The Convergence of Semi-Classical and Quantum Mechanics: Examining Atomic Radiation through the Lens of the Correspondence Principle

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This paper represents my own work in accordance with University regulations.

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Abstract

The semi-classical, or Bohr-Sommerfeld, model of a hydrogen atom concerns a "de Broglie" electron in an elliptical orbit in the Coulomb-law field of a proton. This model led to the later quantum mechanical analysis of Schrodinger and Heisenberg, whose equations model electrons with a probabilistic wave function. Semi-classical and quantum approaches have their own derivations of parameters describing the properties of the atom from quantized energy structure to radiated power. Despite the differences in physical origin between the models, the correspondence principle formulated by Bohr states that in the limit of large quantum numbers, semi-classical and quantum descriptions merge. This thesis considers the comparison of quantum and semi-classical models for frequency and the power radiation of the electronically excited hydrogen atom. Using the mathematics as a base, this paper analyzes correspondence between the semi-classical and non-relativistic quantum models as quantum numbers are increased. It is shown that in the limit of large quantum numbers, there is correspondence for both the frequency and the radiated power of the excited hydrogen atom. Knowing classical mechanics and the de Broglie hypothesis is sufficient indeed to anticipate the qualitative and, in some cases, quantitative decay of highly excited atomic hydrogen.

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Contents

1 Introduction

Traditionally, classical and quantum mechanical models differ greatly in both the mathematics and the fundamental physics from which that math arises. In classical mechanics, particles have a knowable position and momentum. Properties such as energy and angular momentum exist on a continuous spectrum, meaning that a particle can exhibit any real-number value of these characteristics. However, experiments conducted in the early twentieth century began to reveal problems with the application of classical mechanics to all levels of existence. In particular, the classical description breaks down when applied to small, low-energy systems, such as the motion of a photon or an electron. Instead of existing on a continuous spectrum, fundamental properties of small particles are "quantized," meaning they take on discrete, rather than continuous, values. Additionally, the location of these small particles is better described by a wave function, which is an expression of the probability of that particle existing in each location. Another discovery, known as the de Broglie hypothesis, found that all particles in motion, from a photon to an electron to a batted baseball, have wave-like properties. After these experiments, a new branch of physics was founded, called quantum mechanics, named for the quantization of attainable states. An entirely new set of equations was derived from this new information.

The discovery of quantum mechanics provoked the following question: if classical mechanics falls apart for small particles at low energies, what happens to quantum mechanics at high energies? In 1920, physicist Niels Bohr formulated the correspondence principle, which states that, in the limit of large quantum numbers, the quantum description begins to replicate the classical. In other words, when large values of energy are inputted into both the quantum and the classical equations, the results should be the same, even though the equations come from very different physical theories.

Bohr proposed his principle in the context of the radiative decay of a single electron

system, such as a hydrogen atom. The classical model of such a system is an electron in an elliptical orbit around the nucleus. When any charged particle, including an orbiting electron, experiences an acceleration, it loses energy. The radiated power, or energy lost over time, can be modeled as a function of discrete angular frequencies.

In quantum mechanics, an electron exists in probability clouds, called orbitals, described by a wave function. An electron gains and loses energy as it jumps between different orbitals, each of which is defined by a quantized energy level. The energy lost or gained comes from the emission or absorption of a photon, whose energy is determined by its frequency. The power radiated in a quantum mechanical system is proportional to the photon energy, and therefore to photon frequency.

In both models, radiated power is dependent on frequency. Even though these dependencies come from different physical theories, Bohr's correspondence principle suggests that in the limit of high electron energies the different models of power and frequency should coincide. This thesis explores the variation of power versus frequency for quantum and classical mechanics over different values of energy and angular momentum. The hypothesis, consistent with the formulation of the correspondence principle, states that there will be little correspondence between the two models at low energy levels, but correspondence will improve as energy values are increased.

1.1 Reference Table

Symbol	Description	Representation	Approx. Value	Units
e	Electron Charge	—	$1.60 * 10^{-19}$	C
m	Electron Mass	—	$9.11 * 10^{-31}$	kg
ϵ_0	Vacuum Permittivity	—	$8.85 * 10^{-12}$	$\frac{s^2C^2}{kgm^3}$
c	Speed of Light in Vacuum	—	$3.00 * 10^{8}$	$\frac{m}{s}$
h	Planck Constant	—	$6.63 * 10^{-34}$	Js
$\hbar$	Reduced Planck Constant	$\frac{h}{2\pi}$	$1.05 * 10^{-34}$	Js
a_0	Bohr Radius	$\frac{4\pi\epsilon_0\hbar^2}{me^2}$	$5.29 * 10^{-11}$	m
v_0	Velocity at Bohr Radius	$\frac{e^2}{4\pi\epsilon_0\hbar}$	$2.19 * 10^{6}$	$\frac{m}{s}$
R	Rydberg Constant	$\frac{me^4}{2(4\pi\epsilon_0)^2\hbar^2}$	$2.18 * 10^{-18}$	J
Ry	Rydberg Frequency	$\frac{me^4}{4\pi(4\pi\epsilon_0)^2\hbar^3}$	$3.29 * 10^{15}$	s^{-1}
n	Principal Quantum Number	—	—	—
l	Angular Momentum Quantum Number	—	—	—
n', l'	Quantum Numbers of Final State	—	—	—
a_n	Semimajor Axis	n^2a_0	—	m
b	Semiminor Axis	$n\sqrt{l(l+1)}a_0$	—	m
ϵ	Eccentricity	$\sqrt{1-\frac{b^2}{a^2}}$	—	—

Table 1: A table of fundamental quantities and their units and values. This paper uses the SI unit system.

2 Background

Niels Bohr first presented his model of the atom in an article titled "On the Constitution of Atoms and Molecules" in 1913 [1]. At the time, the most recently proposed atomic model was formulated by Ernest Rutherford, who described a positively charged nucleus surrounded by a system of orbiting electrons. Bohr sets up his paper as an improvement on Rutherford's model. He begins by pointing out that classical electrodynamics does a poor job of describing atomic systems when applied to Rutherford's model. He notes in particular that the theory does not allow for stable electron orbits or a discontinuous atomic emission spectrum. He writes, "It seems necessary to introduce in the laws in question a quantity foreign to classical electrodynamics" [1]. The quantity he is referring to is Planck's constant, h.

Bohr concludes that the allowed values of energy of an electron are quantized, meaning they take on values discretized by a quantity that he calls τ, but we will call n. To account for stable orbits, he proposes that a lower limit to these values of energy exists, called the ground state, corresponding to $n = 1$. Finally, he explains that the discontinuous atomic emission spectrum exists because of electrons transitioning between the discretized energy levels. He claims that the energy change of a transitioning electron is equal to the frequency of the emitted photon multiplied by h.

In his paper, Bohr made a significant contribution to the advancement of atomic models. He formulated the principal quantum number, n, and discretized the allowed energy states. However, his model had a few shortcomings. Most importantly, he considered only circular systems, rather than a generalized elliptical orbit. A year and a half later, Physicist Arnold Sommerfeld applied Bohr's findings to the more general case [2].

Like Bohr before him, Sommerfeld acknowledges his predecessor's advancements. Following the same method as Bohr but for elliptical orbit, he derives a similar formula

for energy with an additional factor of $1 - \epsilon^2$, where ϵ is the eccentricity, a geometric property of the orbit. Every ellipse has an eccentricity; a circle's eccentricity is zero, while that of an elongated ellipse will approach one. This new parameter describing the electron's orbit was dependent on yet another discretized quantity, which Sommerfeld calls n' and we will call l. This Bohr-Sommerfeld model of an atom is not quite classical because of its dependence on quantum numbers, and it is not quite quantum mechanical because it assumes classical particle motion. As a result, it is often called a semi-classical model for its unification of the classical and the quantum.

Bohr formulated his correspondence principle by relating his quantized atomic model to the classical Rutherford model of an electron orbiting a nucleus. He first used the term "correspondence principle" in an address to the German Physics Society to examine the correlation of frequencies in each model [3]. For the classical model, he used the angular frequency of orbit,

$$\omega = \sqrt{\frac{2(4\pi\epsilon_0)^2 W^3}{\pi^2 e^4 m}}, \tag{1}$$

where W is the magnitude of the total energy of the orbit, e is the electron charge, and m is its mass. It turns out that it is equivalent to model this motion as circular and elliptical because eccentricity vanishes in the result.

He combines this classical frequency with the energy of the n^{th} state from his own model,

$$E_n = \frac{Kh}{n^2}, \tag{2}$$

where Kh is the energy of the ground state. Substituting E_n for W in the frequency equation gives,

$$\omega_n = \frac{1}{n^3}\sqrt{\frac{2(4\pi\epsilon_0)^2) h^3 K^3}{\pi^2 e^4 m}}. \tag{3}$$

Now that he has the classical angular frequency, he compares it with the frequency

value from his model, which comes from the frequency of the emitted radiation. It is proportional to the energy lost by a transitioning electron, given by,

$$\nu = K\left(\frac{1}{n'^2} - \frac{1}{n^2}\right). \tag{4}$$

For large and roughly equivalent n and n', ν can be expressed as,

$$\nu = \frac{2K(n - n')}{n^3}. \tag{5}$$

Bohr achieves a $\frac{1}{n^3}$ dependence for both ω and ν for large n, demonstrating their correspondence. As Robert Rynasiewicz reminds us in a paper summarizing Bohr's address, "Although one arrives at this convergence in frequencies, the *mechanisms* of radiation remain utterly distinct" [4]. In other words, the theory works, not because the physics merges, but because the math does.

A recent paper by Deepak Dhar examines the correspondence principle in a new context [5]. Since Bohr's initial statement of the correspondence principle, there have been significant advancements in quantum theory. Schrödinger wrote his equation, and more accurate atomic models have been produced. The current model is that an electron has a space and time dependent wave function that describes its probability of existing at a every point in space-time. The classical model is a simplification of the semi-classical model, which is a simplification of the modern, wave function-based electron model. Dhar's paper views the connection between the modern atomic model and the semi-classical model, just as Bohr viewed the correspondence principle as a limit connecting his model to the classical.

Dhar considers a wave function that corresponds to the circular state. He constructs a wave packet by summing the wave function over possible energy values. Because of the circular approximation, the wave packet simplifies to r and θ dependence, with a sharp

peak at $r = n^2 a_0$ and $\theta = \frac{\pi}{2}$. Unsurprisingly, this represents a circle in three dimensional space for every value of n. He plugs his wave packet into the Schrödinger equation and ends up with a time dependent term reading,

$$e^{-2\pi E_l t/h}, \tag{6}$$

where $E_l = -\frac{E_G}{(l+1)^2}$, and E_G is the energy of the ground state. This term is similar to the time dependence of a one-dimensional wave packet, with frequency E_l. Dhar expands E_l to first order around E_{l^*}, where l^* represents the peak value of l:

$$E_l \approx E_{l^*} + (l - l^*)\hbar \frac{\partial E_l}{\partial l} = E_{l^*} + (l - l^*)\hbar\omega^*. \tag{7}$$

This produces a formula for ω^* in terms of a partial derivative of E_l, and so

$$\omega^* = \frac{2E_G}{(l+1)^3} = \frac{2E_G}{n^3}, \tag{8}$$

which has the same n dependence as equation 5. In particular, for the circular case, $n = n' + 1$ in equation 5, so the frequencies reduce exactly. Frequency correspondence has again been illustrated for large n, this time between a wave function model and the semi-classical model.

Bohr's theory has been examined with regard to many different systems and quantum properties. A paper by Kleppner et. al. examines the correspondence principle as it relates to electron charge density in the classical and quantum mechanical models [6]. Classically, an electron is orbiting on an elliptical path. The probability of finding the electron within a given volume is dependent on the amount of time the electron spends in that volume, which is inversely proportional to its velocity for that segment of the orbit. Quantum mechanically, an electron's location is mapped by a wave function that gives the probability of finding the electron within the given volume. Figure 1, borrowed

from the article, represents the results of the analysis.

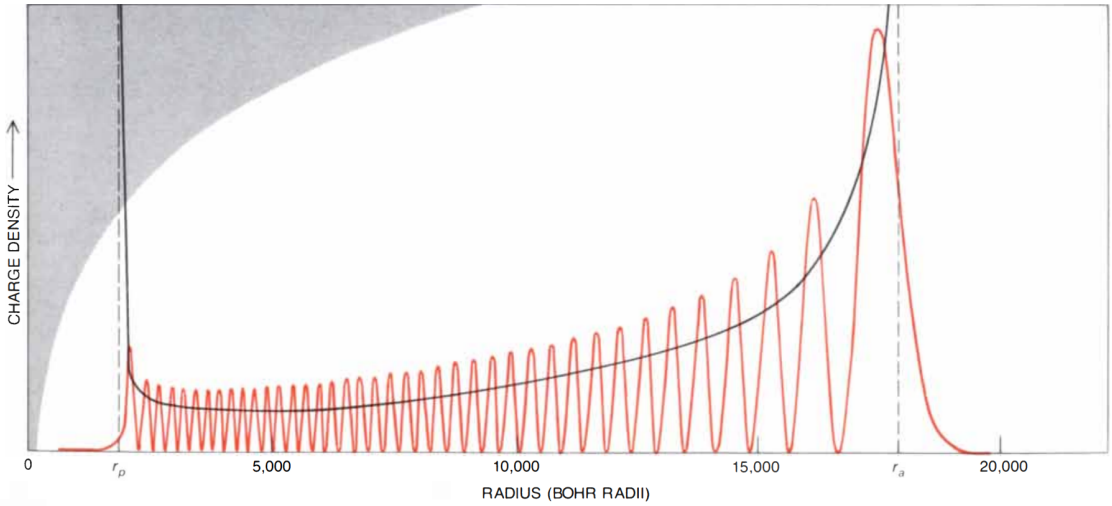

Figure 1: A graph of charge density versus radius for $n = 100$, $l = 50$. The red represents the quantum result, while the black represents the classical result.

Although certain features of the two graphs are incompatible, such as the classical model's tendency toward infinity at the extreme values of radius, as well as the sinusoidal shape of the quantum result, there is still correspondence. For intermediate values of radius, the amplitude of the sinusoid trends in the same direction as the Bohr result.

The correspondence principle is evidently visible when applied to many electron properties, including frequency. The other atomic property investigated in this thesis is radiated power. Can the principle be applied here as well? Bohr seemed to think so. While he did not explicitly carry out the calculations, he noted, "If we now inquire into a deeper meaning of the correspondence established, we are naturally led to expect first that the correspondence arises not only in an agreement of the frequencies of spectral lines determined by the two methods, but will remain valid also for their intensities" [3]. The intensity of a spectral line is proportional to the power radiated. Bohr goes on to connect that power with the amplitude of the periodic orbit in the classical sense. In other words, he did believe that there should be correspondence for power radiation.

