

Lecture/laboratory #1

Resistors

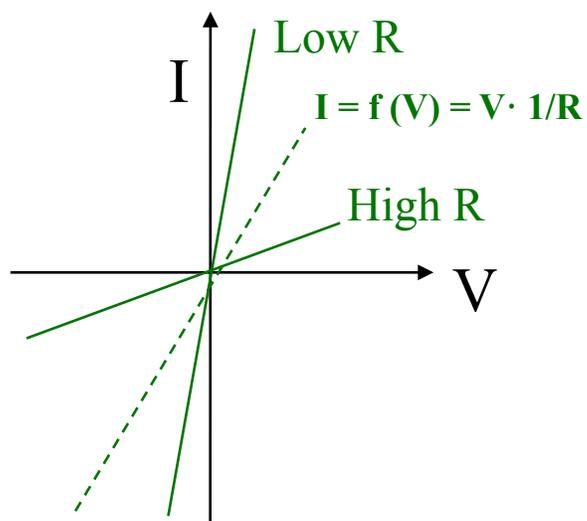
$$V = RI ; \quad I = (1/R)V = \sigma V$$

$$\left. \begin{array}{l} V(t) = A \sin \omega t \\ I(t) = B \sin \omega t \end{array} \right\} \rightarrow R = V/I = A/B \leftarrow \text{perfect resistor}$$

No perfect resistors:

- Inductance inherent in wires (creation magnetic field B)
- Capacitance
- Changes resistance with temperature

I-V Chart



Resistors in Series and Parallel Voltage Dividers

In series: $R = R_1 + R_2 + \dots$

Resistors in series provide larger resistor R

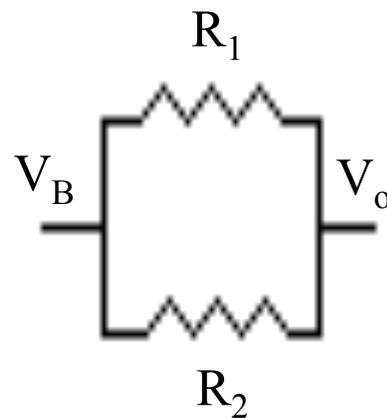
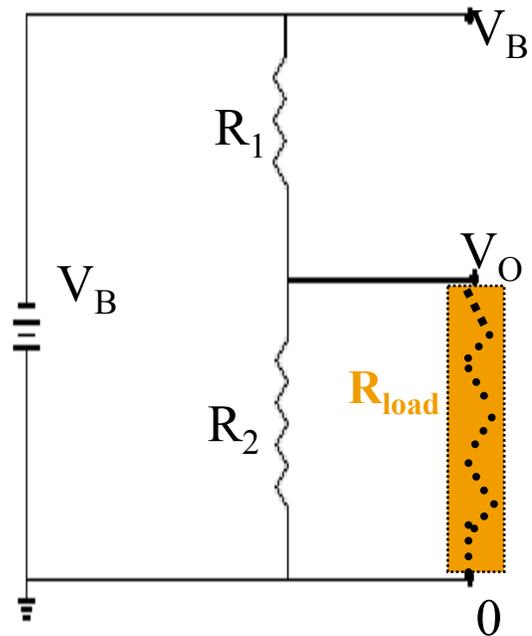
Two resistors in parallel:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R = \frac{R_1 R_2}{R_1 + R_2}$$

$1/R = \sigma \leftarrow$ conductivity

$$\sigma = \sigma_1 + \sigma_2$$

Resistors in parallel provide smaller resistor R



Resistors in Series and Parallel Voltage Dividers(cont)

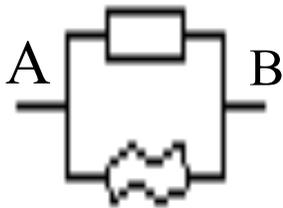
Both follows from Kirchhoff laws:

1. The sum of the currents into point (node) in the circuit equal the sum of the currents out (conservation of charge)

$$\sum I_{node} = 0 \rightarrow \text{For a series circuit the current is the same everywhere}$$

2. The sum of the voltage drops around any closed circuit is zero

$$\sum_{ckt} \Delta V = 0 \quad \text{The voltage drops from A to B is the same for each path and equals voltage between A and B}$$



Demonstration of application of Kirchhoff's law to parallel resistors:

$$I_1 + I_2 - I = 0$$

$$I_1 = \frac{V_B - V_o}{R_1}; \quad I_2 = \frac{V_B - V_o}{R_2}; \quad I = \frac{V_B - V_o}{R}$$

$$\Downarrow$$
$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$$

Power Loss in Resistors:

$$P = VI = RI^2 \text{ (Watt = Joule/sec)}$$

Voltage Dividers:

$$I = V_B / (R_1 + R_2); \quad V_o = IR_2 = V_B - IR_1$$

$$V_o = V_B R_2 / (R_1 + R_2) \leftarrow \text{Output voltage is fraction of input } V$$

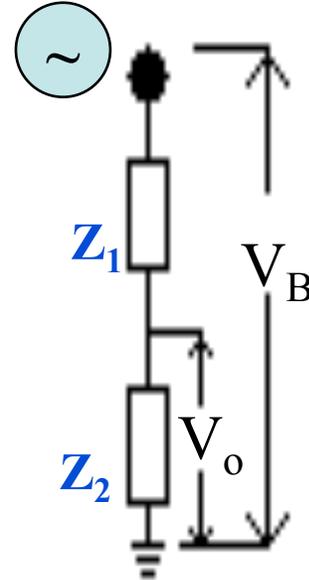
Demonstration of application of Kirchhoff's law to parallel resistors:

In more general form:
R replaced by impedance Z

$$V_o = V_B (Z_2 / (Z_1 + Z_2))$$

Impedance:

Capacitor $\rightarrow Z_c = 1/(i\omega C)$;
 $\omega = 2\pi\nu$; $i = \sqrt{-1}$
 ν is frequency
 $\nu = 1/\tau$

