Lecture/laboratory #2

Outline

1. Review of major points of previous lecture
2. Review some problems in 1st laboratory
   - Connection between lectures and laboratory works
3. Voltage and current as complex numbers
4. RC filters; differentiators and integrators; resonances

Review of major points of 1st lecture

(a) Capacitor vs. inductor

C: $$I = C\frac{dV}{dt}$$

$$V(t) = V_0 \sin\omega t$$

$$I(t) = CV_0\omega\cos\omega t =$$

$$= I_0 \sin(\omega t + 90°)$$

L: $$V = L\frac{dI}{dt}$$

$$V(t) = LI_0\omega\cos(\omega t + 90°) =$$

$$= V_0^* \sin(\omega t + 180°)$$

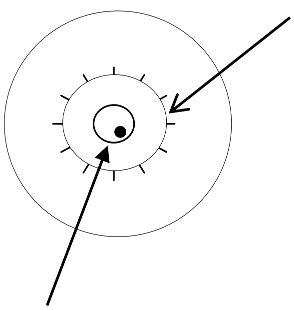
Review of major points of 1st lecture(cont.)

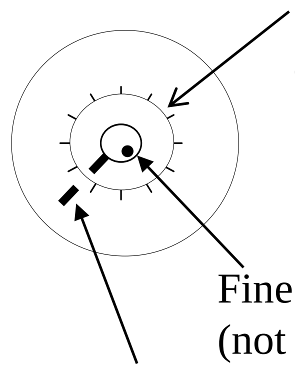
(b) Oscilloscope

y – t Mode: y – signal (vertical)
t – time (horizontal)

y: Channels: A, B

- Position (voltage offset)





“Triggering: Internal (auto) → Level of triggering

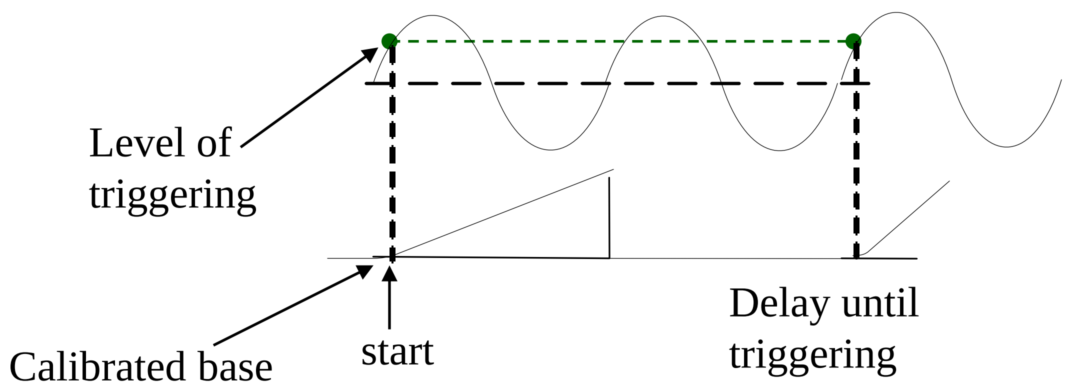
DC – For low frequency signal. Use it always when constant offset is not important

Oscilloscope (cont.)

AC – strips constant voltage → signal is going through capacitor.
Use when frequency is sufficiently high and you want to avoid constant voltage contribution

External trigger - level of triggering → Ex $\begin{cases} 1 \\ :10 \end{cases}$

Internal triggering



For x – y mode (e.g. for curve tracers) use DC coupling

Check channels or probe by using calibrated voltage = 1.2 V (left upper corner of the front panel).

Signals from two channels can be added to subtracted ("+" and "-" switch)

Oscilloscope (cont.)

If you want to compare two signals, gain on each channel (its sensitivity) has to be set up the same.

Very often your signal through coaxial cable is grounded by external shielding of the cable, hence in reality you measure

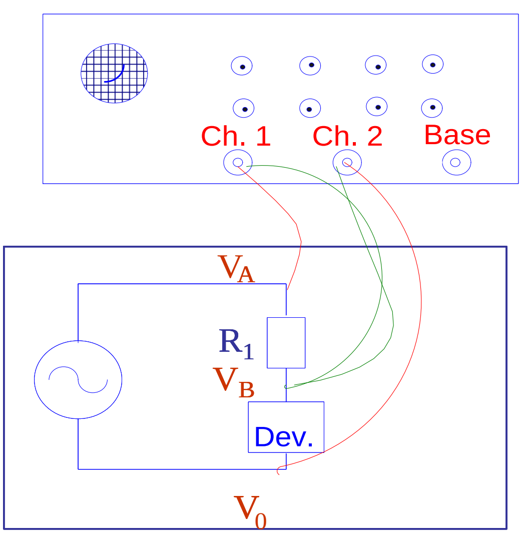
$V_A - V_B$ as

$(V_A - V_{Gr}) - (V_B - V_{Gr})$

Impedance of the scope

Input impedance: is quite high, but can be increased by using probe $\Rightarrow$ voltage divider