There is an obvious failure in power correspondence for the lowest energy state. As a paper by Seidl and Lipas notes, classically, an orbiting electron has no lower limit on its energy [7]. As a result, it will continue radiating energy through its orbit, gradually losing radius until it spirals into the nucleus. The paper calculates this finite lifetime. In quantum mechanics, every electron has a ground state, and it is forbidden from having energy below this value. In other words, in the ground state, an electron has a non-zero classical radiation and a zero quantum mechanical radiation, which indicates an extreme lack of correspondence. As energy levels are increased, however, perhaps correspondence is still possible.

A 2005 paper by Horbatsch et. al. evaluates the branching ratios for radiative decay of a hydrogen atom [8]. The branching ratios are calculated from the quantum mechanical lifetime and the transition probability, which makes them proportional to radiated power. The paper also introduces a classical analog of the branching ratio, which includes a classically calculated transition probability and lifetime. The authors do acknowledge the failure of their analysis for $l = 0$, which makes sense based on the finite classical lifetime. The paper demonstrates similarity between the classical and quantum calculations for the decay from the $n = 90$, $l = 15$ state, which indicates correspondence. Therefore, there is evidence that a power-correlated quantity converges for large n. It remains to be seen, however, what happens with varying l.

Despite the many applications of the principle, there is evidence that it is not universal. A 1999 study by Bo Gao at the University of Toledo reveals flaws in the principle for systems in which the potential is proportional to $\frac{1}{r^n}$ for $n > 2$ [9]. He examines in particular the vibration quantum number ν, which is analogous to energy states, for a system with a $\frac{1}{r^3}$ potential dependence. The classical calculations result in an even spacing between successive values of ν for $[-\epsilon_s(\nu)]^{1/6}$, where ϵ_s is a constant.

In other words,

$$[-\epsilon_s(\nu-1)]^{1/6} - [-\epsilon_s(\nu)]^{1/6} = \delta, \tag{9}$$

where the classical $\delta_c \approx 1.11$ is constant for all values of ν. This same quantity is calculated for the quantum mechanical approach, for which the spacing δ_q in equation 9 is not consistent and depends on the value of ν. This discrepancy in and of itself does not violate the correspondence principle. However, a closer look at the quantum results reveals something surprising. For the highest possible value of ν, the value of δ_q is approximately 1.32. As ν decreases, so does δ_q, and in the limit of small ν, δ_q approaches δ_c. The correspondence principle asserts that in the limit of *high* quantum numbers, the quantum mechanical approaches the classical. In Gao's paper, the correspondence is reversed, occurring in the limit of *low* quantum numbers.

While Gao does demonstrate flaws in the correspondence principle, he does so in a rather contrived manner. Most common electromagnetic potentials do not vary inversely with distance cubed. For instance, the vector potential of a uniform magnetic field is proportional to distance, and the magnetic potential from a current loop goes like r^{-2} for large r relative to the diameter of the loop. The electric potential of a stationary point charge, which is a good approximation for a nucleus from the perspective of an orbiting electron, varies with r^{-1}. Therefore, while Gao's results should be kept in mind as evidence that the correspondence principle is not universally applicable, it still can apply to the power radiation and frequency of a transitioning electron in the field of a positively charged nucleus.

Once the correspondence principle has been explored for the hydrogen atom, it remains to be seen if the results can apply to more complex atomic systems. The Kleppner et. al. paper introduces the concept of a Rydberg atom, defined as an atom where an "electron is excited to an exceptionally high energy level" [6]. A Rydberg atom can be an element as simple as lithium, with three protons and three electrons,

or as massive as cesium, with fifty-five of each. When an atom is described as having an excited electron, it means that the single electron of the outer shell is at a high energy state, regardless of the total number of electrons in the atom. According to the Bohr model, an electron's distance from the nucleus is proportional to the square of its energy state. In other words, a high energy state implies a large orbital radius. If the total number of protons in a Rydberg atom is Z (which will also be the total number of electrons), then, from the perspective of the excited electron at a large distance, the positive charges of the nucleus cancel with the negative charges of the inner electrons until the total charge appears to be +1. As a result, the excited electron of a Rydberg atom behaves like that of a hydrogen atom.

But can the hydrogen results be directly transferred to Rydberg atoms? A paper by Hezel et. al. cautions against this [10]. The paper introduces the notion of quantum defect, which is a property of a Rydberg electron that is inversely proportional to its angular momentum state. As l increases, the quantum defect decreases. The defect has an impact on the electron's energy state, which is a property not shared by the hydrogen atom. In other words, while a Rydberg atom is a good approximation of a hydrogen atom for large n, it is not perfect. For the correlation to be more exact, large values of l are required, as well. So the results for hydrogen are most applicable to other atoms in the limit of large n *and* large l.

3 Classical Radiation

The classical description of the physical world is an approximation that applies at the high energy case. The following is a derivation of the equations that describe the radiation from an electron in an elliptical orbit. More specifically, they describe how that radiation relates to the electron's frequency.

3.1 Larmor Formula

The Larmor formula is an equation that relates instantaneously radiated power, or energy lost per unit time, to the acceleration of a non-relativistic point charge, scaled by a few constants. The formula:

$$\frac{dE}{dT} = \frac{2e^2\dot{v}^2}{3(4\pi\epsilon_0)c^3}. \tag{10}$$

In this equation, e is the elementary charge, $\dot{v}$ is the particle's acceleration, and c is the speed of light.

3.1.1 Larmor Derivation

The derivation of the Larmor formula[1] begins with a classical electrodynamic statement of the fields from a point charge, e, in arbitrary motion:

$$\mathbf{E}(\mathbf{r},t) = \frac{e}{4\pi\epsilon_0}\frac{R}{(\mathbf{R}\cdot\mathbf{u})^3}[(c^2-v^2)\mathbf{u} + \mathbf{R}\times(\mathbf{u}\times\dot{\mathbf{v}})] \tag{11}$$

$$\mathbf{B}(\mathbf{r},t) = \frac{1}{c}\mathbf{R}\times\mathbf{E}. \tag{12}$$

In these equations, bold face is used for vector notation, and the magnitude of the vector is the non-bold faced version of the same text. A "hat" over a vector indicates a unit vector in the same direction. $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field, $\mathbf{r}$ is the vector from the origin to the point being tested, $\mathbf{R}$ is the vector from the source to the

[1] This derivation follows a similar analysis given in Chapter 11 of Griffiths' *Introduction to Electrodynamics* [11].

test point, and $\mathbf{u} = c\mathbf{R} - \mathbf{v}$. The Poynting vector, which gives energy flux per unit time, reads,

$$\mathbf{S} = \epsilon_0 c^2 (\mathbf{E} \times \mathbf{B}). \tag{13}$$

As Griffiths notes, this quantity does not give the total radiation flux, as it also includes the energy carried by a particle. As a result, he draws a large sphere of radius R, centered on the particle, which the news of the radiation will reach before the particle does. This allows him to simplify the expression for electric field, because only terms that vary with $\frac{1}{R^n}$, with $n < 2$, will contribute for large R. The electric field can be written,

$$\mathbf{E}(\mathbf{r}, t) = \frac{e}{4\pi\epsilon_0} \frac{R}{(\mathbf{R} \cdot \mathbf{u})^3} [\mathbf{R} \times (\mathbf{u} \times \dot{\mathbf{v}})]. \tag{14}$$

The charge can be considered instantaneously at rest at the time the integral is being taken, which simplifies the expression for electric field to

$$\mathbf{E} = \frac{e}{4\pi\epsilon_0 c^2 R} ((\hat{\mathbf{R}} \cdot \dot{\mathbf{v}})\hat{\mathbf{R}} - \dot{\mathbf{v}}). \tag{15}$$

The Poynting vector can also be simplified:

$$\mathbf{S} = \epsilon_0 c (E^2 \hat{\mathbf{R}} - (\hat{\mathbf{R}} \cdot \mathbf{E})\mathbf{E}) \tag{16}$$

$$= \epsilon_0 c E^2 \hat{\mathbf{R}}, \tag{17}$$

because the electric field is perpendicular to $\hat{\mathbf{R}}$. Finally, $\mathbf{S}$ can be expressed in terms of the angle between the acceleration vector and $\hat{\mathbf{R}}$:

$$\mathbf{S} = \frac{e^2 \dot{v}^2}{16\pi^2 \epsilon_0 c^3} \left(\frac{\sin^2 \theta}{R^2} \right) \hat{\mathbf{R}}. \tag{18}$$

This expression, combined via dot product with the surface area vectors of our sphere, can be integrated to produce the instantaneous power as a function of the acceleration:

$$P = \int \mathbf{S} \cdot d\mathbf{a} \tag{19}$$

$$= \frac{e^2 \dot{v}^2}{16\pi^2 \epsilon_0 c^3} \int \frac{\sin^2 \theta}{R^2} R^2 \sin\theta d\theta d\phi \tag{20}$$

$$= \frac{2e^2 \dot{v}^2}{3(4\pi\epsilon_0)c^3}. \tag{21}$$

The Larmor formula is easy to interpret in the circular case, where acceleration is constant and directly proportional to orbital frequency. However, in an elliptical orbit, acceleration varies depending on a particle's location in its orbit, which complicates the classically radiated power.

3.2 Semi-Classical Power Equation

The elliptical result of power versus frequency is an extension of the Larmor Formula that analyzes acceleration in non-circular orbits. The following formula is stated from Jackson [12]. It is derived from the Larmor formula, but is applied more generally to the elliptical case.

$$P_k(\omega_0) = \frac{e^2}{3\pi\epsilon_0 c^3}(k\omega_0)^4 \frac{a^2}{k^2}[(J_k'(k\epsilon))^2 + \frac{1-\epsilon^2}{\epsilon^2} J_k^2(k\epsilon)]. \tag{22}$$

In this equation, k discretizes the power spectrum by taking on integer values greater than 0, J_k is the Bessel function of order k, and a, ϵ, and ω_0 are the semi-major axis, eccentricity, and orbital frequency, respectively, and require derivation. Despite its increased complexity, notice also the similarities between this and the Larmor formula: a dependence on e^2, c^{-3}, and a term $(\omega_0^4 a^2)$ with units of acceleration squared.

3.2.1 Parameters

The parameter derivation begins with a statement of the Virial Theorem in the lowest energy state, that kinetic energy is negative one-half times potential energy,

$$\frac{1}{2}mv_0^2 = \frac{e^2}{8\pi\epsilon_0 a_0}, \tag{23}$$

where m is electron mass, v_0 is velocity, and a_0 is the Bohr radius. Next, bringing in some quantum mechanics, the de Broglie wavelength, $2\pi a_0$, is $\frac{h}{mv_0}$, or

$$v_0 = \frac{\hbar}{ma_0}. \tag{24}$$

Substituting back into equation 23 gives the Bohr radius and, eventually, v_0:

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{me^2} \tag{25}$$

$$v_0 = \frac{e^2}{4\pi\epsilon_0\hbar}. \tag{26}$$

The orbital radius and velocity of the n^{th} energy level are given by:

$$a_n = n^2 a_0 \tag{27}$$

$$v_n = \frac{v_0}{n}. \tag{28}$$

The energy of the n^{th} energy level is given by -$\frac{R}{n^2}$, where R is the Rydberg constant:

$$R = \frac{e^2}{8\pi\epsilon_0 a_0} \tag{29}$$

$$= \frac{me^4}{2(4\pi\epsilon_0)^2\hbar^2}. \tag{30}$$

Now that all of these constants have been established, we can begin to define a, ω, and ϵ for an elliptical orbit. The semimajor axis, a, remains the same as the radius of circular

orbit. That is, a is given by equation 27. ω can be found from the period ($\omega = \frac{2\pi}{T}$), given by,

$$\frac{v_0}{n}T = 2\pi a_0 n^2 \tag{31}$$

$$T = n^3 \frac{2\pi(4\pi)^2 {\epsilon_0}^2 \hbar^3}{me^4}. \tag{32}$$

The derivation of ϵ requires the introduction of the additional orbital parameters of angular momentum, L, and semiminor axis, b, because

$$\epsilon^2 = 1 - \frac{b^2}{a^2}. \tag{33}$$

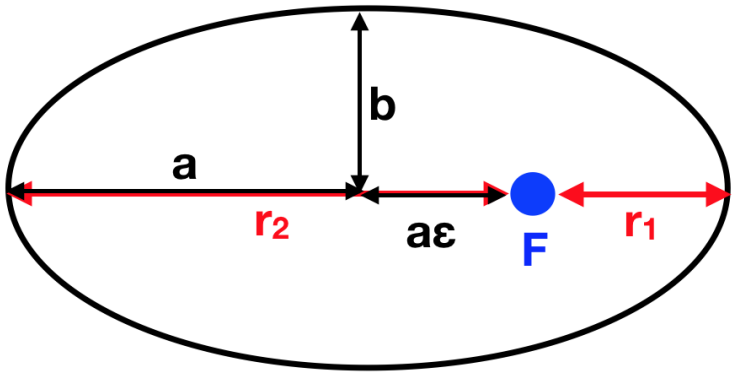

Figure 2: An ellipse with important parameters labeled. F is the focus where the nucleus sits, b is the semiminor axis, and r_1 and r_2 are the distances of closest and farthest approach, respectively.

b is related to L, a relationship that can be derived from the distances of closest and farthest approach, r_1 and r_2. These parameters are outlined in Figure 2. From conservation of angular momentum, and because the velocity and radius are perpendicular at these points,

$$L = mv_1 r_1 = mv_2 r_2. \tag{34}$$

The energy difference between these points must be 0, so,

$$\frac{1}{2}m{v_1}^2 - \frac{e^2}{4\pi\epsilon_0 r_1} - \frac{1}{2}m{v_2}^2 + \frac{e^2}{4\pi\epsilon_0 r_2} = 0. \tag{35}$$

Rearranging gives

$${v_1}^2 - {v_2}^2 = \frac{2e^2}{4\pi\epsilon_0 m}\left(\frac{1}{r_1} - \frac{1}{r_2}\right). \tag{36}$$

Combining equations 34 and 36 gives an expression for the square of the angular momentum in terms of r_1 and r_2:

$$L^2 = \frac{2me^2}{4\pi\epsilon_0(\frac{1}{r_1} + \frac{1}{r_2})}. \tag{37}$$

Now, all that remains for defining ϵ is relating r_1 and r_2 to a and b. From Figure 2, $r_1 = a - a\epsilon$ and $r_2 = a + a\epsilon$. Combining these geometrical facts with equation 33 gives a and b in terms of r_1 and r_2:

$$b = \sqrt{r_1 r_2} \tag{38}$$

$$a = \frac{1}{2}(r_1 + r_2). \tag{39}$$

The term $\frac{1}{r_1} + \frac{1}{r_2}$ from equation 37 can now be rewritten as $\frac{r_1+r_2}{r_1 r_2} = \frac{2a}{b^2}$, giving angular momentum in terms of only a, b and constants:

$$L^2 = \frac{me^2}{4\pi\epsilon_0}\frac{b^2}{a}. \tag{40}$$

Inserting some more quantum mechanics, the operator L^2 has eigenvalues $l(l+1)\hbar^2$, where l is the electron's angular momentum state, which will be explored more closely in the following sections. Combining this fact with equations 33 and 40 gives a neat result for the eccentricity only in terms of the electron's angular momentum and energy

states:

$$\epsilon = \sqrt{1 - \frac{l(l+1)}{n^2}}. \tag{41}$$

3.2.2 Jackson Formula Derivation

Now that the parameters have been established, all that remains is to derive the Jackson formula.[2] As mentioned in the previous section, it begins with a statement of the Larmor formula.

$$P = \frac{e^2 \dot{v}^2}{6\pi\epsilon_0 c^3} \tag{42}$$

This formula is a statement of instantaneous power radiated, which is periodic with period equal to the orbital period, $\frac{2\pi}{\omega_0}$. As a result, the frequency spectrum of the Larmor formula is discrete and can be evaluated using a Fourier series at the discrete harmonic frequencies ω_0. The remainder of this derivation will be an attempt to replace the $\dot{v}$ term with this frequency term.

