\,u\,S.$$

We hence perceive that the force, with which the individual discs in the circuit tend to move, is proportional, both to their innate electroscopic force, and to the magnitude of the current; and that this force alters its direction at that place of the circuit where the electricity passes from the one into the opposite state. And here occurs the circumstance which must not be overlooked, that this expression still holds, even when the electroscopic force u of the element M is changed in the moment of action, by any causes whatsoever, into any other abnormal U, while the electroscopic forces of the adjacent particles continue the same; only that in this case the value U must be substituted for u in the expression $2\,i\,u\,S$. It must also be observed, that the expression $2\,i\,u\,S$ which we have found refers to the whole extent of the section ω, which belongs to that part of the circuit which we have especially in view; if we wish to reduce this motive force of the circuit to the unit of surface, we must divide that expression by the magnitude of the section ω.

With respect to the causal relation between the law of electric attractions and repulsions, and that of the diffusion of electricity, or respecting the mutual dependence of the functions $\varkappa$ and $\varkappa'$ on each other, we will, for the present, not enter into any further inquiries, as shortly an occasion will present itself for this purpose. We will here content ourselves with the observation, that the above mode of explanation has arisen from the endeavour to render the similarity of the mode of treatment in the doctrines of electricity and heat very obvious.

31. Without pursuing any further these conditions to an external change of place of the parts of a galvanic circuit, let us now turn to those changes in the qualitative state of the circuit which are produced by the electric current, *i. e.* in the internal relation of the parts to each other, and which derive their explanation from the electro-chemical theory of bodies. According to this theory, compound bodies must be considered as a union of constituents which possess dissimilar electric states; or, in other words, dissimilar electroscopic force. But this electroscopic force, quiescent in the constituents of the bodies, differs from that to which our attention has hitherto been directed, inasmuch as it is linked to the nature of the elements, and cannot pass from one to the other, without the entire mode of existence of the parts of the body being destroyed. If we confine ourselves, therefore, in the following considerations, to the case where changes, it is true, occur in the quantitative relation of the constituents, and where consequently chemical changes of the body, composed of these constituents, also occur, but where the constituents themselves undergo no alteration destroying their nature, we are able to show the validity of all the laws above developed of electric bodies with reference to their reciprocal attraction and repulsion, only the transition of the electricity from one particle to the other entirely disappears in the consideration of chemically different constituents. A distinction here exists with reference to electricity exactly similar to that which we are accustomed to define relative to heat, by calling it sometimes latent, sometimes free heat. For the sake of brevity, we will in like manner term that electroscopic force

which belongs to the existence of the particles, which therefore they cannot part with without at the same time ceasing to exist, the *electricity bound* to the bodies, or *latent electricity*, and *free electricity*, that which is not requisite for the existence of the bodies in their individuality, and which therefore can pass from one element to the other, without the individual parts being on that account compelled to exchange their specific mode of existence for another.

32. From these suppositions advanced in electro-chemistry, in connexion with what was stated in § 30, respecting the mode in which galvanic circuits exert a different mechanical force on discs of different electrical nature, it immediately follows that when a disc belonging to the circuit is composed of constituents of dissimilar electric value, the neighbouring discs will exert on these two constituents a dissimilar attractive or repulsive action, which will excite in them a tendency to separate, which, when it is able to overcome their coherence, must produce an actual separation of constituents. This power of the galvanic circuit, with which it tends to decompose the particles into their constituents, we will call its *decomposing force*, and now proceed to determine more minutely the magnitude of this force.

Employing for this purpose all the signs introduced in § 30, we will, moreover, imagine each disc to be composed of two constituents, A and B, and designate by m and n the latent electroscopic forces of the constituents A and B, supposing the disc M to be occupied solely by one of the two, entirely excluding the other, in the same manner as u represents the free electroscopic force present in the same disc, and equally diffused over both constituents. If we now admit, in order to simplify the calculation, that the two constituents A and B, before and after their union, constantly occupy the same space, and designate the latent electroscopic force, corresponding to each chemical equivalent, contained in the disc M, and proceeding from the constituent A, by $m\,z$, then $n\,(1-z)$ expresses the latent electroscopic force present in the same disc M, but originating from the constituent B: for the intensity of the force diffused over a body decreases in the same proportion as the space which the body occupies becomes greater, because by the increased distance of the particles from each other the sum of their actions, restricted to a definite extent, is diminished in