I vs. V curves



$$V_A^* = V_A - V_B \qquad V_B^* = +(V_0 - V_B)$$

$$\boxed{I_R = \frac{V_A^*}{R_1} = \frac{V_A - V_B}{R_1}} \qquad \boxed{V_0 \neq 0}$$

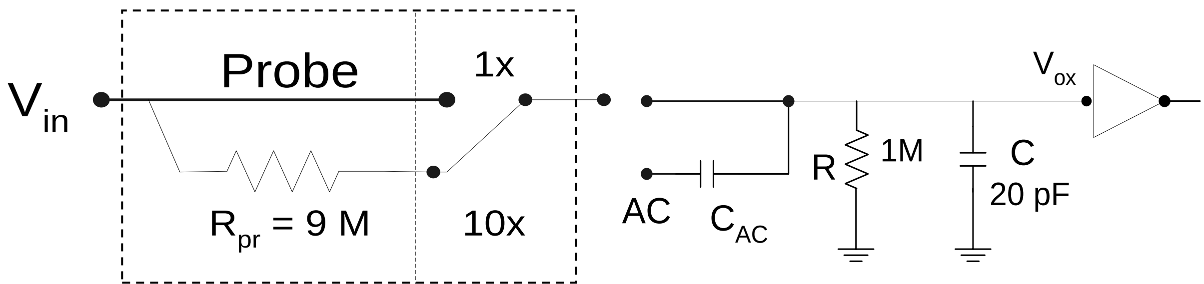
$$-V_B^* = V_B - V_0$$

$$I_{Dev} = -\frac{V_B^*}{R_{Dev}} = \frac{V_B - V_0}{R_{Dev}} = \frac{\Delta V_{Dev}}{R_{Dev}}$$

Impedance of the scope (cont.)

$$I_{Dev} = I_{R_1} \Rightarrow I_{Dev} = f(V_B^*) = f(V_B)$$

Input impedance (cont.)



$$V_{osc} = V_{in}\frac{R}{R_{pr}+R} = V_{in}\frac{1M}{9M+1M} = V_{in}\frac{1}{10}$$

General: $V_{osc} = V_{in}\dfrac{Z_2}{Z_1+Z_2}$ $\rightarrow$ hence important Z_c

$$Z_c = \frac{1}{i\omega c}$$

$$\tau = RC = 10^6 \times 20 \times 10^{-12}\ \text{sec} = 20\mu\text{sec}$$

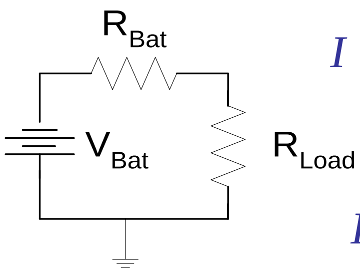
$\omega = 50$ kHz $(Z_c = 1\ M\Omega)$

If $R = 50\ \Omega \Rightarrow \tau = 1$ nsec

$\omega = 1$ GHz $(Z_c = 50\ \Omega)$

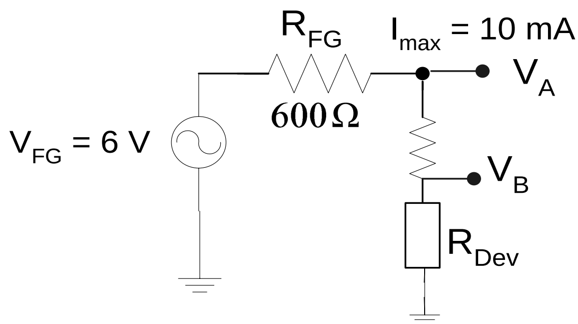
Impedance of the scope (cont.)