The equation of a two dimensional ellipse with one of its foci located at the origin is given by shifting its center down the x-axis by $a\epsilon$ (see Figure 2).

$$\frac{(x + a\epsilon)^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{43}$$

This equation can be simplified by parameterizing x and y with cosine and sine terms, respectively. It is also useful to introduce orbital frequency and time as a function of the same parameter.

$$\frac{x + a\epsilon}{a} = \cos u \tag{44}$$

$$\frac{y}{b} = \sin u \tag{45}$$

$$\omega_0 t = u - \epsilon \sin u \tag{46}$$

[2]This derivation follows a similar analysis given in a solution set to problem 14.22 of J. David Jackson's *Classical Electrodynamics* 3rd edition, [12] Chegg, inc.

The next step is to write x and y in terms of Fourier series,

$$x(t) = \sum_{k=0}^{\infty}(a_k \cos k\omega_0 t + b_k \sin k\omega_0 t) \tag{47}$$

$$y(t) = \sum_{k=0}^{\infty}(a_k \cos k\omega_0 t + b_k \sin k\omega_0 t), \tag{48}$$

where a_k and b_k are coefficients to be determined, and different for x and y. Because x is an even function and y is odd, we can eliminate one of the terms in each sum:

$$x(t) = \sum_{k=0}^{\infty} a_k \cos k\omega_0 t \tag{49}$$

$$y(t) = \sum_{k=0}^{\infty} b_k \sin k\omega_0 t. \tag{50}$$

According to the rules of Fourier series, a_k becomes,

$$\frac{\omega_0}{\pi}\int_0^{\frac{2\pi}{\omega_0}} x(t)\cos k\omega_0 t dt. \tag{51}$$

The next step is to plug in for $x(t)$ and $\omega_0 t$ from equations 44 and 46. Also, because $\frac{du}{dt} = \frac{\omega_0}{1-\epsilon\cos u}$,

$$a_k = \frac{a}{\pi}\int_0^{2\pi}(1-\epsilon\cos u)(\cos u - \epsilon)\cos(ku - k\epsilon\sin u)du \tag{52}$$

$$= \frac{a}{k\pi}\int_0^{2\pi}(\cos u - \epsilon)\frac{d}{du}(\sin(ku - k\epsilon\sin u))du. \tag{53}$$

This integral can be solved via integration by parts:

$$(\cos u - \epsilon)\sin(ku - k\epsilon\sin u)\Big|_0^{2\pi} + \int_0^{2\pi}\sin(ku - k\epsilon\sin u)\sin(u)du. \tag{54}$$

The first term in equation 54 is zero. In order to solve the integral on the right, we must

utilize the following integral rule, sometimes called Bessel's First Integral:

$$J_k(x) = \frac{1}{\pi}\int_0^{\pi} \cos\left(kv - x\sin v\right)dv. \tag{55}$$

This looks close to equation 54. Differentiating with respect to x on both sides,

$$J_k'(x) = \frac{1}{\pi}\int_0^{\pi} \sin\left(kv - x\sin v\right)\sin\left(v\right)dv. \tag{56}$$

Rewriting equation 54,

$$= 2\int_0^{\pi} \sin\left(ku - k\epsilon\sin u\right)\sin\left(u\right)du \tag{57}$$

$$= 2\pi\left(\frac{1}{\pi}\int_0^{\pi} \sin\left(ku - k\epsilon\sin u\right)\sin\left(u\right)du\right) \tag{58}$$

$$= 2\pi J_k'(k\epsilon). \tag{59}$$

Therefore,

$$a_k = \frac{2a}{k}J_k'(k\epsilon). \tag{60}$$

Similar steps can be followed for b_k:

$$b_k = \frac{\omega_0}{\pi}\int_0^{\frac{2\pi}{\omega_0}} y(t)\sin k\omega_0 t dt \tag{61}$$

$$= \frac{b}{\pi}\int_0^{2\pi} (1-\epsilon\cos u)(\sin u)\sin\left(ku - k\epsilon\sin u\right)du \tag{62}$$

$$= -\frac{b}{k\epsilon\pi}\int_0^{2\pi} \sin u\frac{d}{du}(\cos\left(ku - k\epsilon\sin u\right))du. \tag{63}$$

Integrating by parts,

$$= \frac{b}{k\epsilon\pi}\int_0^{2\pi} \cos\left(ku - k\epsilon\sin u\right)\cos\left(u\right)du. \tag{64}$$

Integrating by parts again transforms the integral into

$$\left(\cos u \int \cos\left(ku - k\epsilon \sin u\right)\right)\Big|_0^{2\pi} + \int_0^{2\pi} \left(\int \cos\left(ku - k\epsilon \sin u\right)\right) \sin\left(u\right) du, \tag{65}$$

for which the second term integrates out to zero and the first term leaves us with, according to Bessel's First Integral,

$$b_k = \frac{2b}{k\epsilon} J_k(k\epsilon) = \frac{2a\sqrt{1-\epsilon^2}}{k\epsilon}. \tag{66}$$

Now we can use our expressions for a_k and b_k to write $x(t)$ and $y(t)$ more completely.

$$x(t) = \sum_{k=0}^{\infty} \frac{2a}{k} J_k'(k\epsilon) \cos k\omega_0 t \tag{67}$$

$$y(t) = \sum_{k=0}^{\infty} \frac{2a\sqrt{1-\epsilon^2}}{k\epsilon} J_k(k\epsilon) \sin k\omega_0 t. \tag{68}$$

Taking derivatives with respect to time and squaring,

$$\ddot{x}_k^2 = 4\frac{a^2}{k^2}(k\omega_0)^4 J_k'^2(k\epsilon)(\cos k\omega_0 t)^2 \tag{69}$$

$$\ddot{y}_k^2 = 4\frac{a^2}{k^2}\frac{1-\epsilon^2}{\epsilon^2}(k\omega_0)^4 J_k^2(k\epsilon)(\sin k\omega_0 t)^2. \tag{70}$$

The cycle-average value of $\ddot{q}^2$, denoted $\langle \ddot{q}^2 \rangle$, can be found by taking the average value of each of its components over one orbital period. In other words,

$$\langle \ddot{x}_k^2 \rangle = \langle 4\frac{a^2}{k^2}(k\omega_0)^4 J_k'^2(k\epsilon) \rangle \langle (\cos k\omega_0 t)^2 \rangle \tag{71}$$

$$\langle \ddot{y}_k^2 \rangle = \langle 4\frac{a^2}{k^2}\frac{1-\epsilon^2}{\epsilon^2}(k\omega_0)^4 J_k^2(k\epsilon) \rangle \langle (\sin k\omega_0 t)^2 \rangle, \tag{72}$$

in which every term reproduces itself, except for the trigonometric terms, which each produce $\frac{1}{2}$. Finally, we can combine our coordinate accelerations with the Larmor for-

mula.

$$P_k = \frac{e^2(\langle \ddot{x}_k^2 \rangle + \langle \ddot{y}_k^2 \rangle)}{6\pi\epsilon_0 c^3} \tag{73}$$

$$= \frac{e^2}{6\pi\epsilon_0 c^3}[2k^2\omega_0^4 a^2 (J_k'(k\epsilon))^2 + 2k^2\omega_0^4 a^2 \frac{1-\epsilon^2}{\epsilon^2} J_k^2(k\epsilon)] \tag{74}$$

$$= \frac{e^2}{3\pi\epsilon_0 c^3}(k\omega_0)^4 \frac{a^2}{k^2}[(J_k'(k\epsilon))^2 + \frac{1-\epsilon^2}{\epsilon^2} J_k^2(k\epsilon)] \tag{75}$$

This semi-classical formula has a heavy quantum influence, in that the spectrum of possible frequency and power values is not continuous. Instead, these values are discretized by k. In addition, the variables ω_0, a, and ϵ depend on the quantum numbers n and l. One aspect of this equation that sets it apart from the quantum, however, is that the discretization is evenly spaced. As we will see in the following section, the spacing between fundamental quantum quantities is not even.

4 Quantum Radiation

The quantum mechanical description of electron radiation is significantly different, at least in its inception. In quantum mechanics, electrons exist in discrete states, rather than in a continuous spectrum as classical mechanics would imply. These states are governed by variables known as "quantum numbers."

4.1 Quantum Numbers

4.1.1 Energy

An electron's energy level, denoted by n, is referred to as the principal quantum number. An electron can exist only in distinct energy states. All of the energy values in between the allowed states are quantum mechanically forbidden. The most common way for an electron to jump between energy levels is by emitting or absorbing a single photon, whose energy is exactly the magnitude of the difference in energy between the two electron states. If a photon with an energy that does not match a difference between two states hits the atom, it will pass right through, unabsorbed by the electrons. Because a photon's energy determines its frequency, every atom has an absorption and an emission signature made apparent through either the only absent or the only present photon wavelengths in a measurement.

For hydrogen, the lowest energy possible is assigned an n value of one. This state, known as the ground state, is given by the negative Rydberg constant $-R$, about -13.6 eV or $-2.18 * 10^{-18}$ J. The energy of state n is then given in terms of this ground state value by $-\frac{R}{n^2}$, or the energy of the ground state scaled by the square of the principle quantum number. Unlike the semi-classical power equation, in which frequency values are evenly spaced, the energy levels (and therefore photon frequency values) in quantum mechanics are choppy and inconsistent. Every energy value is closer in absolute terms to one level up than it is to one level down. As a result, the largest gap between two energy

levels is between the $n = 1$ and $n = 2$ states. From there, every distance is smaller than the last. Not only that, but the decrease in differences between energy levels slows down as n increases. This can be viewed through a correspondence limit lens: for larger n, the spacing between energy levels and wavelengths gets smaller and smaller, resembling the continuous classical approximation.

4.1.2 Angular Momentum

The straightforwardness of the energy levels is complicated slightly by the spin properties of photons. A photon has an intrinsic spin angular momentum of $\pm\hbar$, so, by conservation of angular momentum, a transitioning electron must change its angular momentum state. The second quantum number, denoted by l, governs the orbital angular momentum. Just like its energy, an electron's angular momentum is quantized. Its allowed angular momentum states are limited by its energy state. For a principal quantum number n, the allowed values of l are 0, 1, 2, ..., $n - 1$. The angular momentum quantum number can not equal or exceed the energy.

As was just mentioned, an electron that transitions from one energy level to another via single photon emission must also move either up or down one angular momentum state, which limits its transition possibilities. See Figure 3 for a depiction of the allowed transitions between the first four energy and angular momentum states. Each dash is a possible state. The energy states correspond to the rows, while the angular momentum states correspond to the columns. Each angular momentum state is assigned a letter: s for $l = 0$, p for $l = 1$, d for $l = 2$, etc.

4.1.3 Magnetism

The final quantum number considered in this thesis is the magnetic (m) quantum number. Physically, m represents the orientation of the angular momentum vector whose total value is given by the angular momentum quantum number. m affects the energy

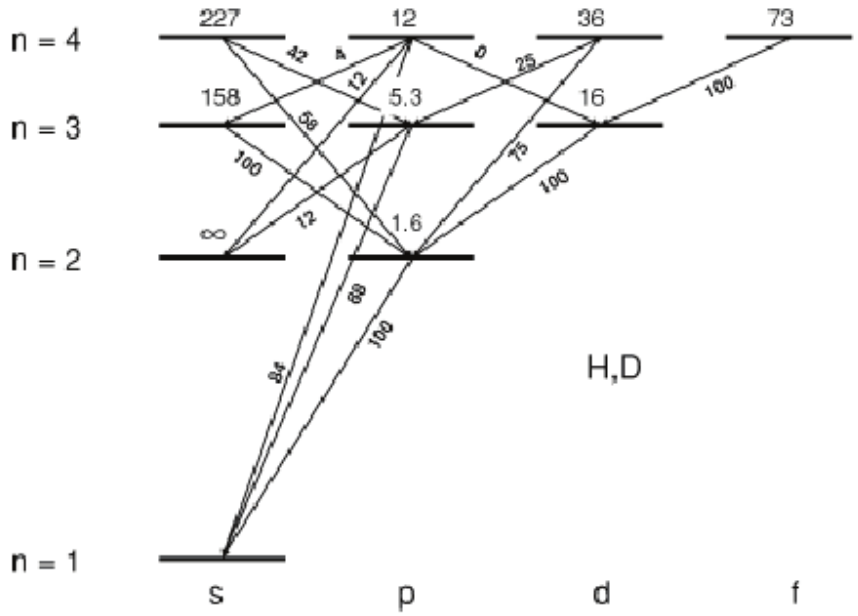

Figure 3: A Grotrian diagram borrowed from [13]. Notice the uneven spacing between energy levels, especially between $n = 1$ and the rest. This is something that approximately vanishes at high n. Also, transitions are only allowed between adjacent angular momentum states, and values of l greater than or equal to n are forbidden.

of an electron if it is placed in an external magnetic field. However, our theoretical hydrogen atoms exist in vacuum. Bethe and Salpeter note, "The life time of a state is independent of its magnetic quantum number and depends only on n and l." [14] The life time of a state is inversely proportional to its transition probability, a value that connects frequency of emitted photons to power, as we will see in the next section. We will also see the dependence of frequency on only n. However, because m can take on every integer value from $-l$ to l, it increases the degeneracies of the angular momentum states of the electron. As such, although m will not be considered explicitly, it has an implicit effect on the power results.