the same proportion. But when two constituents combine, by both reciprocally penetrating one another, each extends beyond the entire space of the compound, on which account the intensity of the force proper to each constituent decreases by combination, in the same proportion as the space of the compound is greater than the space which each constituent occupied before the combination. Consequently if z denote the relation of the space which the constituent A, in the disc M, occupied previous to combination to that space which the compound in the disc M occupies; and also, since we admit that both constituents, before and after the combination, occupy the same extent of space, $1-z$ will denote the same relation relatively to constituent B; then, since m and n designate the electroscopic forces of the constituents A and B previous to combination, $m\,z$ and $n\,(1-z)$ will represent the latent electroscopic forces of the constituents A and B, which correspond to each chemical equivalent of the disc M; and, at the same time, it follows from the above, that the variable values z and $1-z$ cannot exceed the limits 0 and 1.

In order to ascertain the portion of the free electricity u pertaining to each constituent, we will assume that it is distributed over the single constituents in proportion to their masses. If, therefore, we represent respectively by α and β the masses of the constituents A and B, on the supposition that one alone, to the exclusion of the other, occupies the entire disc, then $\alpha\,z$ and $\beta\,(1-z)$ will represent the masses of the constituents A and B united in the disc M; consequently the portions

$$\frac{\alpha\,u\,z}{\alpha\,z+\beta(1-z)}, \quad \text{and} \quad \frac{\beta\,u\,(1-z)}{\alpha\,z+\beta\,(1-z)}$$

of the free electricity u appertain to the constituents A and B; instead of which, for the sake of conciseness, we will write

$$\alpha\,\mathrm{U}\,z, \text{ and } \beta\,\mathrm{U}\,(1-z).$$

If we now take into consideration what was stated in § 30, respecting the motive force of the galvanic circuit, it is immediately evident that the tendency of the constituent A to move along the circuit, is expressed by

$$2\,i\,(m+\alpha\,\mathrm{U})\,z\,\mathrm{S},$$

or that of the constituent B by

$$2\,i\,(n+\beta\,\mathrm{U})\,(1-z)\,\mathrm{S}.$$

In both cases a positive value of the expression shows that the pressure takes place in an opposite direction to that of the abscissæ; a negative value, on the contrary, indicates that the pressure is exerted in the direction of the abscissæ. To deduce from these individual tendencies of the constituents the force with which both endeavour to separate from each other, we must remember that this force is given by the twofold difference between the quantities of motion which each constituent would of itself assume, were it associated to the other by no coherence, and those quantities of motion which each constituent must assume were it strongly combined to the other. We thus readily find for the decomposing force of the circuit the following expression:

$$4\,i\,.\,z\,(1-z)\,.\,\frac{m\,\beta - n\,\alpha}{\alpha\,z + \beta\,(1-z)}\,.\,S,$$

from which we learn that the decomposing force of the circuit is proportional to the electric current, and also to a coefficient dependent on the chemical nature of each place of the circuit.

If this expression has a positive value, it indicates that the separation of the constituent A takes place in a contrary direction to that of the abscissæ, that of the constituent B in the direction of the abscissæ; but if this expression has a negative value, it denotes a separation in the reverse direction. It is besides evident, at first sight, that the decomposing force of the circuit is constantly determined by the absolute value of the expression.

If $\alpha = \beta$, the decomposing force of the circuit changes into

$$4\,i\,.\,z\,(1-z)\,(m-n)\,.\,S.$$

If $m\,z + n\,(1-z) = 0$, *i. e.* if the latent electroscopic forces, existing in the united constituents, are equal and opposed; or, what is the same, if the body, situated in the disc M, is perfectly neutral, in which case m and n have constantly opposite values, we obtain, for the decomposing force of the circuit, the following expression:

$$4\,i\,.\,\frac{m\,n}{m-n}\,.\,S.$$

The form of the general expression found for the decomposing force of the circuit shows that this force disappears; first, when $S = 0$, *i. e.* when no electric current exists; secondly, when $z = 0$, or $z = 1$, *i. e.* when the body to be decomposed is not compound; thirdly, when $m\,\beta - n\,z = 0$, *i. e.* when the densities of the constituents are proportional to the latent electroscopic forces which they possess, which circumstance can never occur with constituents of opposite electric nature.