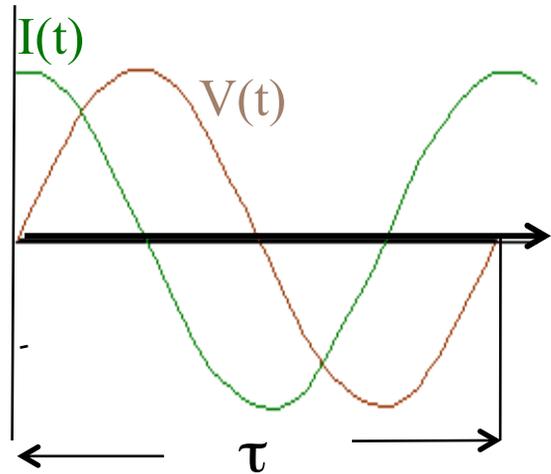


Inductor $\rightarrow Z_L = i\omega L$

Hence: $\begin{matrix} Z_R = R \\ Z_c = 1/(i\omega C) \\ Z_L = i\omega L \end{matrix} \Rightarrow V = ZI$

However:

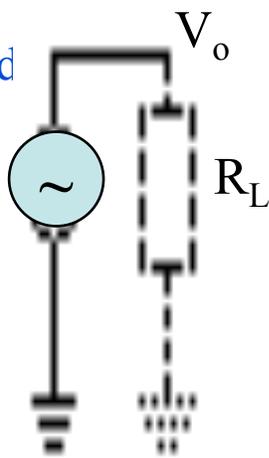
$$\overline{P} \neq ZI^2 \rightarrow \text{e.g. } \overline{Z_c I^2} = 0$$



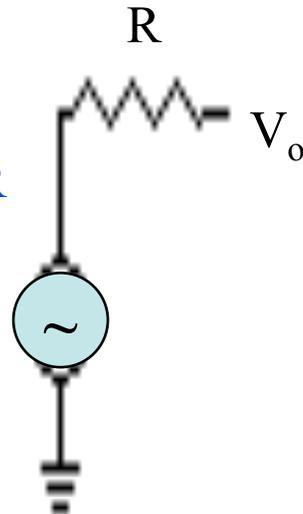
$$\overline{P} = \overline{VI} = \frac{1}{\tau} \int_0^{\tau} V(t)I(t)dt = 0$$

Voltage Sources

Ideal:
 R_L should
be high



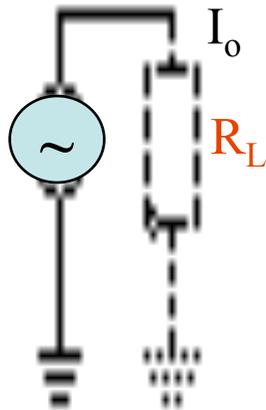
Actual:
Internal
resistance R



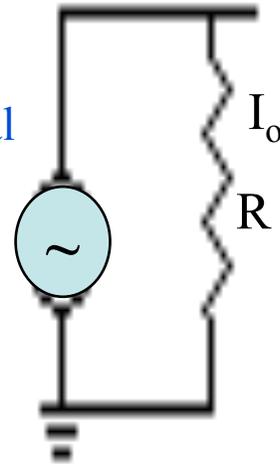
$$V_{\text{const.}} = IR_L \text{ for different } I \text{ and } R_L \text{ (for limited } I \text{ and } R)$$

Current Sources

Ideal:
 R_L should
be low



Actual

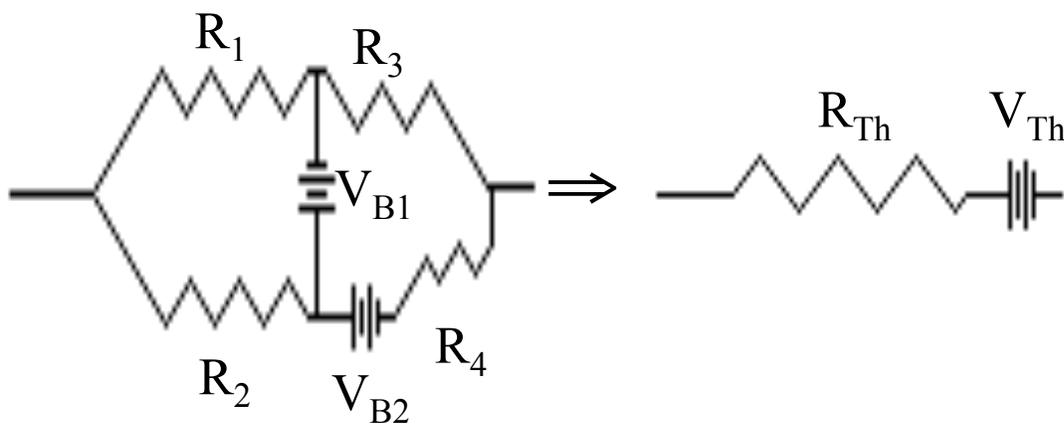


$$I_{\text{const.}} = V/R_L \text{ for different } V \text{ and } R_L$$

Equivalent Ckt.: Thévenin's Theorem

Thévenin's Theorem:

Any two-terminal network of resistors and voltage sources is equivalent to a single resistor R_{Th} in series with a single voltage source V_{Th}



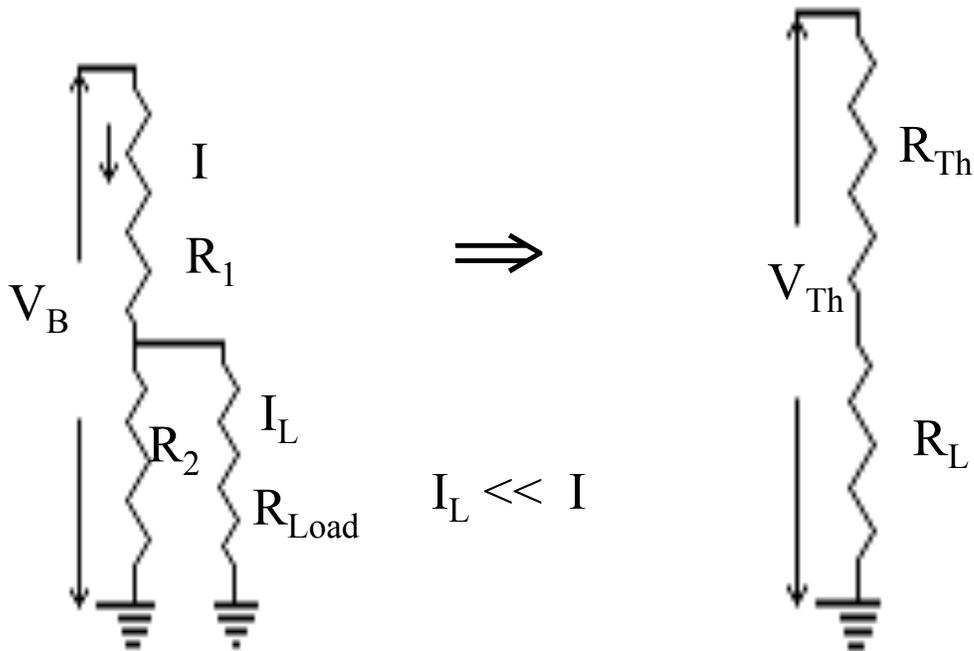
$$V_{Th} = V \text{ (Open Circuit Voltage)}$$