4.1.4 Fine and Hyperfine Structure

The last two quantum numbers, s and I, describe the spin properties of the electron and the nucleus, respectively. A moving charge, such as an orbiting electron, produces an internal magnetic field, which affects the system. This field interacts with both the nuclear spin and the electron spin to split degenerate energy levels. When electron spin causes the split in energy level, it is known as fine structure. When the effect comes from the nucleus, it is known as hyperfine structure. Both effects are small and can be ignored with minimal error for the purposes of this thesis.

4.2 Quantum Power Equation

Now that we have an understanding of the fundamentals that compose a quantum system, we can derive the mathematics that define frequency and power in a quantum sense. Frequency, as already mentioned, is a property of the emitted photon that is proportional to its energy. The quantum interpretation of power is the energy lost by transitioning from one state to the next, multiplied by the probability (in inverse seconds) of that transition happening. The sum of all of these probabilities for a particular state gives the inverse of the lifetime of that state.

Mathematically,

$$P = \hbar\omega A_{if}. \tag{76}$$

In the above equation, ω is the angular frequency of the emitted photon. Two equations govern the relationship between ω and other fundamental properties of a photon:

$$\omega = 2\pi\nu \tag{77}$$

$$\lambda\nu = c, \tag{78}$$

where λ and c are wavelength and speed of light, respectively, in appropriate units. When

λ is measured in centimeters, the units of its inverse are often called wave numbers.

Scaled by the reduced Planck constant, $\hbar$, ω becomes the energy of the emitted photon. This energy is the difference in energy between the initial (n) and final (n') electron states:

$$\hbar\omega = R\left(\frac{1}{n'^2} - \frac{1}{n^2}\right). \tag{79}$$

The Rydberg constant R is the electron ground state energy, given by

$$\frac{me^4}{2(4\pi\epsilon_0)^2\hbar^2} \approx 2.18 * 10^{-18} J. \tag{80}$$

Finally, A_{if} is known as the Einstein coefficient of spontaneous emission. There are three Einstein coefficients; the other two govern stimulated emission and absorption. A_{if} can be thought of as representing the number of transitions per second from a higher energy level to a lower level. The following equation, taken from a mix of Bethe and Salpeter [14] and Wiese [15], demonstrates the dependence of the transition probability on the frequency of the emitted photon, ω, and a unitless quantity, f, known as the oscillator strength of a transition:

$$A_{if} = 8 * 10^9 \left(\frac{\omega}{2\pi Ry}\right)^2 f. \tag{81}$$

In this equation, Ry is the Rydberg frequency, with a value of $3.29 * 10^{15}\ s^{-1}$. The quantity f is given by yet another equation that depends on yet another thus-far-undefined value:

$$f = \frac{1}{3}\frac{\omega * \max(l, l')}{(2l+1)2\pi Ry} {d_{if}}^2. \tag{82}$$

Where $\max(l, l')$ is the larger of the initial and final angular momentum states and d_{if}

is the dipole moment of a certain transition. Finally,

$$d_{if} = \int_0^\infty R_{nl} R_{n'l-1} r^3 dr. \tag{83}$$

The R_{ij}'s are the radial equations corresponding to the initial and final electron states, which are dependent on associated Laguerre functions given by the solution to the time independent Schrödinger equation,

$$\hat{H}\Psi = E\Psi, \tag{84}$$

where $\hat{H} = -\frac{\hbar}{2m}\nabla^2 - \frac{e^2}{4\pi\epsilon_0 r}$ is the Hamiltonian of the electron in a hydrogen atom, Ψ is the electron's wave function, and E is the system's energy level. This differential equation can be solved by separation of variables in spherical coordinates to produce three terms that are θ dependent, ϕ dependent, and r dependent, respectively. The r dependent term is the following:

$$R_{nl}(r) = -\frac{(n-l-1)!^{\frac{1}{2}}}{(n+l)!^{\frac{3}{2}}(2n)^{\frac{1}{2}}}\left(\frac{2}{n}\right)^{\frac{3}{2}} e^{-\frac{r}{n}}\left(\frac{2r}{n}\right)^l L_{n+1}^{2l+1}\left(\frac{2r}{n}\right). \tag{85}$$

In this equation, $L_a^b = \frac{d^b}{d\rho^b}(e^\rho \frac{d^a}{d\rho^a}(e^{-\rho}\rho^a))$ are the associated Laguerre functions. Finally, the integral for the dipole moment can be evaluated to produce a general formula for every electron transition. The solution is complex, so I quote the result from Bethe and Salpeter [14].

$$d_{if} = \frac{(-1)^{n'-1}}{4(2l-1)!}\sqrt{\frac{(n+l)!(n'+l-1)!}{(n-l-1)!(n'-l)!}}\frac{(4nn')^{l+1}(n-n')^{n+n'-2l-2}}{(n+n')^{n+n'}} * \tag{86}$$

$$\left[F\left(-n_r, -n_r', 2l, -\frac{4nn'}{(n-n')^2}\right) - \left(\frac{n-n'}{n+n'}\right)^2 F\left(-n_r-2, -n_r', 2l, -\frac{4nn'}{(n-n')^2}\right)\right], \tag{87}$$

Where

$$F(\alpha,\beta,\gamma,x)=\sum_{\nu}\frac{\alpha(\alpha+1)...(\alpha+\nu-1)(\beta)...(\beta+\nu-1}{\gamma...(\gamma+\nu-1)\nu!}x^{\nu} \tag{88}$$

is the hypergeometric function, $n_r = n - l - 1$, and $n'_r = n' - l$. Thus completes the puzzle that started with a simple $P = \hbar\omega A_{if}$. Substituting gives,

$$P=\frac{\hbar\omega^4 * \max(l,l')}{3\pi^3(2l+1)Ry^3}d_{if}^2 * 10^9 \tag{89}$$

$$=\frac{m^4e^{16} * \max(l,l')}{48\pi^3(2l+1)(4\pi\epsilon_0)^8\hbar^{11}Ry^3}{d_{if}}^2\left(\frac{1}{n'^2}-\frac{1}{n^2}\right)^4 * 10^9. \tag{90}$$

A table of values of the important quantum parameters for every transition up to $n = 15$ is given in Appendix E. What immediately stands out in the quantum power equation is the dependence on ω^4, which it shares with the semi-classical power equation. This correlation bodes well for correspondence.

4.3 Selection Rules

There are a few "selection rules" in the transition of an electron to different states. The rule most relevant for our purposes is that a transitioning electron must change its angular momentum quantum number by exactly ± 1. It cannot jump up or down the l ladder very far, nor can it retain its value of l.

Because frequency is dependent only on the initial and final energy levels and not on l, for a given final state with angular momentum l, there is another state, $l+2$, for which the frequency of the transition is the same. However, because power is dependent on the values of l of both the initial and final states, the power radiated during the transition is not the same. This produces a two-fold degeneracy for every value of frequency besides the two largest. In other words, almost every transition frequency is associated with two different values of power, one for $\Delta l = +1$ and one for $\Delta l = -1$. In order to calculate the total power for each frequency value, the two individual powers must be summed.

5 Mathematical Analysis

5.1 Relativistic Consideration

Before moving on to comparing the semi-classical and quantum results, a quick note on relativity is essential. As mentioned before, the Larmor formula, from which our semi-classical power equation is derived, is the power radiated by the acceleration of a non-relativistic charged particle. In addition, the quantum equations do not consider relativity. So if the electron is relativistic, the entire analysis would fall apart.

The electron's velocity in each energy level is given by,

$$v_n = \frac{v_0}{n} = \frac{e^2}{4\pi\epsilon_0\hbar n} \tag{91}$$

$$= \frac{e^2}{4\pi\epsilon_0\hbar nc}c \tag{92}$$

$$= \frac{1}{137.2n}c. \tag{93}$$

For the lowest value of n, $n = 1$, this formula produces a Lorentz factor of

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{94}$$

$$= \frac{1}{\sqrt{1 - \frac{c^2}{(137.2c)^2}}} \tag{95}$$

$$= 1.00002656 \approx 1, \tag{96}$$

so the effect due to relativistic considerations is negligible. At higher n, which this paper is more focused on, this effect practically disappears. $n = 100$, for example, gives an error term due to the Lorentz factor on the order of 10^{-9}.

5.2 Frequency

The correspondence of the quantum and the semi-classical can be examined for both power and frequency. The frequencies are given by, in the semi-classical and quantum cases,

$$\omega_c = \frac{me^4}{(4\pi\epsilon_0)^2\hbar^3}\frac{1}{n^3} \tag{97}$$

$$\omega_q = \frac{me^4}{(4\pi\epsilon_0)^2\hbar^3}\frac{1}{2}\left(\frac{1}{n'^2} - \frac{1}{n^2}\right). \tag{98}$$

These two equations are similar in their usage of fundamental constants, but different in a few ways. The semi-classical dependence on energy level is n^{-3}, while the quantum dependence is $n'^{-2} - n^{-2}$, with an additional factor of $\frac{1}{2}$. A brief mathematical analysis will prove that they converge for large n. Rewriting $n'^{-2} - n^{-2}$ gives,

$$\frac{n^2 - n'^2}{n^2 n'2}. \tag{99}$$

Introducing δ, where $\delta = n - n'$, allows us to write n' in terms of n and a constant:

$$\frac{n^2 - (n-\delta)^2}{n^2(n-\delta)^2} = \frac{2n\delta - \delta^2}{n^4 - 2n^3\delta + n^2\delta^2}. \tag{100}$$

The value of δ can range from 1, which describes emission of a photon with the lowest possible frequency, up to $n-1$, which is the case of decay from the p to the s state. For $\delta = n - 1$, equation 100 becomes $\frac{n^2-1}{n^2}$, which is not the $\frac{2}{n^3}$ that was anticipated. However, as δ decreases, the equation approaches the desired value, until δ reaches 1, for which equation 100 turns into,

$$\frac{2n-1}{n^4 - 2n^3 - n^2}, \tag{101}$$

which, in the limit of large n, becomes $\frac{2}{n^3}$. Even still, for small n, there is significant error, underlining the effect on correspondence of increasing the values of quantum numbers.

It is this lowest frequency, ω_0 corresponding to $\delta = 1$, for which it is most essential for the semi-classical and quantum equations to correlate, because, while the quantum frequencies are given by the emission spectra of the hydrogen atom, the semi-classical frequencies are produced in a more rudimentary way. They are calculated from $k\omega_0$, where $k = 1, 2, 3...$ is an integer. This produces equal spacing between the frequency values, which is not the case for the quantum frequencies. However, if ω_0 is the same for both approaches, the shapes of the graphs should be similar for larger values of δ and larger values of k, even if the frequencies fail to correspond exactly.

As an example, let us consider the frequency values for decay from the p state for $n = 100$. In units of ω^*, where $\omega^* = \frac{me^4}{(4\pi\epsilon_0)^2\hbar^3}$, the semi-classical ω_0 is 10^{-6}, and the quantum is $1.02 * 10^{-6}$. The quantum equation reaches maximum frequency during decay to the ground state. In the same units, this frequency, ω_{98}, is 0.50, or $5 * 10^5\omega_0$. Because k increases by one for every successive semi-classical frequency value, it would take $5 * 10^5$ points to reach this frequency, far more than the 99 points on the quantum graph. In other words, the disconnect between large frequencies in the semi-classical and quantum cases is not a problem. In fact, the *hope* is that a large δ would lead to inaccuracies, because the correlation should exist between the ω_0 values. What this tells us is that the semi-classical graph simply needs far more points than does its quantum counterpart.

5.3 Semi-Classical Power Versus Time

As an evaluation of the data, we can look at the instantaneous power emission. Thus far, because of the Fourier series applied to the coordinate variables, we have been studying the cycle-averaged power for fundamental frequency modes in the semi-classical case.

Such an analysis is certainly useful for comparison with the quantum results, and will be used for the remainder of this paper. However, converting to power as a function of time allows for greater intuition about semi-classical radiation.

The first step is to look at displacement from the origin as a function of time. Going back to the derivation of the semi-classical power equation,

$$x(u) = a(\cos u - \epsilon) \tag{102}$$

$$y(u) = b \sin u \tag{103}$$

$$\omega_0 t = u - \epsilon \sin u. \tag{104}$$

Therefore, r, defined as distance from the origin, can be written,

$$r = \sqrt{x^2 + y^2} \tag{105}$$

$$= \sqrt{a^2(\cos u - \epsilon)^2 + (b \cos u)^2}. \tag{106}$$

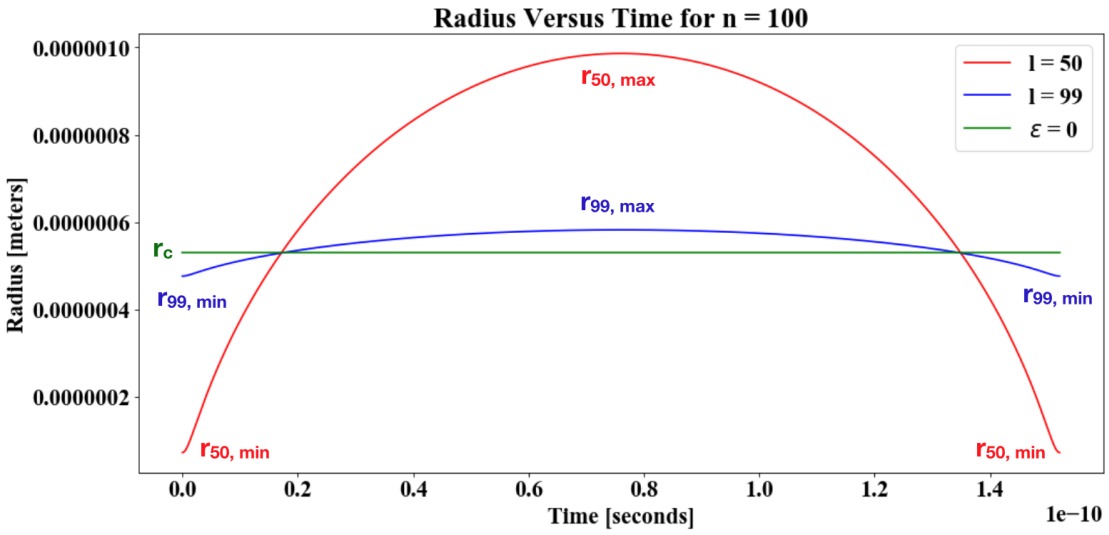

Figure 4: Distance from the origin versus time for one orbital period at the $n = 100$ energy level

We now have r as a function of u, and u defined implicitly as a function of t in the expression for frequency, so a plot of r as a function of t can be made for every n and l value. The plot for $n = 100$ is shown in Figure 4.