All the expressions here given for the decomposing force of the circuit refer to the entire section belonging to the respective place; if we wish to reduce the value of the decomposing force to the unity of surface, the expression must be divided by the magnitude of the section, to which attention has been already called in § 30, in a similar example.

33. If this decomposing force of the circuit is able to overcome the coherence of the particles in the disc, a coherence produced by their electric opposition, this necessarily occasions a change in the chemical equivalent of the particles. But such a change in the physical constitution of the circuit must, at the same time, react on the electric current itself, and give rise to alterations in it, with which a more accurate acquaintance is desirable, and which we will therefore spare no trouble to acquire.

For this purpose we will imagine a portion of the galvanic circuit to be a homogeneous fluid body, in which such a decomposition actually takes place; then, at all points of this portion, the elements of one kind will tend to move with greater force towards one side of the circuit than those of the other kind; and since we suppose that, by the active forces, the coherence is overcome, it follows, if we pay due attention to the nature of fluid bodies, that the one constituent must pass to one side, those of the other constituent, on the contrary, towards the other side of the portion, which necessarily produces on one side a preponderance of the constituent of one kind, and on the other side a preponderance of the other kind of constituent. But as soon as a constituent is predominant on one side of any disc, it will oppose by its preponderance the movement of the like constituent in the disc towards the same side, in consequence of the repulsive force existing between both; the decomposing force, therefore, has now not merely to overcome the coherence between the two constituents in the disc, but also the reacting force in the neighbouring discs. Two cases may now occur; the decomposing force of the electric current either constantly overcomes all the forces opposed to it, and then evidently the action terminates by a total separation of the constituents, the entire mass of the one passing to the one end of the portion, and the entire mass of the other constituent being impelled towards the other end of this portion; or such a relation takes place between the forces in action, that the forces opposing the separation ultimately maintain the decomposing force in equilibrium; from this moment no further decomposition will occur, and the portion will be, in a remarkable state, a peculiar distribution of the two constituents occurring, into the nature of which we will now inquire. If we call Z the decomposing force of the current in any disc of the portion in the act of decomposition, Y the magnitude of the reaction by which the neighbouring discs oppose the decomposition by the electric current, and X the force of the coherence of the two constituents in the same disc, then evidently the state of a permanent distribution within the supposed portion, will be determined by the equation

$$X + Y = Z;$$

and it is already known, from the preceding paragraph, that

$$Z = 4\,i\,z\,(1-z)\,\frac{m\,\beta - n\,\alpha}{\alpha\,z + \beta\,(1-z)}\,.\,S;$$

or if we substitute $\varkappa\,\omega\,\frac{d\,u}{d\,x}$ for S,

$$z = 4\,\varkappa\,\omega\,\frac{d\,u}{d\,x}\,.\,i\,z\,(1-z)\,\frac{m\,\beta - n\,\alpha}{\alpha\,z + \beta\,(1-z)}.$$

Before we proceed further, we will add to what has been above said the following remarks. At the limits of the portion in question, we imagine the circuit so constituted, that insuperable difficulties there oppose themselves to any further motion; for it is obvious that otherwise the two extreme strata of both constituents, which it is evident could never of themselves arrive at equilibrium, would quit the portion in which we have hitherto supposed them, and either pass on to the adjacent parts of the circuit, or from any other causes separate entirely from the circuit. We will not here follow the last-mentioned modification of the phænomenon any further, although it frequently occurs in nature, as sufficiently shown by the decomposition of water, the oxidation of the metals on the one side, and a chemical change of a contrary kind occurring on the metals at the other side of the portion hitherto less observed, but placed entirely beyond doubt by *Pohl's* remarkable experiments on the reaction of metals. Besides, we will direct our attention to a difference which exists between the distribution of electricity above examined, and the molecular movement now under consideration. If, for instance, the same forces, which previously effected the conduction of the electricity, and there, as it were, incorporeally without impediment strove against each other, here enter into conflict with masses, by which their free activity is restricted, a restriction which, whether we regard the electricity *de se ipso* as something material or not, must render their present velocities, beyond comparison, smaller than the former ones; therefore we cannot in the least expect that the permanent state, which we at present examine, will instantaneously occur like that above noticed, arising from the electric distribution; we have rather to expect that the permanent state resulting from the chemical equivalent of both constituents, will make its appearance only after a perceptible, although longer or shorter time.