Output impedance of function generator



$$I = \frac{V_{Bat}}{R_{Bat} + R_L}$$

$$I_{\max} = \frac{V_{Bat}}{R_{Bat}}$$



If $I \to I_{max} \Rightarrow$ non-linear output

Laboratory: (a) Verify oscilloscope input impedance

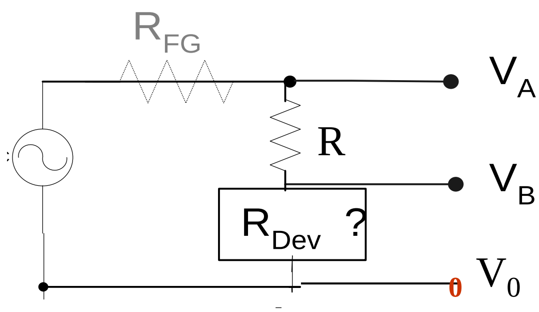
(b) Verify function generator output impedance

Comments on 1st laboratory work

Common mistakes:

Often Y-t mode has been used instead of x –y mode : time base was on; V_B was not put on x: you should remember that you have to measure $I = f(V_B)$, where $f(V_B) = 1/R\,(V_A - V_B)$. Hence, by measuring $V_A - V_B = IR$ you measure ***current*** with coefficient of proport. R. Therefore V_B should be on channel B (with “-”) and *x* (channel with time base)

Comments on 1st laboratory work (cont.)

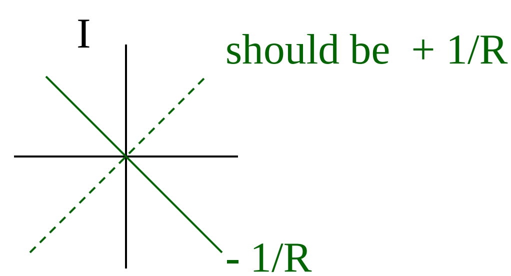


$\rightarrow$ to channel A (y_1)

$\rightarrow$ to channel B (y_2) (“-”)

$\rightarrow$ to channel x

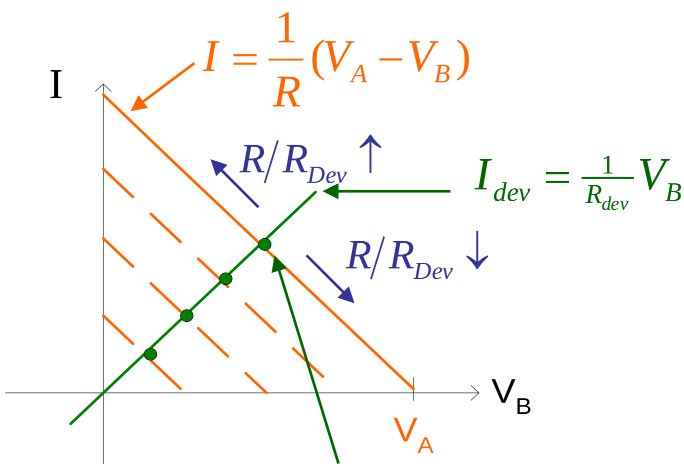
- Using AC for channel A or B (or both) for X – Y plots $\Rightarrow$ at lower ν impedance Z_c start to play significant role and you see ellipsoidal curve instead of line for R
- Too high R and R_{Dev} (comparable with impedance of the scope), hence scope impedance play important role in voltage divider
- Wrong polarity on one of the channels $\Rightarrow$ “negative” resistance



Comments on 1st laboratory work (cont.)

Curve Tracers

R_{dev} = Resistor (R_{dev})



oper. points: $I = \frac{V_A - V_B}{R} = \frac{V_B}{R_{dev}}$

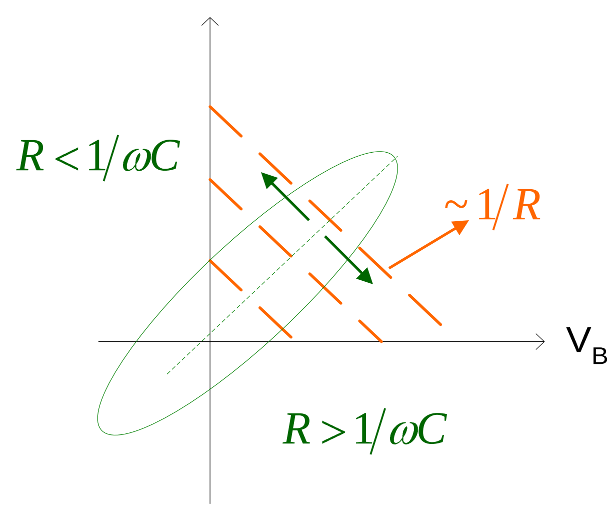
Device: Capacitor ($R_{dev} = Z_c$)