Three different eccentricities are considered over one orbital period, $\frac{2\pi}{\omega_0} \approx 1.5{*}10^{-10}$ seconds. The first, in red, corresponds to $l = 50$. The second, in blue, corresponds to what we have been referring to as "circular orbit," which is the highest possible angular momentum state for a given energy state, or $l = 99$ in this case. The last one, in green, represents a geometrically circular orbit, with eccentricity set to zero. While the $l = 99$ case is similar to the green, they do not quite overlap, and we can refer to the blue plot as "nearly circular."

To achieve some intuition about what each of these curves represents in terms of a real orbit, we can refer to Figure 5. When ϵ is zero, the orbit is perfectly circular, and

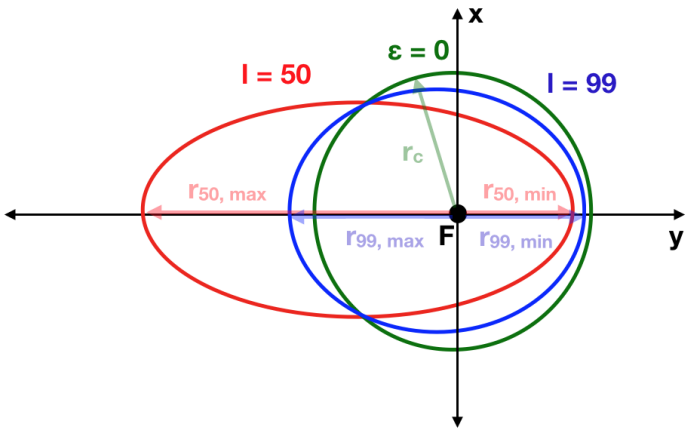

Figure 5: Three possible orbital paths for an electron with $n = 100$, with important radii marked. The nucleus sits at the origin, which is a focus of the two elliptical orbits and the center of the circular orbit.

the radius of that orbit, r_c, is constant. For $l = 99$, the orbit is almost circular. The eccentricity is 0.1, and the distance between r_{min} and r_{max} is small. The $l = 50$ result

is the most elliptical, and therefore has the largest variation between its minimum and maximum distances from the origin. As a test, the average of r_{min} and r_{max} for each of these curves should be equal to the semimajor axis for an orbit at this energy level, given by $n^2 a_0$, and indeed they are.

$$\frac{1}{2}(r_{50,min} + r_{50,max}) = \frac{1}{2}(7.24 * 10^{-8} + 9.86 * 10^{-7}) = 5.29 * 10^{-7} m \tag{107}$$

$$\frac{1}{2}(r_{99,min} + r_{99,max}) = \frac{1}{2}(4.76 * 10^{-7} + 5.82 * 10^{-7}) = 5.29 * 10^{-7} m \tag{108}$$

$$\frac{1}{2}(r_c + r_c) = r_c = 5.29 * 10^{-7} m \tag{109}$$

$$n^2 a_0 = 5.29 * 10^{-7} m. \tag{110}$$

The results for distance can be extended to describe the velocity v, defined as $\sqrt{v_x^2 + v_y^2}$, or,

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} \tag{111}$$

$$= \sqrt{\left(\frac{dx}{du}\frac{du}{dt}\right)^2 + \left(\frac{dy}{du}\frac{du}{dt}\right)^2} \tag{112}$$

$$= \frac{du}{dt}\sqrt{(a\sin u)^2 + (b\cos u)^2} \tag{113}$$

$$= \frac{\omega_0}{1 - \epsilon\cos u}\sqrt{(a\sin u)^2 + (b\cos u)^2}, \tag{114}$$

and a graph, like the one for distance from the origin, can be plotted for velocity as a function of time. See Figure 6.

The value of v for the circular orbit is $2.19 * 10^4 \frac{m}{s}$, which matches up with $\frac{v_0}{n}$. When distance achieves a maximum, velocity achieves a minimum, and vice versa. This is due to the conservation of angular momentum, which is proportional to a cross product of the radial distance and the velocity. When one increases, the other must decrease. A fact of circular orbit, just like its constant radius, is its constant velocity (in magnitude).

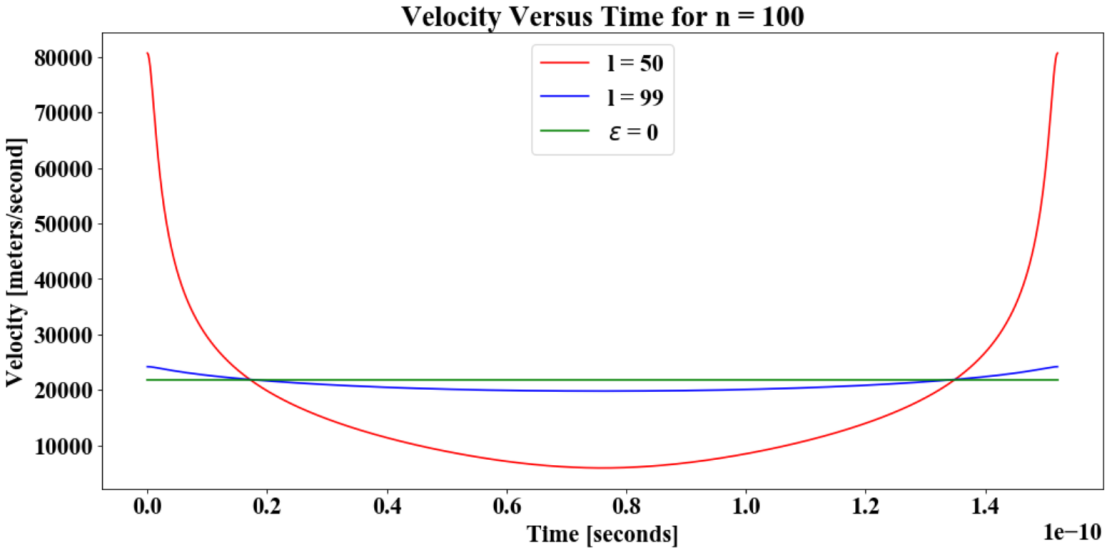

Figure 6: Velocity versus time for one orbital period at the $n = 100$ energy level

However, the direction of the velocity is changing, providing an acceleration, even in the circular case.

One more time derivative of the coordinate variables can be taken to find the acceleration, $\dot{v}$, defined by $\sqrt{\ddot{x}^2 + \ddot{y}^2}$.

$$\dot{v} = \sqrt{\left(\frac{d^2x}{dt^2}\right)^2 + \left(\frac{d^2y}{dt^2}\right)^2} \tag{115}$$

$$= \sqrt{\left[a\cos u\left(\frac{du}{dt}\right)^2 + a\sin u\frac{d^2u}{dt^2}\right]^2 + \left[b\sin u\left(\frac{du}{dt}\right)^2 - b\cos u\frac{d^2u}{dt^2}\right]^2}, \tag{116}$$

where,

$$\frac{du}{dt} = \frac{\omega_0}{1 - \epsilon\cos u} \tag{117}$$

$$\frac{d^2u}{dt^2} = -\frac{\epsilon\omega_0^2\sin u}{(1 - \epsilon\cos u)^3} \tag{118}$$

Using this result, a graph of acceleration versus time can be produced, like that in Figure 7.

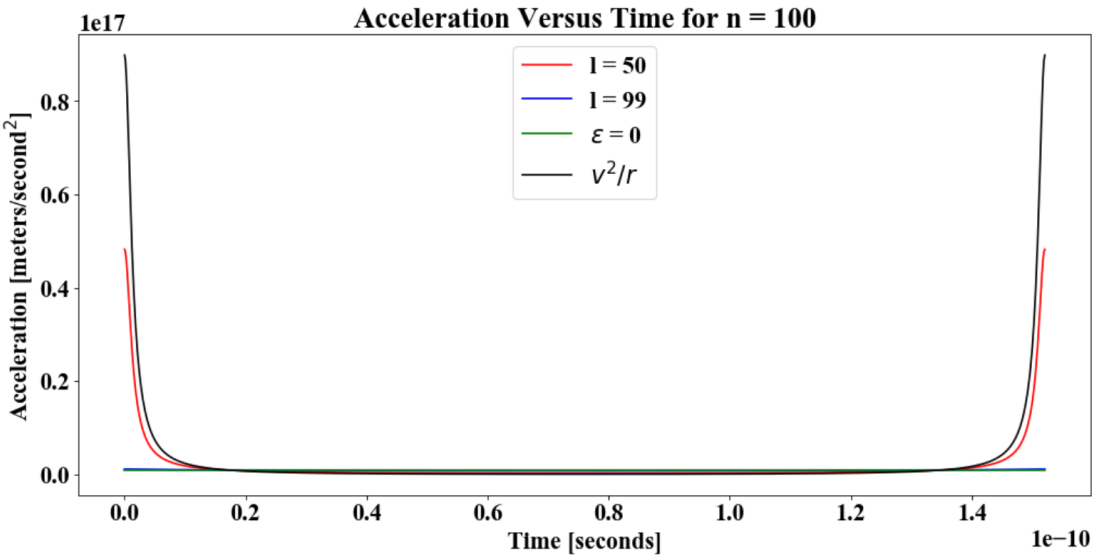

Figure 7: Acceleration versus time for one orbital period at the $n = 100$ energy level

The magnitudes of the circular values are too small to see here, because of how sharply peaked the acceleration is at its maximums. A plot of $\frac{v^2}{r}$ is included because it should track with the acceleration, and it does, especially for the less extreme values. The circular values of v and r should also reproduce the circular acceleration in $\frac{v^2}{r}$.

$$\frac{v_c^2}{r_c} = 9.04 * 10^{14} \frac{m}{s^2} \tag{119}$$

$$a_c = 9.04 * 10^{14} \frac{m}{s^2}. \tag{120}$$

Now, all that remains is to find the square of this acceleration, because the Larmor formula defines instantaneous power as proportional to acceleration squared. A final graph, this one of acceleration squared versus time, is shown in Figure 8.

On this plot, the $l = 99$ and $\epsilon = 0$ cases were even less visible, so we excluded them entirely. The purpose of this plot is to give qualitative intuition of instantaneous power emission. The magnitude of the power is sharply peaked towards the beginning and end of each period, and it flattens out in the middle. Referring back to the figure

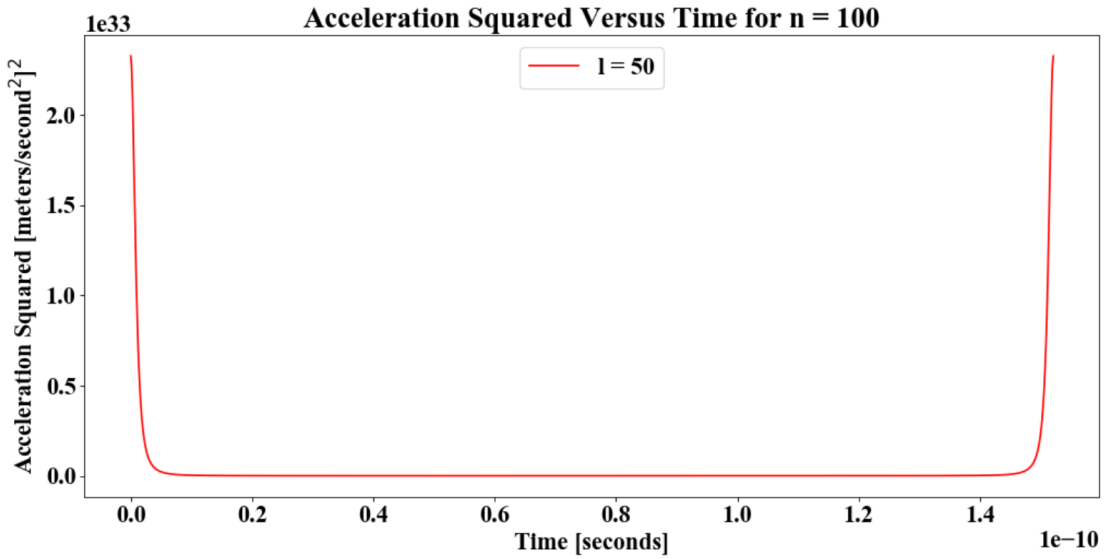

Figure 8: Acceleration squared versus time for one orbital period at the $n = 100$ energy level

of the elliptical orbit, the power is intense for small values of r, and it peaks at r_{min}. It weakens as r increases, and experiences a minimum at r_{max}. The semi-classical radiation formula presented as a function of frequency depends on the coefficients of the Fourier series produced by this graph.

6 Circular Orbit

6.1 A Quantum–Semi-Classical Connection

Now that we have a greater understanding of what l and n actually represent, we can take another look at the eccentricity formula:

$$\epsilon = \sqrt{1 - \frac{l(l+1)}{n^2}} \tag{121}$$

A few interesting facts can be deduced from examining the circular case. For circular orbits, the semimajor and semiminor axes are equivalent, so the eccentricity ($\sqrt{1 - \frac{b^2}{a^2}}$) is 0, meaning that $\frac{l(l+1)}{n^2} = 1$. Quantum mechanically, the circular case in terms of l and n is when l is at its largest allowed value for a given energy level, meaning $l = n-1$. The fraction under the square root becomes $\frac{n^2-n}{n^2} = 1$, which isn't quite true at smaller n (although $l = n-1$ is still the closest that ϵ gets to being 0), but is true as n grows larger. This is just another example of the correspondence limit allowing for more accurate, and converging, calculations. For instance, for $n = 2$, $\frac{n^2-n}{n^2} = 0.5$, while a much larger n such as $n = 100$ gives $\frac{n^2-n}{n^2} = 0.99$.

Another notable feature of the orbital eccentricity provides an interesting justification for the limits placed on an electron's angular momentum state. In order for the orbit's eccentricity to be real, $n^2 \geq l(l+1)$. Because n and l carry integer values, this places the following restriction on l: $l \leq n-1$. This quantum mechanical fact has been implied with a semi-classical justification.

To test the accuracy of the equations thus far, it is worthwhile to look further into the specific case of circular orbit.

6.2 Frequency

The quantum and semi-classical frequencies for circular orbit can be found by simplifying the equations for the elliptical case. It requires a brief glance at the Grotrian diagram to understand how many frequency values are at play. In the quantum case, a circular orbit corresponds to the decay from the lowest energy state of any particular angular momentum state to its only allowed resulting state, which is the lowest energy state of the next lowest angular momentum state. Therefore, there is a single value of frequency to consider. A classical way of thinking about it is that a circular orbit corresponds to a constant acceleration, which is consistent with a single orbital frequency.