After these remarks, we will now proceed to the determination of the separate values X and Y.

34. To obtain the value X, we have merely to bear in mind that the intensity of coherence is determined by the force with which the two adjacent constituents attract or repel each other by virtue of their electric antagonism, and consequently, as was shown in § 30, proportional to the product of the latent electroscopic forces $m\,z$ and $n\,(1-z)$ possessed by the constituents of the disc M, and is, moreover, dependent on a function to be deduced from the size, form, and distance, which we will designate by $4\,\phi$. Accordingly, when we restrict the coherence to the magnitude of the section ω,

$$X = -4\,\phi\,m\,n\,z\,(1-z)\,\omega.$$

We have placed the sign $-$ before the expression ascertained for the strength of the coherence, since a reciprocal attraction of the constituents only occurs when m and n have opposite signs; when m and n have the same signs, the constituents exert a repulsive action on each other, which no longer prevents, but promotes the decomposing force. After this remark it will at first sight be evident that a positive or negative value must be ascribed to the function ϕ, according as the expression taken for the decomposing force z is positive or negative; the sign of the function ϕ, therefore, changes when the direction of the decomposition is transposed from the one constituent to the other. The nature of the function ϕ is as little known to us as the size and form of the elements on which it is dependent; nevertheless, we may, in our inquiries, regard its absolute value as constant, since the size and form of the corporeal particles, acting on each other, must be conceived to be unchangeable so long as the two constituents remain the same, and the supposition that the two constituents constantly maintain for every chemical equivalent the same volume, renders attention to the mutual distance of the chemically different particles unnecessary, as regard has already been paid, when determining the electroscopic forces in the disc M, to the relative distances of the elements of each constituent.

35. To determine the magnitude of the reaction Y, which in the disc M opposes the latent electricity of the neighbouring discs to the decomposing force, we have nothing further to do than to substitute in the expression for z, instead of u, the sum of all the latent electroscopic forces in the disc M. Since now the sum of these latent forces is $m z + n (1 - z)$, we obtain for the determination of the force Y, which is called into existence by the change in the chemical equivalent of the constituents, and which opposes the decomposition, after due determination of its sign, the following equation:

$$\mathrm{Y} = 4 \varkappa \omega \frac{d z}{d x} \cdot i (n - m) \cdot z (1 - z) \cdot \frac{m \beta - n \alpha}{\alpha z + \beta (1 - z)}.$$

If now we substitute for x, y, and z, the values found in the equation

$$\mathrm{X} + \mathrm{Y} = \mathrm{Z},$$

we obtain, after omitting the common factor $4 z (1 - z)$, and multiplying the equation by $\frac{\alpha z + \beta (1 - z)}{i (m \beta - n \alpha)}$, as the condition of the permanent state in the chemical equivalent of the two constituents, the equation

$$0 = \varkappa \omega \frac{d u}{d x} + \frac{\phi m n}{i (m \beta - n x)} \times \cdot [\alpha z + \beta (1 - z)] \omega$$
$$- \varkappa \omega (n - m) \frac{d z}{d x},$$

which, when we put

$$\frac{\phi m n}{i (m \beta - n \alpha)} = \psi = \frac{\varkappa \phi m n}{\varkappa' (m \beta - n \alpha)},$$

passes into

$$0 = \varkappa \omega \frac{d u}{d x} + \psi \omega [\alpha z + \beta (1 - z)] - \varkappa \omega (n - m) \frac{d z}{d x}. \quad (\text{ö})$$

This equation undergoes no change, as indeed is required by the nature of the subject, when m, α, z, and n, β, $1 - z$ are respectively interchanged, and, at the same time, the sign of ϕ is changed, as according to the remark made in the preceding paragraph, must take place, since by this transformation the direction of the decomposition is transferred from one constituent to the other.