$V_A = f_1(\omega), \quad V_B = f_2(\omega)$

$$f_2(\omega) = (V_{B_0})\sin \omega t$$

$\Rightarrow Z_c = f_3(\omega)$

$$I = C\frac{dV_B}{dt} = C\omega V_{B_0} \cos \omega t$$



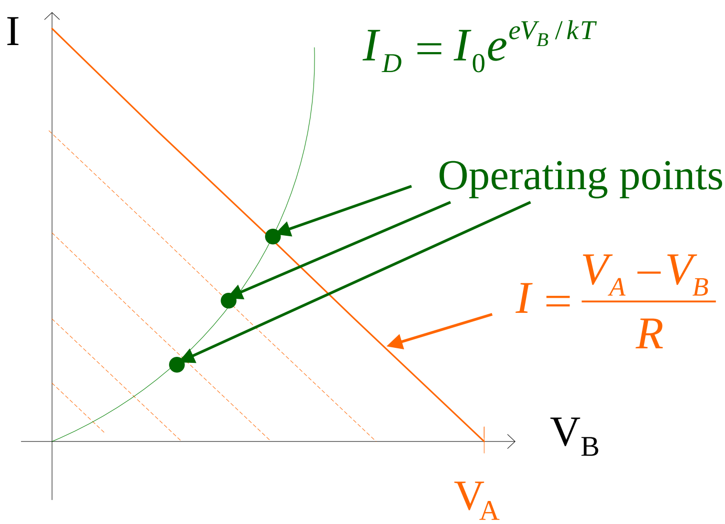
oper. points: $\frac{V_A - V_B}{R} = \frac{V_B}{Z_c}$

Comments on 1st laboratory work (cont.)

$$y \Rightarrow \left(\frac{V_{A_0} - V_{B_0}}{R} \right) \sin \omega t \quad ; \quad x \Rightarrow \omega C \times V_{B_0} \cos \omega t$$

$$\omega C = \frac{1}{R} \Rightarrow V_{B_0} = \tfrac{1}{2} V_{A_0} \Rightarrow \ circle$$

Device: Diode



Operating points:

$$\frac{V_A - V_B}{R} = I_0 e^{eV_B/kT}$$

Voltage and Current as Complex Numbers

(closely follows Horowitz & Hill)

In Lecture 1 we discussed phase shift between voltage and current in circuit with capacitor, driven by a sine wave at some frequency. When the circuit contains only linear elements (resistors, capacitors, inductors) the magnitude (but not phase) of the currents everywhere in the circuit is still proportional to the magnitude of the driving voltage. Therefore it is possible to generalize Ohm's law if we will provide information not only about amplitudes but also about phase shifts.

It is possible to use, as we did up to now, trigonometric functions

$$V(t) = V_0 \sin(\omega t + \phi)$$

However it is simpler (and more elegant) to use complex numbers $\Rightarrow$ it is easier to add and subtract complex numbers than trigonometric functions. Because V and I are real quantities, we need procedure for converting actual I & V to their representation in complex numbers and transfer back to actual quantities. So the procedure is as follow:

> Voltages and currents are represented by the complex quantities **V** and **I** in such a way that voltage and current amplitudes and phase shifts are represented by complex number $\underline{V_0 e^{i\phi}}$ and $\underline{I_0 e^{i\phi}}$. Actual voltage and current are recovering from complex representation

Voltage and Current as Complex Numbers (cont.)

by multiplying them by $e^{i\omega t}$ and taking the real part:

$V(t) = \mathrm{Re}(\mathbf{V}e^{i\omega t})$; $I(t) = \mathrm{Re}(\mathbf{I}e^{i\omega t})$

In explicit form:

$$V(t) = \mathrm{Re}(\mathbf{V}e^{i\omega t}) = \mathrm{Re}(\mathbf{V})\cos\omega t - \mathrm{Im}(\mathbf{V})\sin\omega t$$

$$I(t) = \mathrm{Re}(\mathbf{I}e^{i\omega t}) = \mathrm{Re}(\mathbf{I})\cos\omega t - \mathrm{Im}(\mathbf{I})\sin\omega t$$

Example: $\mathbf{V} = 5\,i \rightarrow$ what is real voltage vs. time ?:

$$V(t) = \mathrm{Re}(\mathbf{5i}e^{i\omega t}) = -5\sin\omega t$$

Application to capacitors and inductors

Now we can apply Ohm's law to capacitors and inductors as for resistors using reactance. In order to find reactance we can write

$$V(t) = Re(V_0 e^{i\omega t}) \quad , \quad \varphi = 0$$

For capacitor:

$$I(t) = C\frac{dV}{dt} = -V_0 C\omega\sin\omega t = \mathrm{Re}\left(\frac{V_0 e^{i\omega t}}{1/i\omega C}\right) = \mathrm{Re}\left(\frac{V_0 e^{i\omega t}}{\mathbf{Z_C}}\right)$$

Application to capacitors and inductors (cont.)