In terms of quantum numbers, the circular case is the decay from n, l to $n-1$, $l-1$, where $l = n-1$. Therefore,

$$\omega_c = \omega^* \frac{1}{n^3} \tag{122}$$

$$\omega_q = \omega^* \frac{1}{2}\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right). \tag{123}$$

ω_q becomes,

$$\omega^* \frac{1}{2} \frac{2n-1}{n^4 - 2n^3 - n^2}. \tag{124}$$

In the limit of large n, this becomes $\omega^* \frac{1}{n^3}$, and the only value of frequency for the quantum and semi-classical cases converge. Figure 9 shows the frequency values for every circular decay from $n = 2$ to $n = 50$. In order to make the correspondence more clear, the base ten logarithm values of the frequencies are depicted. Table 2, a table of the true values, is included in Appendix B for reference.

Unsurprisingly, the correspondence between the semi-classical and quantum frequencies improves with increasing n, which the mathematical analysis predicted. This is a good check on the reasonableness of the frequency equations.

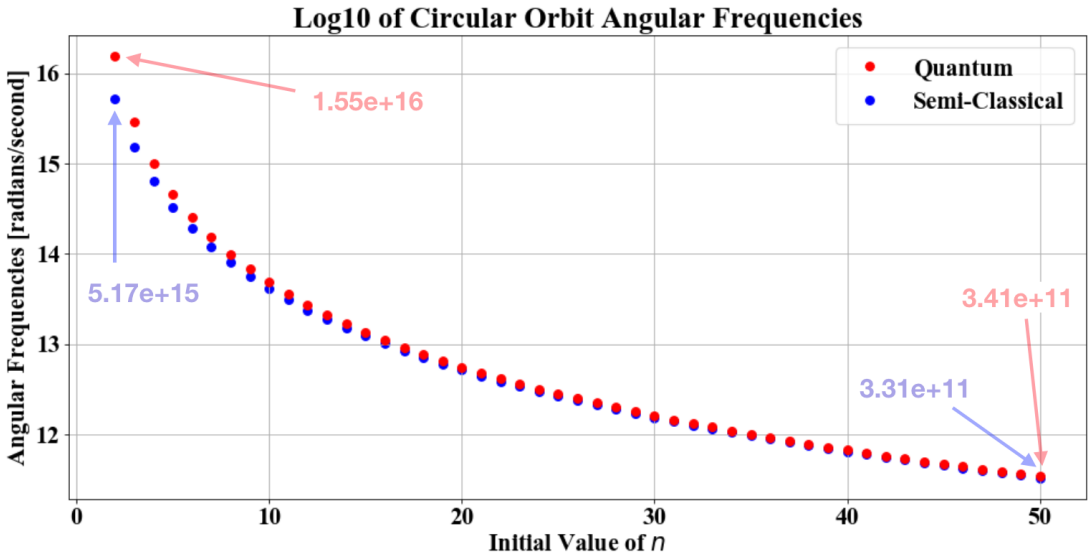

Figure 9: Angular frequency versus n value for circular orbit

6.3 Semi-Classical Power

Next, we will take a look at the semi-classical power equation in the circular case. Of course, it should reduce exactly to the Larmor formula, from which it was derived. Rewriting the Larmor formula by plugging in $\omega^2 a$ for $\dot{v}$,

$$P = \frac{2e^2\dot{v}^2}{3(4\pi\epsilon_0)c^3} \tag{125}$$

$$= \frac{e^2\omega^4 a^2}{6\pi\epsilon_0 c^3}. \tag{126}$$

But what does a circular orbit mean for the Jackson equation in terms of its parameters, particularly ϵ and k? One of two things can happen with the ϵ term. In a purely theoretical sense, the eccentricity can be set to 0. However, in practice, the eccentricity *approaches* 0, but mathematically never quite makes it because of the l and n dependence of b and a. This value of eccentricity can be termed "nearly circular." Both of these possibilities will be explored in the next sections. What happens with k is consistent for both explanations. As mentioned in the previous section, the allowed values

of frequency are limited by the allowed decays of the electron's energy and angular momentum state. There is a single frequency at which the electron radiates power, and, because k discretizes the frequency, $k = 1$.

6.3.1 Setting Eccentricity to Zero

For circular orbit, the Jackson equation can be rewritten as:

$$P_k(\omega) = P_1(\omega) = \frac{e^2}{3\pi\epsilon_0 c^3}\omega^4 a^2[(J_1'(\epsilon))^2 + \frac{1-\epsilon^2}{\epsilon^2}J_1^2(\epsilon)], \tag{127}$$

where we set $\epsilon = 0$. The term outside the brackets is unaffected by the new eccentricity. Inside the brackets, the square of the derivative of the Bessel function evaluated at zero is 1/4. Because the Bessel function of order one evaluated at zero is zero, the term on the right inside the brackets requires a Taylor series expansion around $\epsilon = 0$. The Bessel function of order k expands as follows:

$$J_k(\epsilon) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\Gamma(k+n+1)}(\frac{\epsilon}{2})^{k+2n}. \tag{128}$$

In the sum, $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$ is the gamma function, whose integer values are $\Gamma(\nu) = (\nu-1)!$. Applying this expression to the Bessel function of order one and taking it to third order in ϵ gives $J_1 \approx \frac{\epsilon}{2} - \frac{\epsilon^3}{16}$. After squaring and combining with the $\frac{1-\epsilon^2}{\epsilon^2}$ term, it gives an additional 1/4. So the term in brackets produces a factor of $\frac{1}{2}$. And the elliptical formula produces, for the circular case,

$$P = \frac{e^2\omega^4 a^2}{6\pi\epsilon_0 c^3}, \tag{129}$$

which matches up with equation 126.

6.3.2 Nearly Circular

The Jackson equation can be written the exact same way as equation 127:

$$P_k(\omega) = P_1(\omega) = \frac{e^2}{3\pi\epsilon_0 c^3}\omega^4 a^2[(J_1'(\epsilon))^2 + \frac{1-\epsilon^2}{\epsilon^2}J_1^2(\epsilon)]. \tag{130}$$

Now, however, instead of setting ϵ to zero, we will plug in values from the parameters section for b^2 and a^2 in the formula $\epsilon^2 = 1 - \frac{b^2}{a^2}$. The eccentricity becomes,

$$\epsilon = \sqrt{1 - \frac{l(l+1)}{n^2}}. \tag{131}$$

For a circular orbit, $l = n - 1$, and terms cancel until the square of the eccentricity is $\frac{1}{n}$. This is where the distinction between this method and simply setting the eccentricity to zero becomes clear. It is true that in the limit of large n the eccentricity does go to zero. However, it is worthwhile to test its behavior for more reasonable values of n to get a more exact result. The Jackson formula can be restated.

$$P_1(\omega) = \frac{e^2}{3\pi\epsilon_0 c^3}\omega^4 a^2[(J_1'(n^{-1/2}))^2 + \frac{1-\frac{1}{n}}{\frac{1}{n}}J_1^2(n^{-1/2})]. \tag{132}$$

The term on the left inside the brackets is still close to 1/4 for most n. The term on the right, following an expansion, becomes,

$$(n-1)(\frac{1}{2\sqrt{n}} - \frac{1}{16n^{3/2}} + ...)^2. \tag{133}$$

This is, again, 1/4, with an error term of $\frac{1}{4n}$. In other words, each method of evaluating eccentricity led to a very similar result, especially at large n. It is the low n terms where they should differ most.

6.4 Quantum Power

What happens to the quantum power result in the circular case? Written in terms of frequency and the dipole moment, it reads,

$$P = \frac{\hbar\omega^4 * \max(l, l')}{3\pi^3(2l+1)Ry^3} d_{if}^2 * 10^9. \tag{134}$$

For a circular orbit, $\max(l, l') = l$ and $l = n - 1$. This leaves the power equation with constants, ω dependence, n dependence, and the d_{if}^2 term. Conveniently, the hypergeometric function on which d_{if} depends can be expanded.

$$F(\alpha, \beta, \gamma, \delta) = 1 + \frac{\alpha\beta}{\gamma}x + \frac{\alpha(\alpha+1)\beta(\beta+1)}{\gamma(\gamma+1)2!}x^2 + ... \tag{135}$$

For both hypergeometric functions involved in calculating d_{if}, our $\beta = l - n'$. For circular orbit, $l = n'$, so β becomes zero, and each hypergeometric function simplifies to 1. The equation for d_{if} becomes,

$$d_{if} = \frac{1}{4}\sqrt{(2n-1)(2n-2)}\frac{(4n(n-1))^{n+1}}{(2n-1)^{2n+1}}. \tag{136}$$

In the limit of large n, the term under the square root becomes $4n^2$. The fractional term can be rewritten as

$$\frac{(4n^2-4n)^{n+1}}{(4n^2-4n+1)^{n+\frac{1}{2}}}, \tag{137}$$

which becomes,

$$\sqrt{4n^2-4n} \approx 2n. \tag{138}$$

In this limit, therefore, $d_{if} \sim n^2$. Rewriting our power equation with this new d_{if},

$$P = \frac{\hbar\omega^4 n^4 (n-1)}{3\pi^3 (2n-1) Ry^3} * 10^9 \tag{139}$$

$$\approx \frac{\hbar\omega^4 n^4}{6\pi^3 Ry^3} * 10^9. \tag{140}$$

Plugging in for everything but ω and n,

$$P_q = P^* * \omega^4 n^4, \tag{141}$$

where $P^* = 1.59 * 10^{-74}$. Replacing constants in the semi-classical power equation for a circular orbit,

$$P_c = \frac{e^2 \omega^4 a^2}{6\pi\epsilon_0 c^3} \tag{142}$$

$$= \frac{e^2 \omega^4 a_0^2 n^4}{6\pi\epsilon_0 c^3} \tag{143}$$

$$\approx P^* * \omega^4 n^4, \tag{144}$$

so the quantum and semi-classical equations for power coincide for large n. See Figure 10 for a comparison of three results at different n: the circular quantum radiation, the nearly circular semi-classical radiation, and the circular semi-classical radiation. The base ten logarithm is depicted so that the graph is easier to interpret. For reference, Tables 3 and 4 in Appendix C have the true values of power spectral density for these graphs.

For low values of n, all three interpretations diverge. The values for the two different semi-classical methods are similar, but still a factor of two off. The quantum result is relatively enormous. However, as n increases, so, too, does the correspondence, until the two semi-classical methods overlap almost perfectly, with the quantum result not much larger.

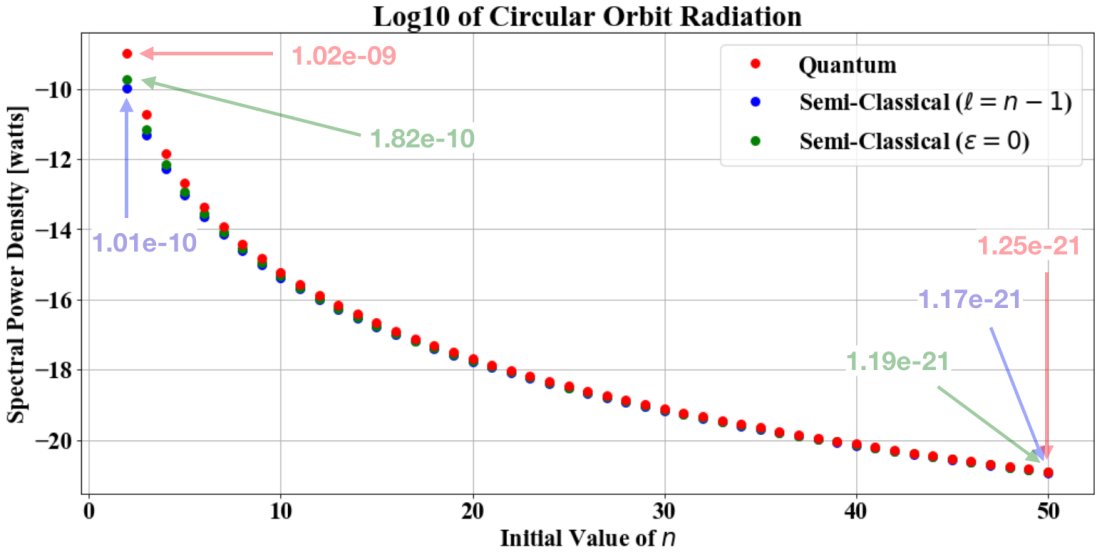

Figure 10: Spectral power density versus n value for circular orbit

This circular analysis, although a special case of the elliptical analysis that is the ultimate goal of this thesis, is useful for a few reasons. First, it helps demonstrate the principles at work in a simplified context. The angular momentum quantum number is eliminated, facilitating visualization of acceleration and power which are both constant for a given n. Also, it provides a check on the accuracy of the equations. If simplifying the elliptical equations for a circular orbit produced problematic results, then those equations would need to be investigated further. Finally, it demonstrates that correspondence is possible, for both frequency and power.

7 Results

7.1 Power Versus Frequency

In order to examine correspondence for large quantum numbers, we use graphs of power versus frequency for transitions from various angular momentum states at the $n = 100$ energy level. The quantum and semi-classical power equations read,

$$P_q = \frac{\hbar\omega^4 * \max(l, l')}{3\pi^3(2l+1)Ry^3} d_{if}^2 * 10^9 \tag{145}$$

$$P_c = \frac{e^2}{3\pi\epsilon_0 c^3}(k\omega_0)^4 \frac{a^2}{k^2}[(J_k'(k\epsilon))^2 + \frac{1-\epsilon^2}{\epsilon^2} J_k^2(k\epsilon)]. \tag{146}$$

The quantum power depends on ω, while the semi-classical power depends on $k\omega$. A full array of these graphs can be found in Appendix A for transition from every ten angular momentum states. Four of them have been extracted to this section for further analysis.

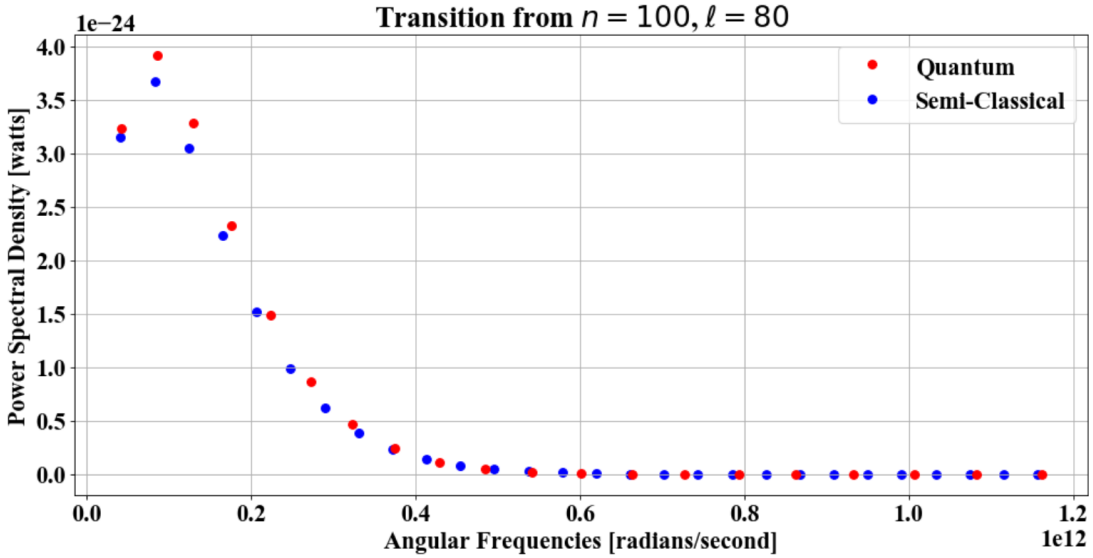

Figure 11: Quantum and semi-classical power versus frequency for the transition from $l = 80$

For the transition from $l = 80$, the power correspondence is clear. The steps between the semi-classical frequency values are slightly smaller because of their k dependence,

while the quantum steps grow because of the inconsistent spacing between quantum energy levels. There are twenty eight points on the semi-classical graph, and only twenty for the quantum. This causes the semi-classical frequencies to lag behind. However, they both still follow almost exactly the same shape, demonstrating the convergence of the semi-classical and quantum results for large quantum numbers. Perhaps unsurprisingly, the greatest correspondence occurs at the first frequency value, which is given by $k = 1$.

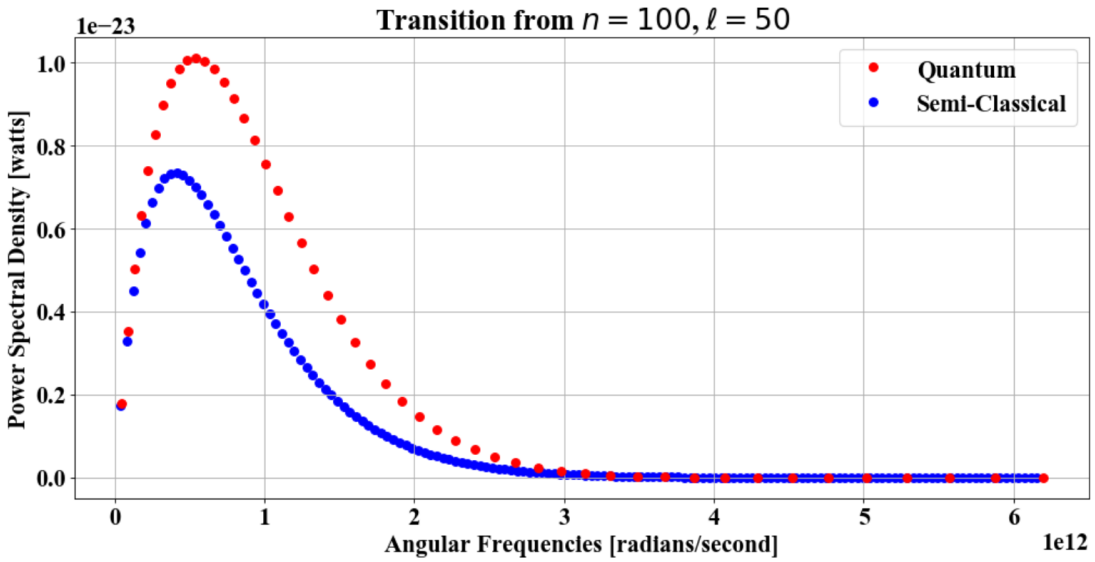

Figure 12: Quantum and semi-classical power versus frequency for the transition from $l = 50$

In the transition from $l = 50$, the correspondence is a little less clear, but still very present. The graphs have the same shape, although the semi-classical is slightly scaled down. Again, there is very good correspondence with the first point in each graph. This time, the value of k reached 151 to keep up with the fifty quantum points.

A trend of the semi-classical power results shrinking with respect to the quantum as l decreases is starting to be visible here, but is even more obvious at $l = 20$.

For this transition (on the following page), the largest magnitudes of the semi-classical power are roughly 7% of their quantum counterparts. A plot of the semi-

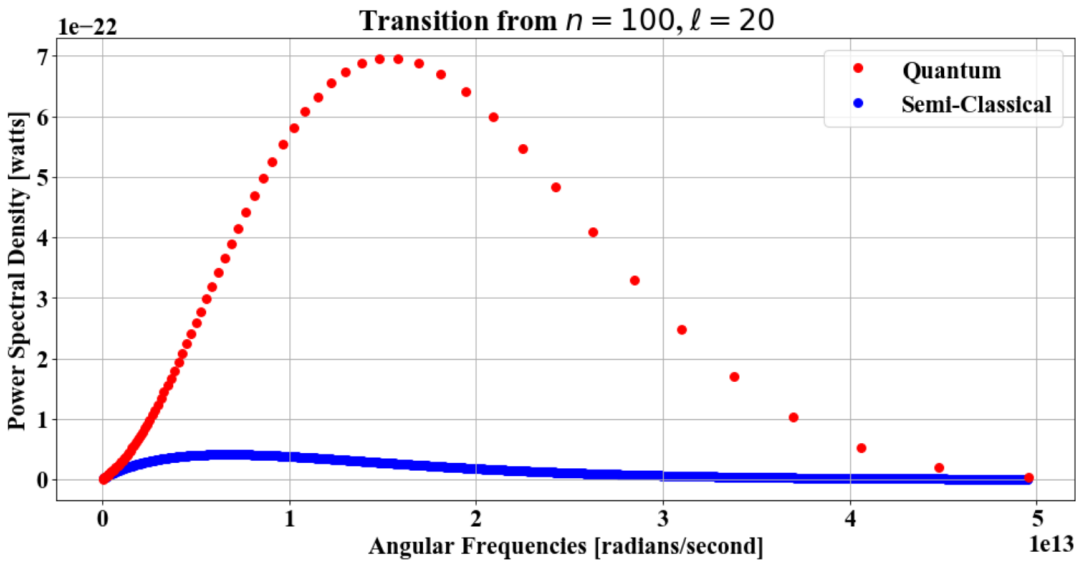

Figure 13: Quantum and semi-classical power versus frequency for the transition from $l = 20$

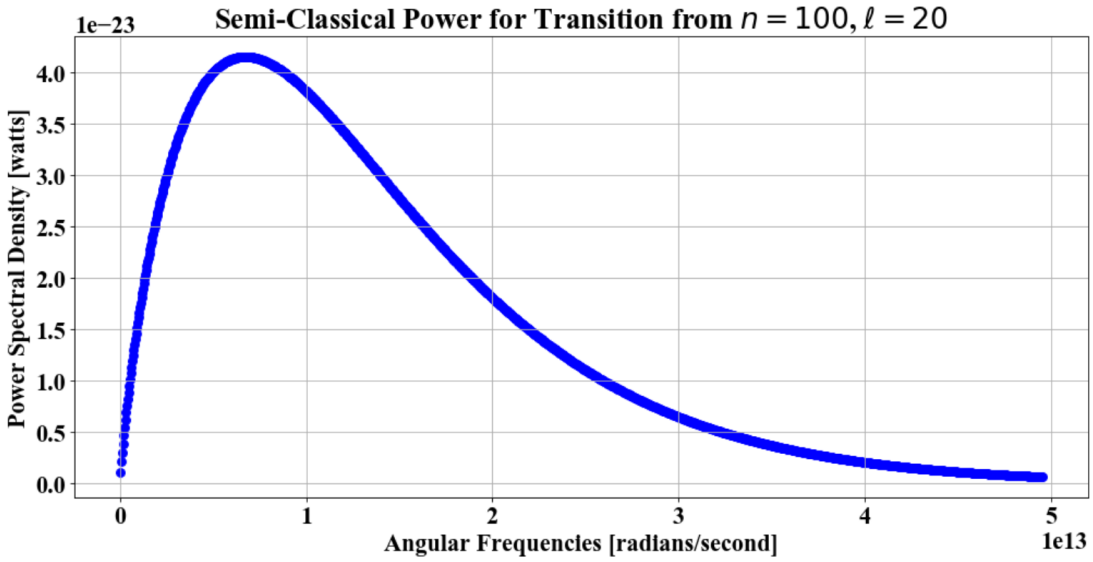

Figure 14: Classical power versus frequency for the transition from $l = 20$

classical power on its own is included because the shape is difficult to see when scaled by the quantum result. A similar trend is being followed for both graphs, but it is difficult to say that there is strong correspondence in the power values. Because of the massive

frequency differences in the lower n states that this transition now has access to, 1200 semi-classical values were needed, compared to eighty quantum values.

7.2 Maximum Power Analysis

A qualitative analysis, though revealing in its own way, is not enough to understand the trends in correspondence as l decreases, especially because of the collapsing semi-classical result. Also, because the frequency values fail to match up exactly, a quantitative analysis describing vertical difference between quantum and semi-classical points is impossible. Therefore, what remains is to analyze how the shapes of the plots change as l changes, which can be done by tracking the frequency value at which the power graphs peak. The result is given in Figure 15 for every five values of l from $l = 5$ to $l = 95$. The log plot technique is used to enhance differences between frequency values. Table 5, in Appendix D, contains the actual values in radians/second.

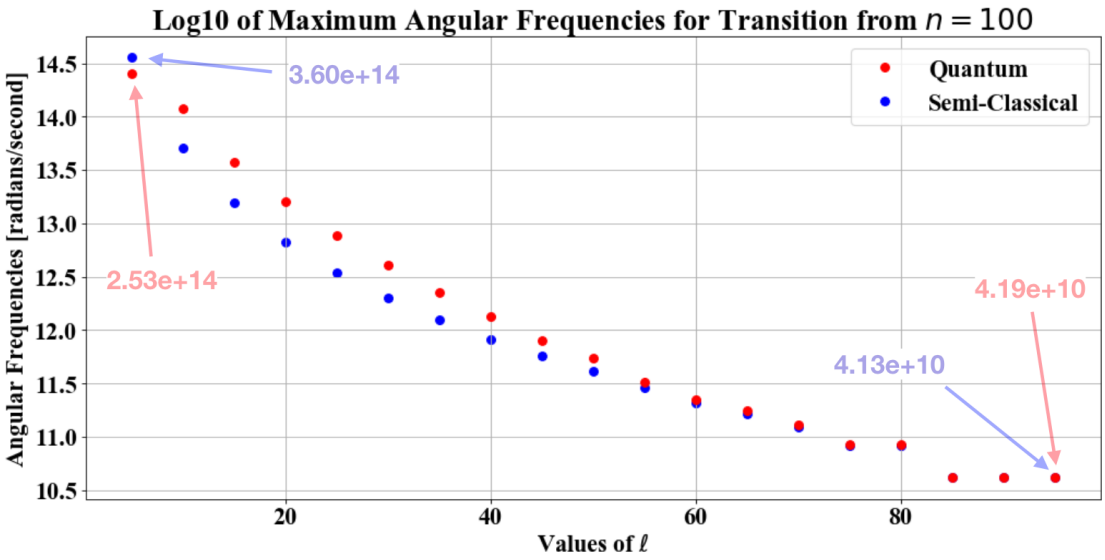

Figure 15: A plot of the angular frequencies at which each curve has its maximum versus the value of l depicted in the curve.

While the qualitative analysis demonstrated that the magnitudes of the quantum and semi-classical power results diverged as l decreased, this quantitative analysis indi-

cates that the shapes of the graphs diverged, as well. In other words, both the magnitude of the peak power radiation and its location as a function of frequency tend to converge for the quantum and semi-classical results with increasing l. This is surprising, as the correspondence limit is usually discussed only in the limit of large n, not large l.

8 Conclusion

This thesis started with simple definitions of the frequency and radiated power of an electron in orbit. Examining the derivations of the power equations for both the quantum and semi-classical models built a connection between physics and mathematics. The physics from which the math arose differed for each model, and the math itself was quite different, as well. However, by increasing quantum numbers in each model of frequency and each model of power, the mathematics began to converge. At first, it seemed that increasing the energy state n was all that was necessary to produce convergence. But as the graphs of power versus frequency indicated, in the end, a large angular momentum l also appeared necessary for convergence. Thus, this thesis demonstrates another quantum number to which the correspondence principle applies.

Bohr's prediction proved fruitful. His correspondence principle certainly has relevance for not just frequency, but power, too.