36. In order to be able to deduce from this equation the mode of the diffusion of the two constituents in the fluid, *i. e.* the value of z, we ought to know the power of conduction $\varkappa$, and the electroscopic force u at each point of the portion in the act of decomposition, the values, however, of which, are themselves dependent on that diffusion. Experience, as yet, leaves us in uncertainty respecting the change of conductibility, which occurs when two fluids are mixed in various proportions with one another, and likewise with respect to the law of tensions, which is followed by different mixtures of the same constituents in various proportion; for, if we do not err, no experiments have been instituted relatively to the latter law, and the law of the change produced in the conducting power of a fluid, by the mixture of another, is not yet decidedly established by the experiments of Gay Lussac and Davy. For this reason we have been inclined to supply this want of experience by hypothesis. We have, it is true, constantly endeavoured to conceive the nature of the action in question, in its connexion with those with whose properties we are better acquainted; but, nevertheless, we wish the determinations given to be regarded as nothing more than fictions, which are only to remain until we become by experiment in possession of the true law.

With regard to what relates to the change in the power of conduction of a body, by mixture with another, we have been guided by the following considerations. We suppose two adjacent parts of a circuit of the same section ω, whose lengths are v and w, and whose powers of conduction are a and b; then, when A is the sum of the tensions in the circuit, and L the reduced length of the remaining portion of the circuit, the magnitude of its current, which results from the above-found formulæ, is

$$\frac{A}{L + \frac{v}{a\,\omega} + \frac{w}{b\,\omega}}.$$

If now a conductor of the length $v + w$, and of the power of conduction $\varkappa$ with the same section, being taken instead of the two former, leaves the current of the circuit unchanged, then must

$$\frac{v}{a\,\omega} + \frac{\omega}{b\,\omega} = \frac{v+w}{\varkappa\,\omega},$$

whence we find

$$\varkappa = \frac{a\,b\,(v+\omega)}{b\,v + a\,w}.$$

But it is perfectly indifferent for the magnitude of the current, whether the entire length v be situated near the entire length w, or any number of discs be formed of the two, which are arranged in any chosen order, if only the extreme parts remain of the same kind, as otherwise a change might result in the sum of the tensions, consequently also in the magnitude of the current. If we extend this law, which holds for every mechanical mixture, likewise to a chemical compound, the above value found for $\varkappa$ evidently gives the conducting power of the compound, where, however, it has been taken for granted that the two parts of the circuit, even after the mixture, still occupy the same volume, for v and w are here evidently proportional to the spaces occupied by the two bodies mixed with each other.

If we now apply this result to our subject, and therefore put, instead of v and w, the values z and $1-z$, which express the relations of space of the two constituents in the disc M, we obtain, when a denotes the conducting power of the one constituent A, and b the same for the constituent B; further, $\varkappa$ the power of conduction of the mixture of the two contained in the disc M, the following expression for $\varkappa$,

$$\varkappa = \frac{a\,b}{a + (b-a)\,z}.$$

37. Having thus determined the power of conduction at each place of the extent in the act of decomposition, there only remains to be ascertained the nature of the function u at each such place; and since all tensions and reduced lengths in the part of the circuit, in which no chemical change occurs, are unalterable and given, it is, in accordance with the general equation given in § 18, which likewise holds for our present case, only requisite for the perfect knowledge of the function u, that we are able to determine the tensions and reduced lengths for each place within the extent in which the chemical change takes place.