$$R_{Th} = V(\text{Open Circuit Voltage}) / I(\text{Short Circuit})$$

Note: *It can be generalized to impedance as:*

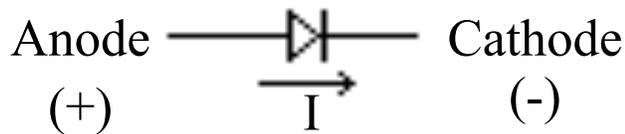
$$R_{Th} \Rightarrow Z_{Th}$$

Application of Thévenin's Theorem to Voltage Divider

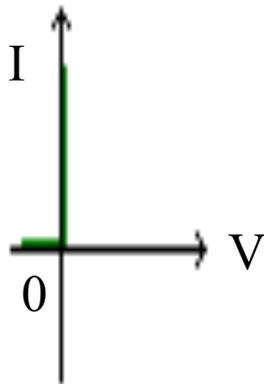


$$\left. \begin{aligned} V(\text{O.C.}) &= V_B \left(\frac{R_2}{R_1 + R_2} \right) \\ I(\text{S.C.}) &= V_B / R_1 \end{aligned} \right\} \Rightarrow \begin{cases} V_{\text{Th}} = V_B \left(\frac{R_2}{R_1 + R_2} \right) \\ R_{\text{Th}} = (R_1 R_2) / (R_1 + R_2) \end{cases}$$

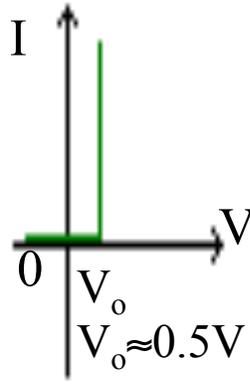
Diodes $I = I_0 e^{eV/kT}$; e is the electrical charge and T is the temperature



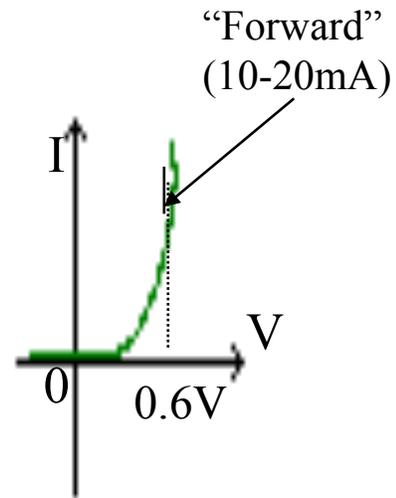
Diodes (cont'd)



0th order approx.



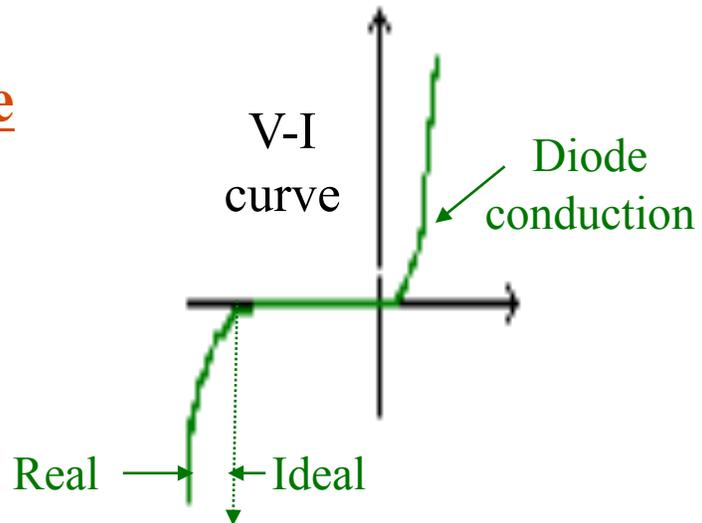
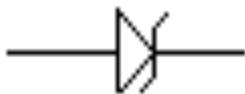
1st order approx.



2nd order approx. (actual)

Current flows for $V_o \geq +0.5V$

Zener Diode

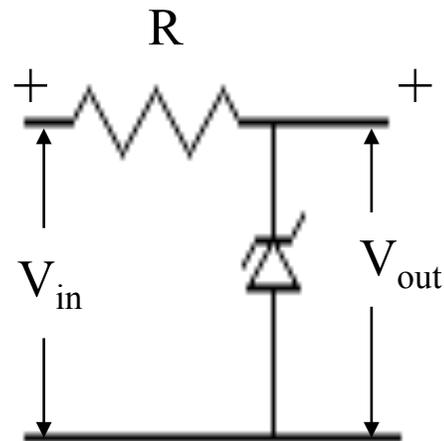


Zener: to create constant voltage inside a circuit (somewhere) by providing them with roughly constant current

Zener Diode(cont.)