Hence, for capacitor $\mathbf{Z_C} = 1/i\omega C$, which was obtained in more rigorous way than in previous lecture.

Similarly for inductor: $\mathbf{Z}_L = i\omega L$

Generalized Ohm's Law:

$\mathbf{I} = \mathbf{V} / \mathbf{Z}$ with impedance $\mathbf{Z}$ as we already know

RC Filters

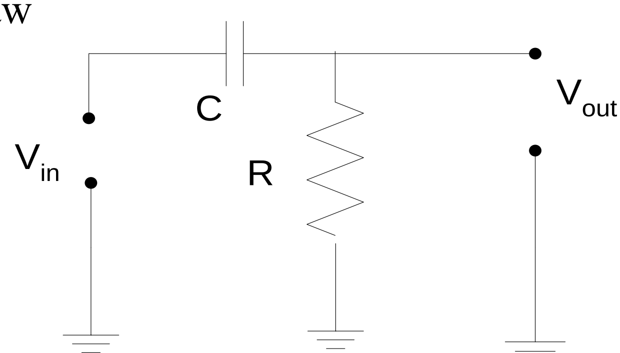
RC filters: for rejection undesired signal frequencies and passing needed one.

This are simple filters → more sophisticated will be later.
RC filter is combination of R and C: for C resistance is $f(\omega)$ $(\mathbf{Z_C} = 1/i\omega C)$, therefore possible voltage divider sensitive to frequency (ω).

From generalized Ohm's law

$$\mathbf{I} = \frac{\mathbf{V}_{in}}{\mathbf{Z}_{tot}} = \frac{\mathbf{V}_{in}}{R + \frac{1}{i\omega C}} = $$

$$= \mathbf{V}_{in} \frac{R - \frac{1}{i\omega C}}{R^2 + \frac{1}{\omega^2 C^2}}$$



High-pass RC filter

(rejecting low ω)

RC Filters – High-Pass filters (cont.)

and

$$\mathbf{V}_{out} = \mathbf{I}\, Z_R = \mathbf{V}_{in} R \left(R - \frac{1}{i\omega C} \right) \Big/ \left(R^2 + \frac{1}{\omega^2 C^2} \right)$$

Neglecting phase (as in most cases)

$$V_{out} = (\mathbf{V}_{out} \times \mathbf{V}^*_{out})^{1/2} = V_{in} \frac{R}{\left(R^2 + \frac{1}{\omega^2 C^2} \right)^{1/2}}$$

Compare with resistive divider

$$V_{out} = V_{in} \frac{R_1}{R_1 + R_2}$$

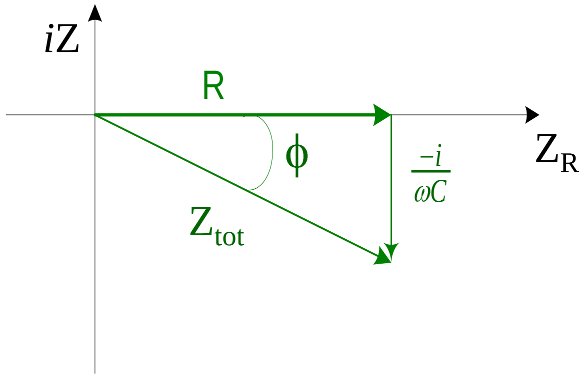
$$\omega = 2\pi\nu \Longrightarrow V_{out} = V_{in} \frac{2\pi\nu RC}{\left[+ (2\pi\nu RC)^2 \right]^{1/2}}$$

This results $\rightarrow$ directly from complex impedance:

$\mathbf{Z}_C$ on imaginary axis (vertical)

Z_R on real axis (horizontal)

RC Filters – High-Pass filters (cont.)



$$\mathbf{Z}_{tot} = R - \frac{i}{\omega C}$$

$$\underbrace{(Z_{tot.}) = \left(R^2 + \frac{1}{\omega^2 C^2}\right)^{1/2}}$$

For $1/\omega C = R$

$$Z_{tot} = \sqrt{2}\,R \quad ; \quad \phi = 45^\circ$$

$$\phi = \tan^{-1}\left(\frac{-1/\omega C}{R}\right)$$

For $\omega = 1/RC$ $\rightarrow$ $V_{out} = 0.7\ V_{in}$ $|20\log(V_{out}/V_{in})| = 3\text{dB}$

$$\boxed{\omega_{3dB} = \frac{1}{RC} \qquad \nu_{3dB} = \frac{1}{2\pi RC}}$$

Frequency response of high-pass filter: