14.9 Bohr's correspondence principle states that in the limit of large quantum numbers the classical power radiated in the fundamental is equal to the product of the quantum energy ($\hbar\omega_0$) and the reciprocal mean lifetime of the transition from principal quantum number n to $(n - 1)$.

(*a*) Using nonrelativistic approximations, show that in a hydrogen-like atom the transition probability (reciprocal mean lifetime) for a transition from a circular orbit of principal quantum number n to $(n - 1)$ is given classically by

$$\frac{1}{\tau} = \frac{2}{3}\frac{e^2}{\hbar c}\left(\frac{Ze^2}{\hbar c}\right)^4 \frac{mc^2}{\hbar}\frac{1}{n^5}$$

(*b*) For hydrogen compare the classical value from (*a*) with the correct quantum-mechanical results for the transitions $2p \to 1s$ (1.6×10^{-9} sec), $4f \to 3d$ (7.3×10^{-8} sec), $6h \to 5g$ (6.1×10^{-7} sec).

14.10 Periodic motion of charges gives rise to a discrete frequency spectrum in multiples of the basic frequency of the motion. Appreciable radiation in multiples of the fundamental can occur because of relativistic effects (Problems 14.7 and 14.8) even though the components of velocity are truly *sinusoidal*, or it can occur if the components of velocity are not sinusoidal, even though periodic. An example of this latter motion is an electron undergoing nonrelativistic elliptic motion in a hydrogen atom.

The orbit can be specified by the parametric equations

$$x = a(\cos u - \epsilon)$$

$$y = a\sqrt{1 - \epsilon^2}\sin u$$

where

$$\omega_0 t = u - \epsilon \sin u$$

a is the semimajor axis, ϵ is the eccentricity, ω_0 is the orbital frequency, and u is the angle in the plane of the orbit. In terms of the binding energy B and angular momentum L, the various constants are

should be

$a = \dfrac{e^2}{2(4\pi\epsilon_0)B}$

+

$\epsilon = \sqrt{1 - \dfrac{2BL^2(4\pi\epsilon_0)^2}{me^4}}$

$$a = \frac{e^2}{2B}, \qquad \epsilon = \sqrt{1 - \frac{2BL^2}{me^4}}, \qquad \omega_0^2 = \frac{8B^3}{me^4}$$

gaussian unit ?

(*a*) Show that the power radiated in the kth multiple of ω_0 is

$$P_k = \frac{\overset{8}{\cancel{4}}e^2}{3c^3}(k\omega_0)^4 a^2 \left\{\frac{1}{k^2}\left[(J_k'(k\epsilon))^2 + \left(\frac{1-\epsilon^2}{\epsilon^2}\right)J_k^2(k\epsilon)\right]\right\} \times \frac{1}{4\pi\epsilon_0}$$

where $J_k(x)$ is a Bessel function of order k.

(*b*) Verify that for circular orbits the general result (*a*) agrees with part (*a*) of Problem 14.9.

14.11 Instead of a single charge e moving with constant velocity ω_0 in a circular path of radius R, as in Problem 14.8, a set of N such charge moves with fixed relative positions around the same circle.

(*a*) Show that the power radiated into the mth multiple of ω_0 is

$$\frac{dP_m(N)}{d\Omega} = \frac{dP_m(1)}{d\Omega}F_m(N)$$