Zener resistance will change with changing of the driving current: dynamic resistance (specification of zener : resistance given at a certain current)

***Note:** dynamic resistance R_{dyn} of a zener diode varies roughly in inverse proportion to current*



Definition equation of $R_{dyn} \Rightarrow \Delta V = R_{dyn} \Delta I$

Example: $R_{dyn} = 10 \Omega$ at $I = 10 \text{ mA}$ and $V = 5 \text{ V}$

If $\Delta I/I = \underline{10\%}$ then

$$\Delta V = 10 \Omega \times 0.1 \times 10^{-2} \text{ A} = 10 \text{ mV}$$

$$\Delta V/V = 10 \text{ mV} / 5 \text{ V} = \underline{0.2\%}$$

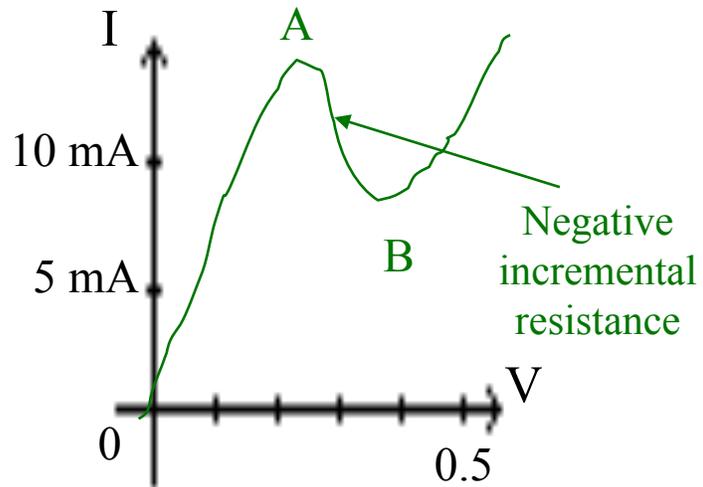
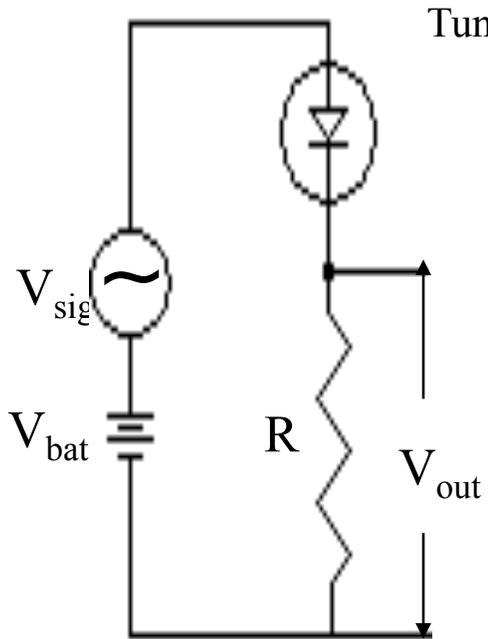
Good voltage regulating ability!

Power loss in zener:

$$P_{zener} = 1 \text{ A} \times 10 \text{ V} = 10 \text{ W}$$

Tunnel Diode

Also called Esaki diode



$$I = V_{out}/R \rightarrow \Delta I = \Delta V_{out} / R$$

$$\Delta I = [\Delta(V_{sig} + V_{bat}) - \Delta V_{out}] / R_{dyn}$$

$$\Delta V_{out}/R = (\Delta V_{sig} - \Delta V_{out})/R_{dyn}$$

$$\Delta V_{out}(1+(R_{dyn}/R)) = \Delta V_{sig}$$

or

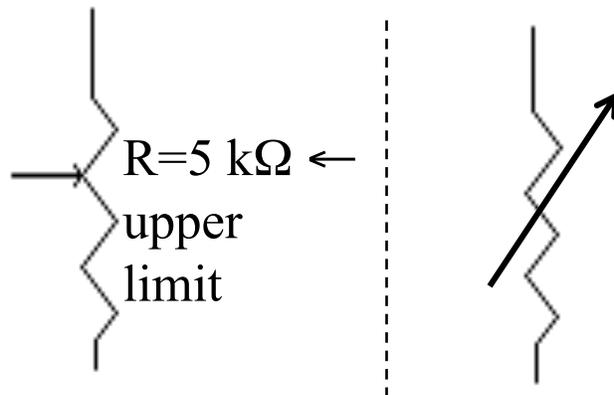
$$\Delta V_{out} = R/(R+R_{dyn}) \Delta V_{sig}$$

Equation of voltage divider

$R_{dyn} < 0$ between A and B.

If $R_{dyn} \approx -R$, then amplifier

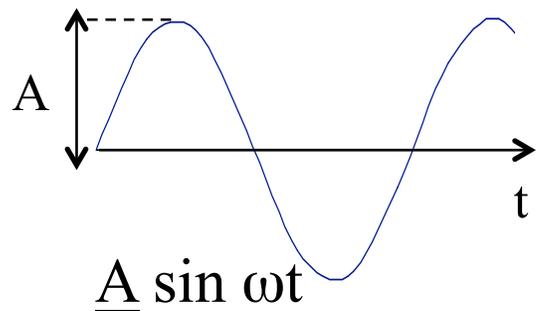
Potentiometer (“Pot”)



Signal Amplitudes

① Peak - to - peak amplitude
(pp amplitude) = $2 A$

② Root - mean- square amplitude
(RMS amplitude) = $(1/\sqrt{2})A =$
 $0.707 A$



Example: In US $\rightarrow V = 117 \text{ V RMS}, \nu = 60 \text{ Hz}$
Amplitude $V_A = 165 \text{ V (330 V PP)}$

Decibels

Logarithmic measure of a ratio of two signals

$$\text{dB} = 20 \log_{10} (A_2/A_1); \quad \text{e.g.} \quad A_2/A_1 = 2 \Rightarrow 20 \times 0.3 = 6 \text{ dB}$$

$$\text{For power ratio:} \quad A_2/A_1 = 10 \Rightarrow P_2/P_1 = (A_2/A_1)^2 = 10^2$$

$$\text{dB} = 10 \log_{10} (P_2/P_1) \quad \rightarrow \quad \text{dB} = 20$$

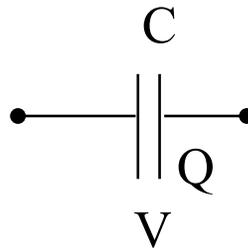
Capacitors

Capacitor: charge (Q) storage

Might be considered as a

frequency-dependent resistor

$$Z_c = 1/(i\omega C)$$

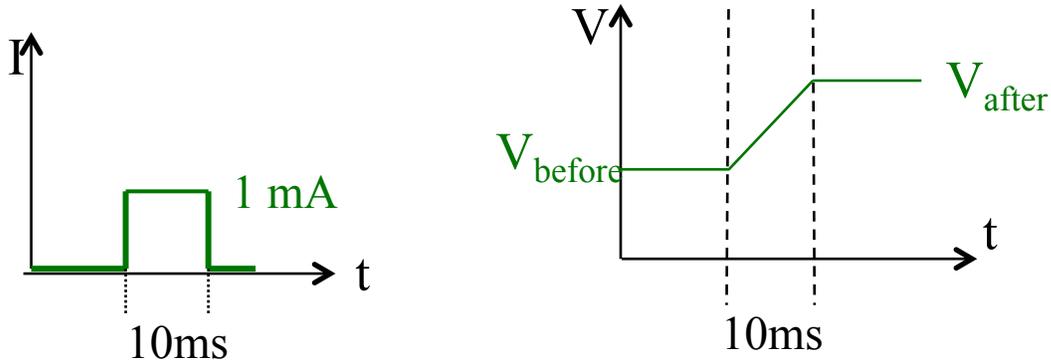


However cannot dissipate power (energy) because V and I are 90° out of phase

A capacitor of C Farads (F) with V volts across will contain Q Coulombs of storage charge $Q = CV$

$$\text{and} \quad dQ/dt = I = C (dV/dt)$$

Capacitors



Example: $C = 1 \mu\text{F}$ (10^{-6}F) $I = 1.0 \text{ mA}$ in 10 ms

$$\frac{dV}{dt} = \frac{I}{C} = \frac{1 \times 10^{-3} \text{ A}}{1 \times 10^{-6} \text{ F}} = 10^3 \text{ V / sec}$$

$$\Rightarrow \Delta V = \int_0^{10 \text{ ms}} \frac{dV}{dt} dt = (10^3 \text{ V / sec}) \times 10^{-2} \text{ sec} = 10 \text{ V}$$

Capacitors in parallel:

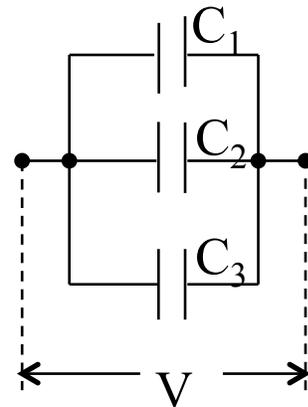
$$C_{\text{tot}} V = CV = Q_{\text{tot}} = C_1 V + C_2 V + \dots$$

$$C = C_1 + C_2 + C_3 + \dots$$

Analogy with impedance:

$$i\omega C = i\omega C_1 + i\omega C_2 + i\omega C_3 + \dots$$

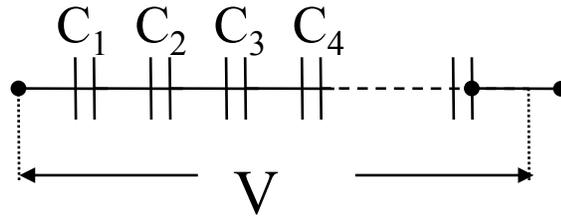
$$\frac{1}{Z_c} = \frac{1}{Z_{c1}} + \frac{1}{Z_{c2}} + \frac{1}{Z_{c3}} + \dots$$



Capacitors in series

Like impedance in series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$



Analogy with impedance:

$$Z_c = Z_{c1} + Z_{c2} + Z_{c3} + \dots$$

$$\frac{1}{i\omega C} = \frac{1}{i\omega C_1} + \frac{1}{i\omega C_2} + \frac{1}{i\omega C_3} + \dots$$

RC Circuits: V and I vs. time

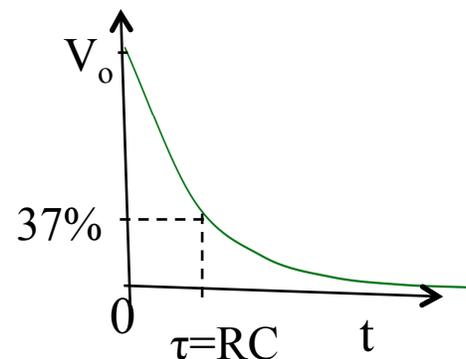
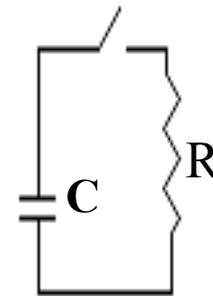
(a) Capacitor C was charged to V_0

At $t = 0$ switch S_w closed and capacitor begin discharges through resistance R:

$$I = C(dV/dt) = -(V/R)$$

$$V = V_0 e^{-t/RC}; \quad RC = \tau \text{ is time constant of ckt.}$$

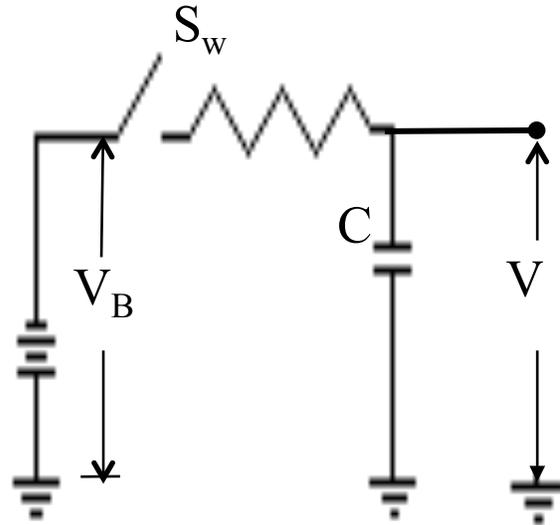
If R in Ω , C in F \rightarrow τ in sec.



RC Circuits: V and I vs. time (cont.)