But evidently the reduced length of the disc M is

$$\frac{d\,x}{\varkappa\,\omega};$$

or if we substitute for $\varkappa$ its value just found,

$$\frac{a + (b-a)\,z}{a\,b\,\omega}\,d\,x;$$

we accordingly obtain the reduced length of any part of that extent, if we integrate the above expression, and take the limits of the integral corresponding to the commencement and end of the part. If now we bear in mind that the integral

$$\int \frac{a + (b-a)\,z}{a\,b\,\omega}\,d\,x$$

may also be written thus:

$$\frac{l}{b\,\omega} + \frac{b-a}{a\,b\,\omega^2}\int z\,\omega\,d\,x,$$

when l represents the length of the part, over which the integral is to be extended, and $z\,\omega\,d\,x$ expresses merely the space which the constituent A in the disc M occupies; consequently $\int z\,\omega\,d\,x$, the sum of all the spaces which the constituent A fills in the part whose reduced length has to be found, it is obvious that the reduced length of the entire portion, in the act of decomposition, remains unchangeable during the chemical change, since, as we have supposed, each constituent maintains, under all circumstances, constantly the same volume. The same result may also be directly deduced from what was advanced in the preceding paragraph; however, this unchangeability only relates to the reduced length of the *entire* portion; the reduced length of a *part* of it does not in general depend merely on the actual length of this part, but likewise on the contemporaneous chemical distribution of the constituents in the extent, and must therefore, in each separate case, be first ascertained in the manner indicated.

38. We have lastly to determine the alteration in the tension of the circuit, which is produced by the chemical alteration of the extent, which has hitherto been considered. For this purpose we assume, till experience shall have taught us better, the position, that the magnitude of the electric tension between two bodies is proportional, first to the difference of their latent electroscopic forces, and secondly to a function, which we will term the *coefficient of the tension*, dependent on the size, position and form of the particles which act on each other at the place of contact. Not only from this hypothesis may be deduced the law which the tensions of the metals observe *inter se*,—nothing further being requisite than to assume the same coefficient of tension between all metals placed under similar circumstances,—but it likewise affords an explanation of the phænomenon, in accordance with which the electric tension does not merely depend on the chemical antagonism of the two bodies, but also on their relative density, and can for this reason exhibit themselves differently, even in different temperatures. For the same reasons which we have already mentioned in § 34 on the determination of the coherence which occurs between the two constituents of a mixed body, we shall likewise admit here, in the circumference of the chemically variable extent as constant, the unknown function dependent on the size, form and position of the particles in contact, and designate it by ϕ'. Since now the latent electroscopic force in the disc M, to which the abscissa x belongs, is expressed by

$$n + (m - n)\, z,$$

and that in the disc M′, to which the abscissa $x + d\,x$ belongs, by

$$n + (m - n)\, z + (m - n)\, d\, z,$$

the tension originating between the discs M and M′ is

$$-\, \phi' (m - n)\, d\, z\,;$$

consequently the sum of all the tensions produced throughout a portion exposed to chemical change

$$-\, \phi' (m - n)\, (z'' - z'),$$

when z' and z'' represent those values of z, which belong to the commencement and end of the extent in question.

But the tension of the circuit undergoes, besides the change just explained, a second one, from the extremities of the chemically changeable portion, which are in connexion with the other chemically unchangeable parts of the circuit, undergoing a gradual change during the decomposition till they arrive at their permanent state, giving rise at those places to an altered tension. If, for instance, we call ζ the value of z, which belongs to all places of the extent in question, before chemical change has begun in it, and designate the coefficient of the tension occurring at the extremities of this extent, supposing that it is the same at both ends, by ϕ'', and moreover express by μ and ν the latent electroscopic forces of those places of the chemically unalterable part of the circuit which are situated adjacent to the chemically changeable extent, the tensions existing at these places can be determined individually. They are, namely, previous to the commencement of chemical change, the following:

$$\phi'' \left[\mu - (n + (m - n)\, \zeta)\right], \text{ and}$$

$$\phi'' \left[(n + (m - n)\, \zeta) - \nu\right]\,;$$

and after the permanent state in the decomposition has been attained, if we, as above, let z' and z'' be those values of z which belong in this state to those places, they are the following:

$$\phi'' \left[\mu - (n + (m - n)\, z')\right], \text{ and}$$

$$\phi'' \left[(n + (m - n)\, z'') - \nu\right],$$

their sum is therefore in one case

$$\phi'' (\mu - \nu),$$

and in the other

$$\phi'' (\mu - \nu) + \phi'' (m - n)\, (z'' - z')\,;$$

consequently the increase of tension at those places is

$$\phi'' (m - n)\, (z'' - z').$$

If we add this change of the tension to that above found, we obtain for the entire difference of the tension, produced by the decomposition until the commencement of the permanent state,

$$(\phi'' - \phi')\, (m - n)\, (z'' - z'),$$

which, if we substitute ϕ for $\phi'' - \phi'$, changes into

$$\Phi\ (n - m)\, (z'' - z').$$

If now we represent by S the magnitude of the current, and by A the sum of the tensions in the circuit, before any chemical change has commenced, by S′ the magnitude of the current, after the permanent state has been attained; lastly, by L the reduced length of the entire circuit, which, as we have seen, remains under all circumstances the same, it results

$$S' = \frac{A - \Phi\,(n-m)\,(z''-z')}{L};$$

or, if we write for $\frac{A}{L}$ its equivalent S,

$$S' = S - \frac{\Phi\,(n-m)\,(z''-z')}{L},$$

so that, therefore, $\frac{\Phi\,(n-m)\,(z''-z')}{L}$ designates the decrease produced in the magnitude of the current by the chemical alteration.

39. After all these intermediate considerations, we now proceed to the final determination of the chemical alteration in the changeable portion, and the change of the current in the whole circuit produced by this chemical alteration, where, however, we have constantly to keep in view only the permanent state of the altered portion. If we substitute in the equation (♁) given in § 35, for $\varkappa\,\omega\,\frac{d\,u}{d\,x}$ its value S′, which, as we have just found, is solely dependent on the fixed and unalterable values of z, and therefore has to be treated in the calculation as a constant magnitude; further, for $\varkappa$ its value $\frac{a\,b}{a+(b-a)\,z}$, given in § 36, this equation changes into

$$0 = S' + \psi\,\omega\,\beta + \psi\,\omega\,(\alpha-\beta)\,z - \frac{a\,b\,\omega\,(n-m)}{a+(b-a)\,z}\cdot\frac{d\,z}{d\,x};$$

or if we place $S' + \psi\,\omega\,\beta = \Sigma$, and $\psi\,\omega\,(\alpha-\beta) = \Omega$, into

$$0 = \Sigma + \Omega\,z - \frac{a\,b\,\omega\,(n-m)}{a+(b-a)\,z}\cdot\frac{d\,z}{d\,x},$$

from which, by integration, we deduce the following:

$$c = \frac{(b-a)\,\Sigma - a\,\Omega}{a\,b\,\omega\,(n-m)}\,x + \log\,\frac{\Sigma + \Omega\,z}{a+(b-a)\,z},$$

where c represents a constant remaining to be determined. If we designate by χ the abscissa of that place of the chemically changed portion for which z has still the same value, which, previous to the commencement of the chemical decomposition, belonged to each place of this portion, for which therefore $z = \zeta$,

and determine in accordance with this statement the constant c, our last equation acquires the following form:—

$$\frac{\Sigma + \Omega\,z}{a+(b-a)\,z} = \frac{\Sigma + \Omega\,\zeta}{a+(b-a)\,\zeta}\cdot e^{\frac{(b-a)\,\Sigma - a\,\Omega}{a\,b\,\omega\,(n-m)}(\chi - x_{,})}$$

where e denotes the base of the natural logarithms. The following consideration leads to the determination of the value χ. Since, namely, ζ represents the space which the constituent A occupies in each individual disc of the changeable portion previous to the commencement of the chemical decomposition, if we denote by l the actual length of this portion, $l\,\zeta$ expresses the sum of all the spaces which the constituent A occupies on the entire expanse of the changeable portion; but this sum must constantly remain the same, since, according to our supposition, no part of either of the constituents is removed from this portion, and both maintain, under all circumstances, the same volume, even after chemical decomposition has taken place; we obtain, therefore,

$$l\,\zeta = \int z\,d\,x,$$

where for z is to be substituted its value resulting from the previous equation, and the abscissæ corresponding to the commencement and end of the changeable portion are to be taken as limits of the integral.

These two last equations, in combination with that found at the end of the previous paragraph, answer all questions that can be brought forward respecting the permanent state of the chemical alteration, and the change in the electric current thus produced, and so form the complete base to a theory of these phænomena, the completing of the structure merely awaiting a new supply of materials from experiment.

40. At the conclusion of these investigations we will bring prominently forward a particular case, which leads to expressions that, on account of their simplicity, allow us to see more conveniently the nature of the changes of the current produced by the chemical alteration of the circuit. If, for instance, we admit $a = b$, and $\alpha = \beta$, the differential equation obtained in the preceding paragraph changes into the following:

$$0 = \Sigma\,d\,x - a\,\omega\,(n-m)\,d\,z,$$

whence we obtain by integration

$$z-\zeta=\frac{\Sigma\,(x-\chi)}{a\,\omega\,(n-m)},$$

when χ designates the value of x, for which $z=\zeta$. Since in this case the value of z constantly changes to the same amount on like differences of the abscissæ, the abscissa χ, which belongs to its mean value ζ, as it was at all places of the changeable portion previous to the commencement of the chemical decomposition, must be referred to the middle of this portion. If, therefore, z' and z'', as above, represent the values of z, which correspond to the commencement and end of the variable portion, and l the actual length of this portion, it follows, from our last equation, that

$$z''-\zeta=+\tfrac{1}{2}\,\frac{l\,\Sigma}{a\,\omega\,(n-m)},$$

and

$$z'-\zeta=-\tfrac{1}{2}\,\frac{l\,\Sigma}{a\,\omega\,(n-m)},$$

and from these two equations results

$$(n-m)\,(z''-z')=\frac{l}{a}\,.\,\Sigma\,;$$

or, if we put, instead of $\frac{l}{a\,\omega}$, by which here nothing further is expressed than the unchangeable reduced length of the chemically variable portion, the letter λ, the following:

$$(n-m)\,(z''-z')=\lambda\,\Sigma.$$

If we place this value of $(n-m)\,(z''-z')$ in the equation found in § 38,

$$S'=S-\frac{\Phi\,(n-m)\,(z''-z')}{L},$$

and at the same time substitute for Σ its value $S'+\psi\,\omega\,\alpha$, we obtain

$$S'=S-\frac{\Phi\,\lambda}{L}\,(S'+\psi\,\omega\,\alpha),$$

an equation, the form of which is extremely well suited to indicate in general the nature of the change of the current produced by the chemical alteration, and the expressions of which coincide exceedingly well with the numerous experiments I have made on the fluctuation of the force in the hydro-circuit, and of which only a small part have been published*.

* Schweigger's *Jahrbuch*, 1825, Part 1 and 1826, Part 2.

ARTICLE XIV.

Selections from a Memoir on the Expansion of Dry Air. By the late Professor F. RUDBERG.

[From Poggendorff's *Annalen*, B. 41. S. 271.]

AMONG the constants in physics there is certainly not one which is usually considered to be determined with greater precision than the expansion of dry air, or of dry gases generally, under a constant pressure, between the standard points of the thermometer scale. The numerous experiments made by Dalton and Gay Lussac, almost at the same time, about the beginning of the present century, appeared to show, beyond all doubt, that the amount of this expansion from 0° to 100° C., under a constant pressure, was 0·375 of the volume of the air at 0°. Their great skill in experimenting, and the magnitude and number of the services they had rendered science, left no room for any doubt as to the accuracy of this result; consequently, for more than thirty years in all computations in which the expansion of gas occurs, it has been assumed to be 0·375.

The constant in question is undeniably of the greatest importance in Physics, since it forms the basis of all methods of measuring temperature; it is used in the explanation of most of the phænomena caused by heat; and lastly, is requisite in the reduction of many observations in Physics and other sciences; as, for example, in determining the velocity of sound, in the measurement of heights by means of the barometer, and in computing astronomical refractions. This being the case, it will no doubt appear surprising, that the value of this constant, which has been employed up to the present time, is erroneous to no small amount, since, as will be shown in this memoir, it appears to be not more than from 0·364 to 0·365, instead of 0·375.

The change of volume produced by heat can be determined, either by heating cold air and measuring the increase of its volume, or by cooling warm air and determining the diminution of its volume. I have adopted the latter method, as being by far the most accurate.

In most of the experiments, a glass globe, having a neck made of thermometer tube A B C (fig. 1.), and capable of con-