(b) At $t = 0$ S_w closed and battery start charging C through R

$$I = C \frac{dV}{dt} = \frac{V_B - V}{R}$$

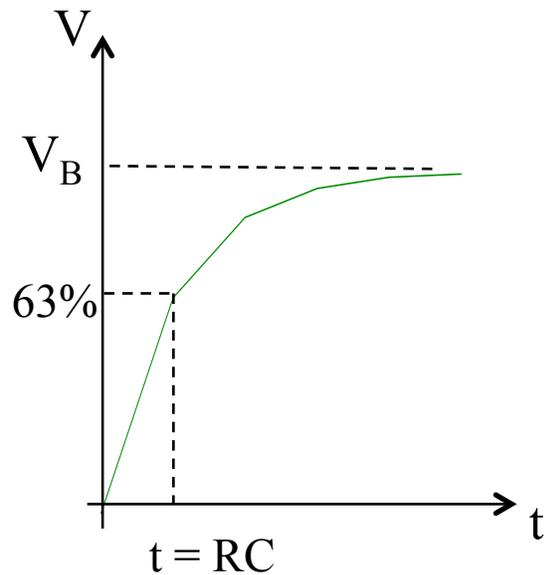


Solution:

$$V = V_B + Ae^{-t/RC}$$

At $t = 0$ $V = 0$, hence

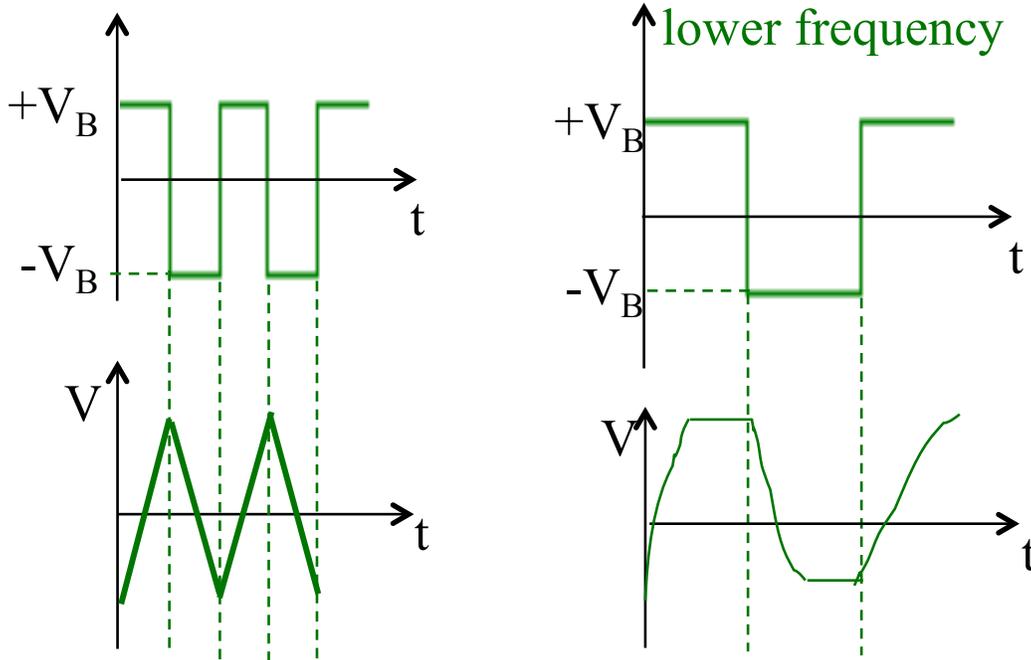
$$V = V_B(1 - e^{-t/RC})$$



Rule of thumb:

Capacitor charges or decays to within 1% of its final value in 5 time constant ($t = 5\tau = 5RC$)

Square waves through a capacitor

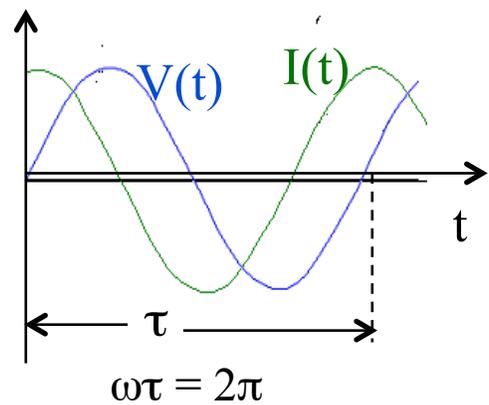


V-I Relation

(Phase Shift)

$$V(t) = V_0 \sin \omega t$$

$$I(t) = C \frac{dV}{dt} = I_0 \cos \omega t; \quad I_0 = V_0 C \omega$$



$$\bar{P} = \frac{1}{\tau} \int_0^{\tau} V(t) I(t) dt = \frac{1}{\tau} I_0 V_0 \int_0^{\tau} \sin \omega t \cos \omega t dt = 0$$

Inductors

Inductors are related to capacitors, but with impedance

$$Z_L = i\omega L$$

In circuit with capacitor rate of change of voltage across it depends on the current through it, while with inductors rate of change of current depends on voltage applied across it.

Therefore defining equation for an inductor L is:

$$V = L \frac{dI}{dt}$$

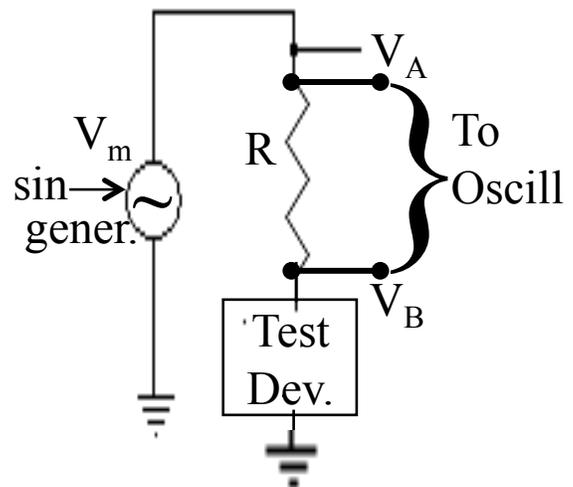
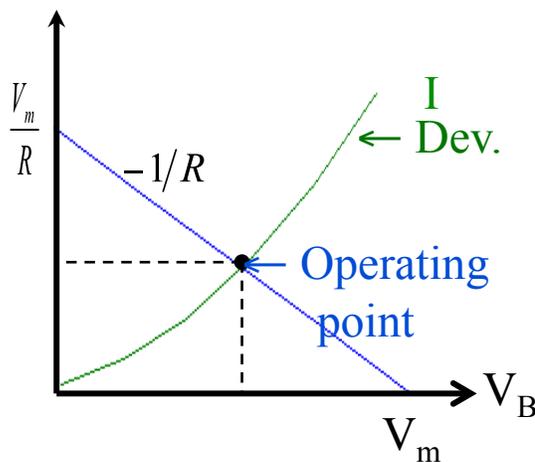
where L is inductance measured in Henrys (H);

$$(\mu\text{H} = 10^{-6} \text{H})$$

Load Line Analysis \Rightarrow Curve Tracer

$$I_{\text{dev}} = \frac{V_A - V_B}{R}$$

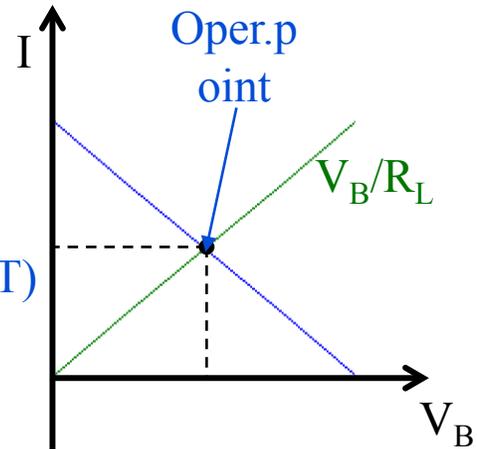
$$V_{\text{dev}} = V_B ; \quad V_A = V_m$$



Load Line Analysis \Rightarrow Curve Tracer(cont.)

In Laboratory

- (a) Dev $\Rightarrow R_L$; $I_{dev} = V_B/R_L$
- (c) Dev $\Rightarrow C$; $I_{dev} = C \frac{dV}{dt}$
- (e) Dev \Rightarrow Diode; $I_{dev} = I_0 \exp(eV_B/kT)$



Sin-generator: $V = V_0 \sin \omega t$

$I_{dev} = VZ_c \leftarrow$ pure imaginary,
 V and I out of phase

Observe on scope I vs. V at different frequency of sin-generator

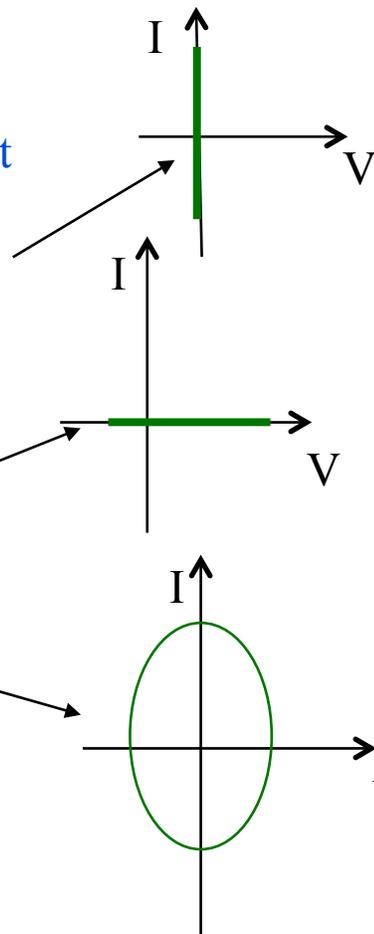
If $\omega \gg 0$ Z_c – small (short circuit)

If $\omega \approx 0$ Z_c – large (open circuit)

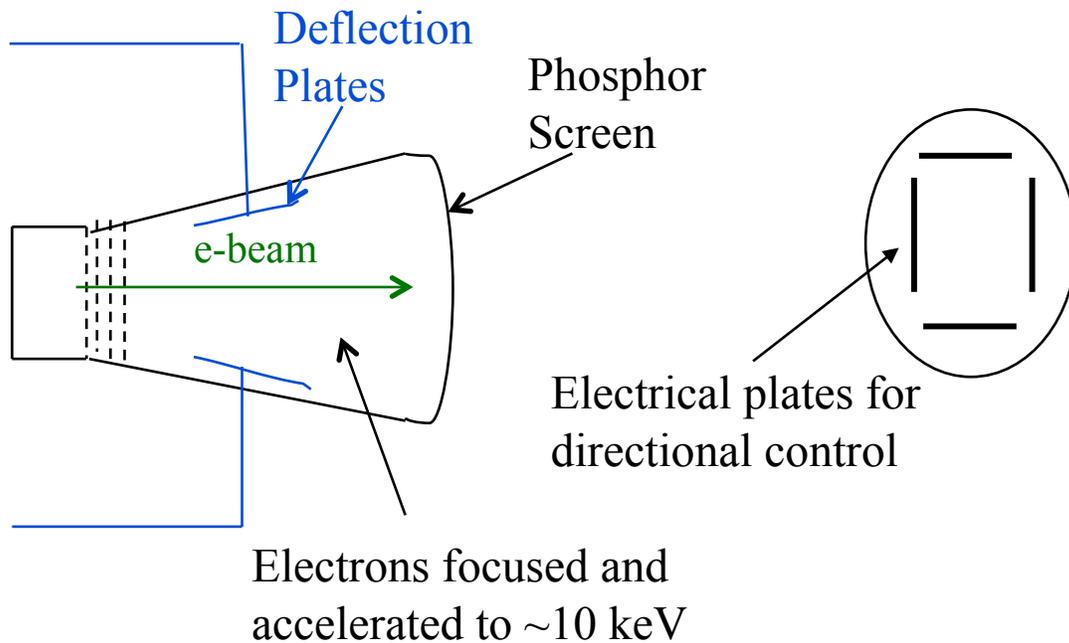
$$\text{If } \omega = 2\pi\nu = \frac{2\pi}{RC}$$

(d) Dev $\Rightarrow L$

Measurements as for capacitor



Principle of Work of Oscilloscope

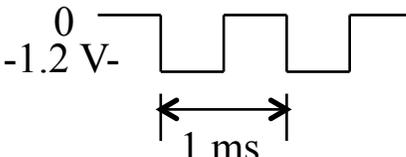


In TV: 3 Magnetic Lenses (Yokes) for direction
(inductance on coils → can't switch back & forth fast)

In Oscill.: Electric deflection → easier to control,
faster than in TV

Principle of Oscill. (cont.)

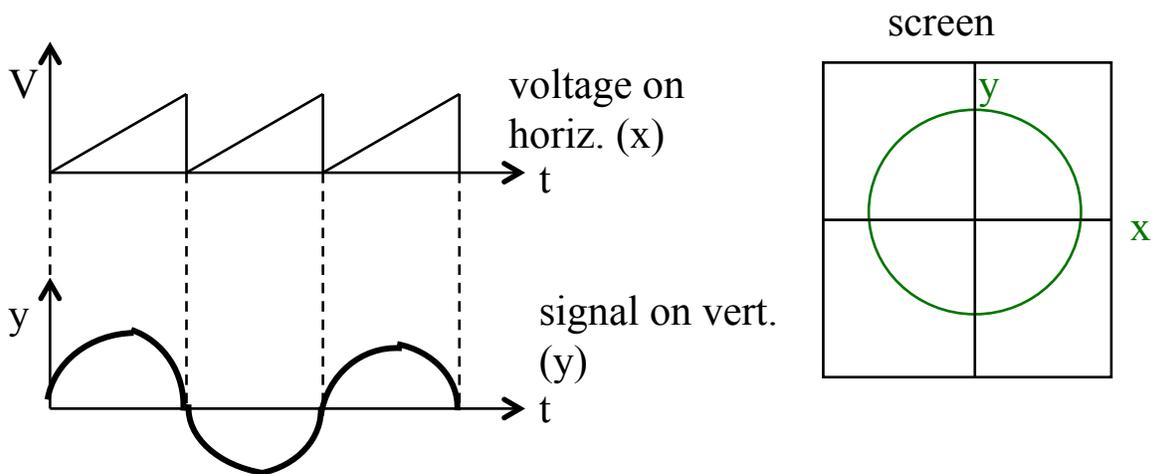
X-Y Mode: 1st channel in vertical } e.g. V(v)
 2nd channel in horizontal }

1 Channel  appears as $\left\{ \begin{array}{l} \frac{1}{2} \text{ time} \rightarrow 0 \text{ V} \\ \frac{1}{2} \text{ time} \rightarrow -1.2 \text{ V} \end{array} \right.$
 two spots

Coupling: For X-Y mode (plot) must be in DC coupled
 mode e.g. one voltage vs. another voltage V(v)

AC – strips constant voltage; only responds on variation

Y-t Mode: time \rightarrow on horizontal plates
 signal \rightarrow on vertical



Principle of Oscill. (cont.)

ATL, CHOP: When mult. Inputs

-ALT: Alternates between A and B channels 

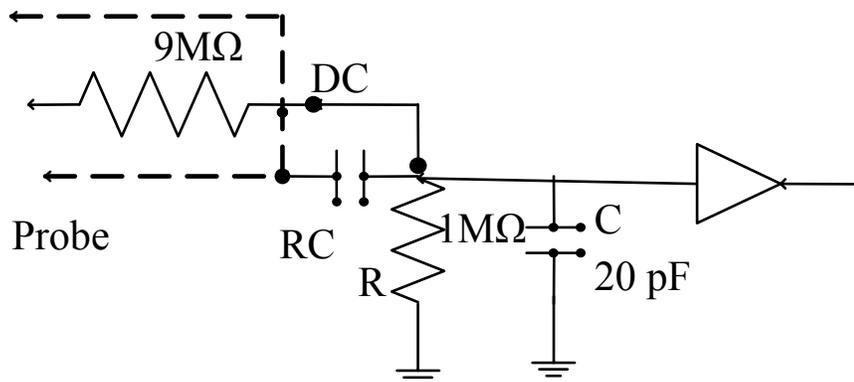
-CHOP: ALT between A and B very fast (for the same base)

To check relations between phases – CHOP only

Slow beam (Low Time Base) – CHOP

Fast Beam - ALT

Impedance of Basic Scope



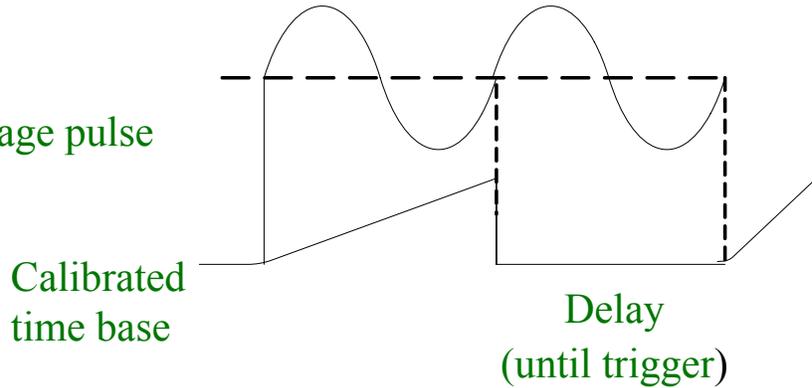
Resistor: $Z_R = R$ Capacitor: $Z_c = \frac{1}{i\omega c}$; $\omega = 2\pi\nu$
 for DC $\omega = \nu = 0 \Rightarrow Z_c = \infty$
 Inductor: $Z_L = i\omega L$

Impedance of Basic Scope

Triggering

Trigger Level: Voltage pulse

Slope: “+” or “-”



Cables

Twisted wire will minimize the far field radiation

Bare wire up to 100 kHz

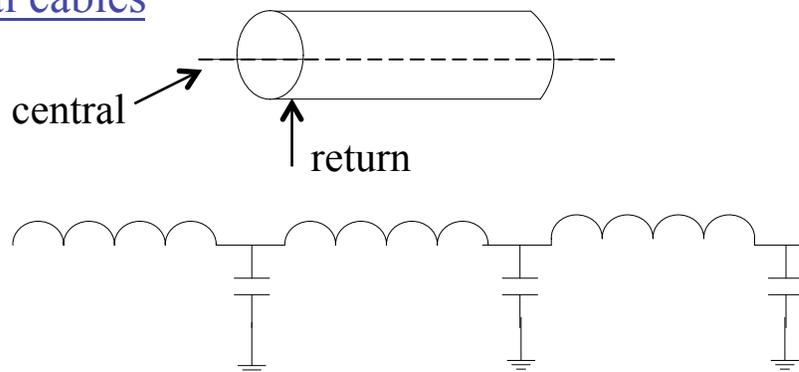
Twisted wire up to 1 MHz

Coaxial wire up to 100 MHz

Wave guide up to 10 GHz

Note: *Ground Loops – Measurement device has to have low impedance to the ground reference.*

Coaxial cables



Distributed inductance - capacitance