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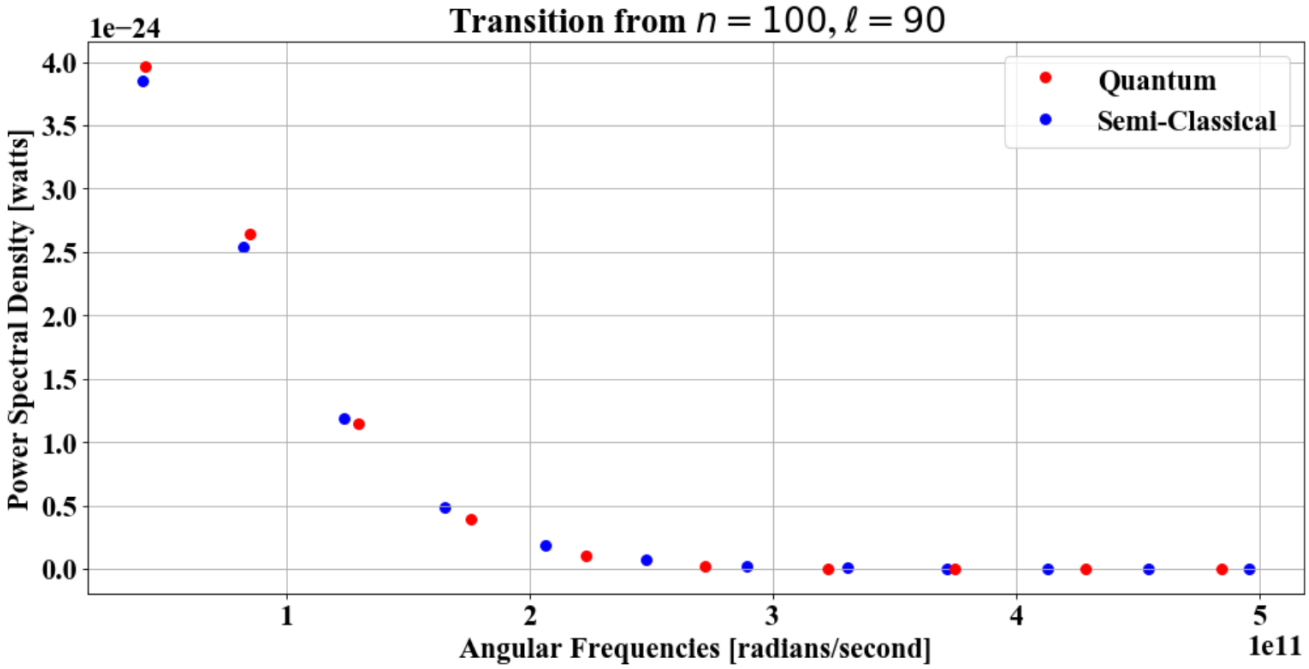
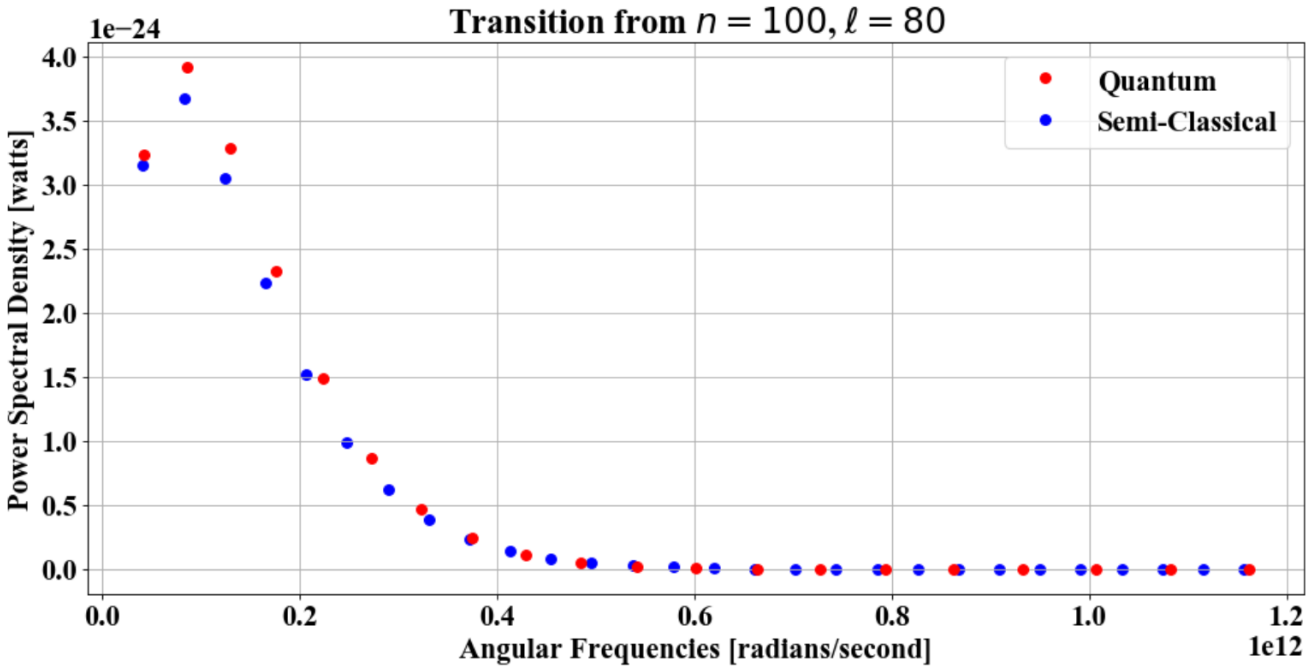
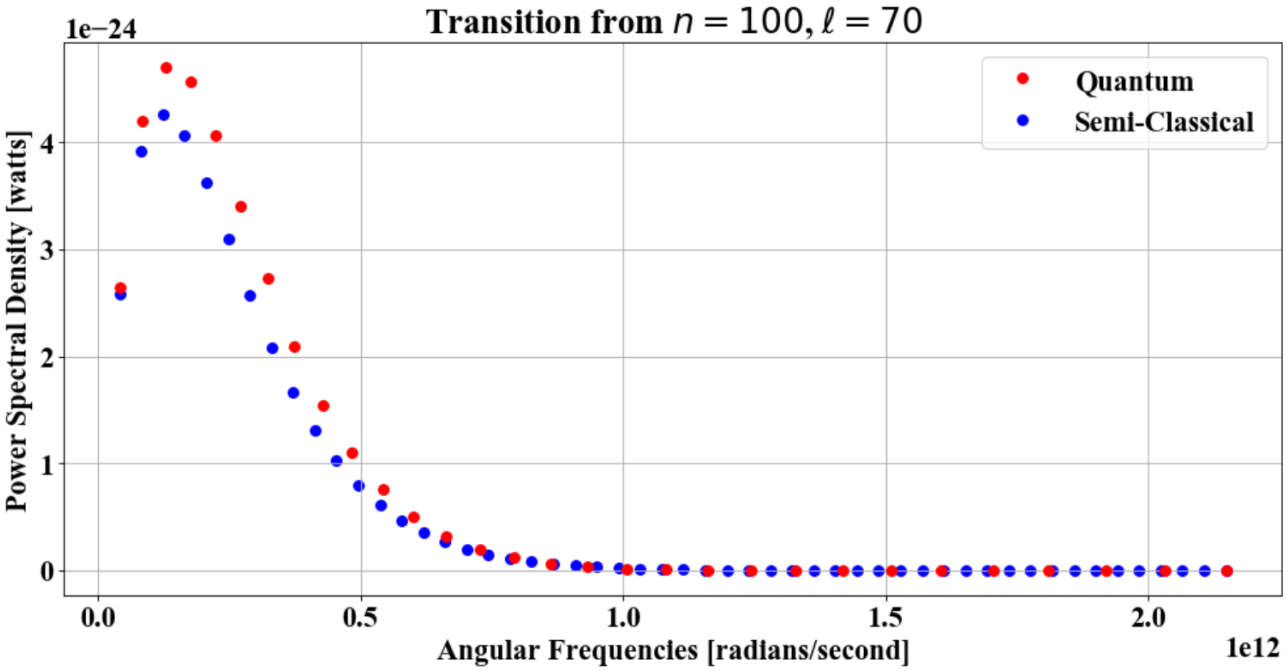
A Power Versus Frequency Graphs






Transition from $n = 100, \ell = 70$
1e−24
Power Spectral Density [watts]
Angular Frequencies [radians/second]
1e12
Quantum
Semi-Classical
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2
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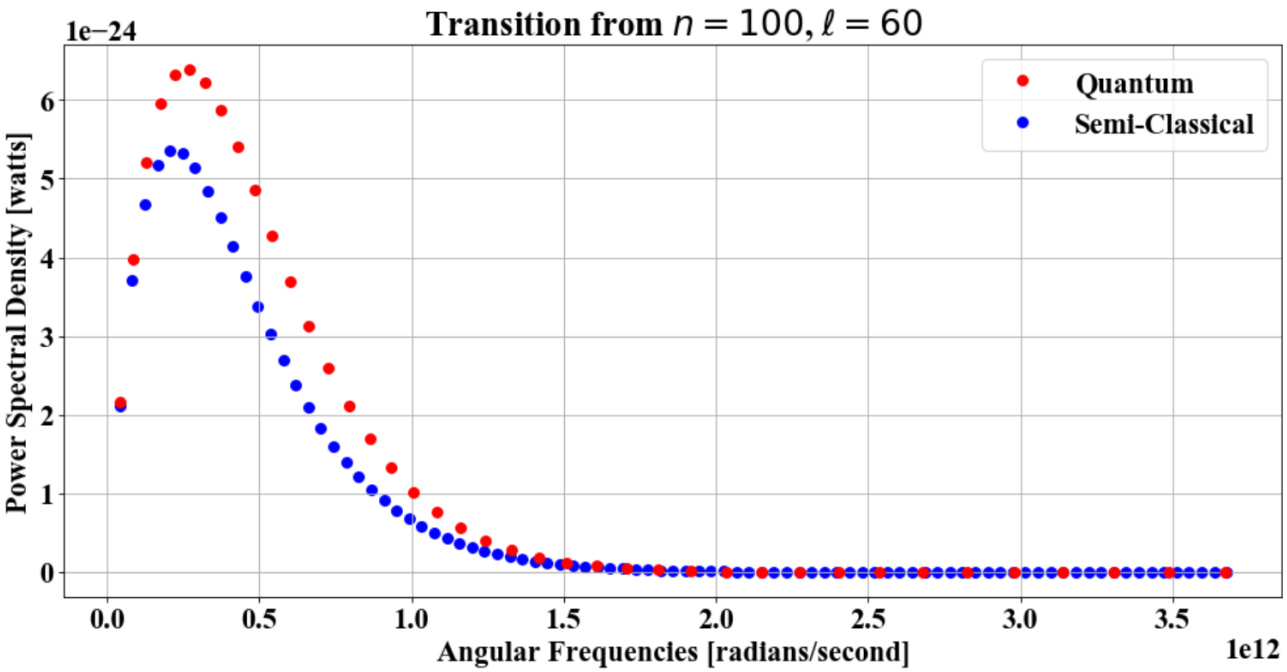
Transition from $n = 100, \ell = 60$
1e−24
Power Spectral Density [watts]
Angular Frequencies [radians/second]
1e12
Quantum
Semi-Classical
0
1
2
3
4
5
6
0.0
0.5
1.0
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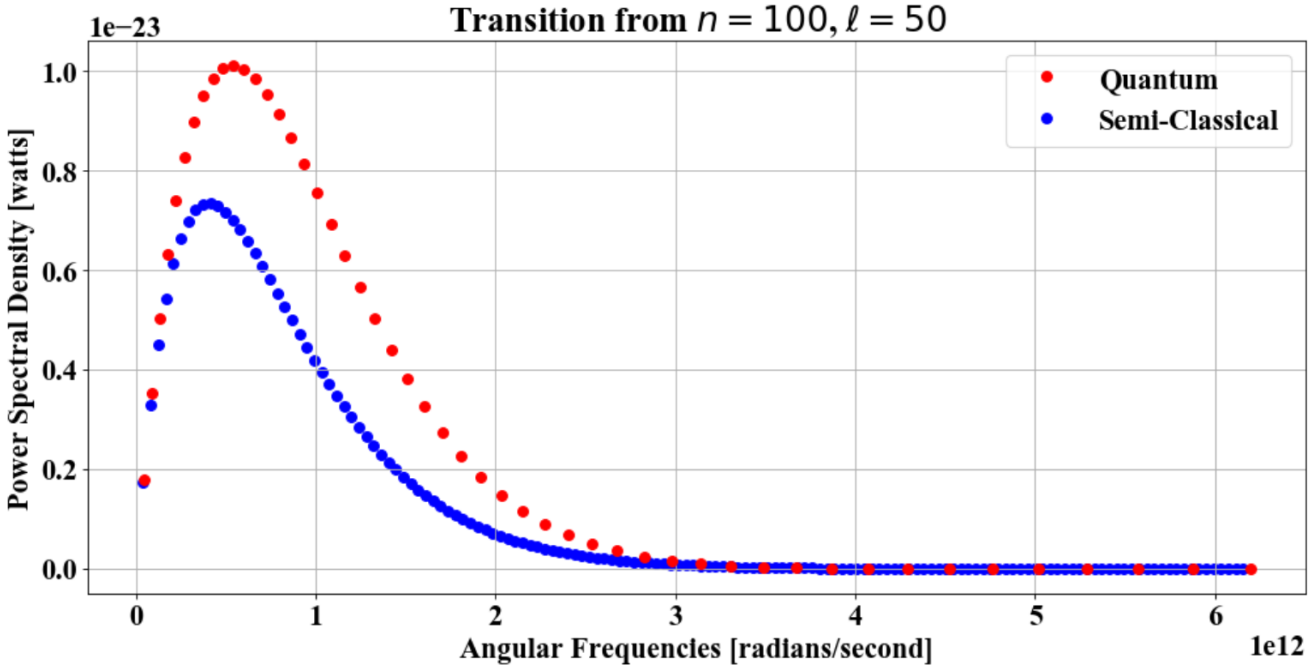
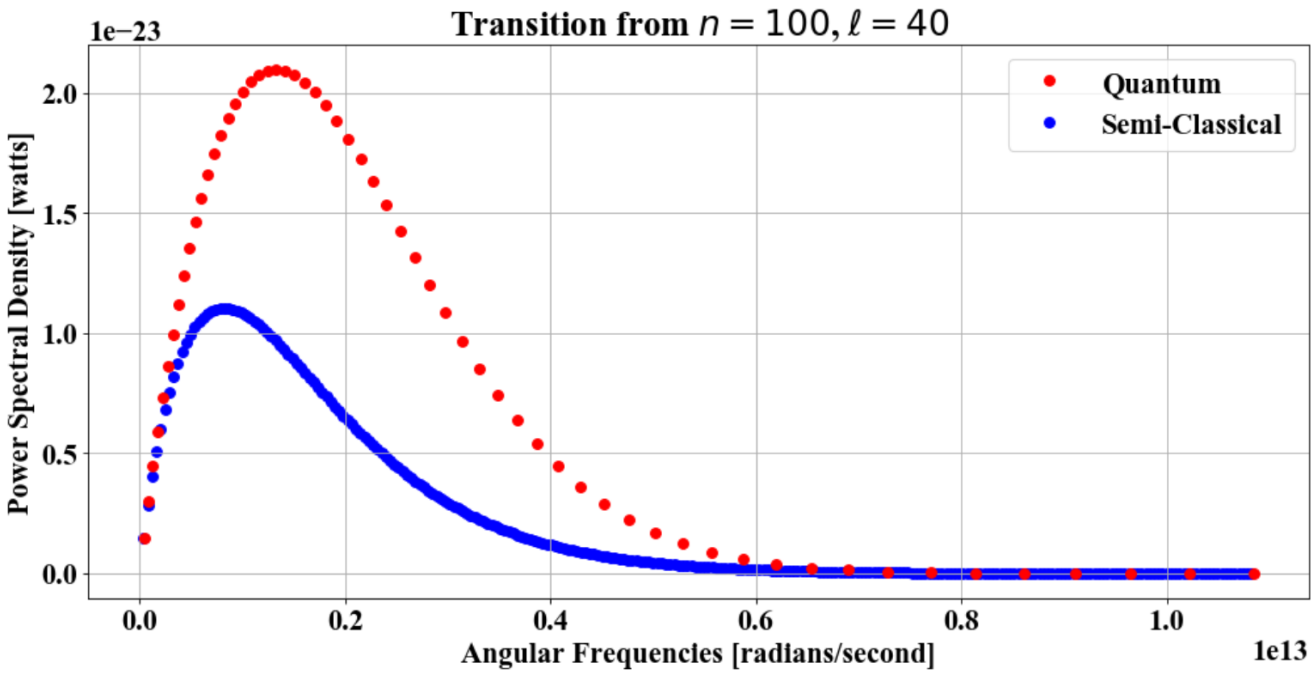
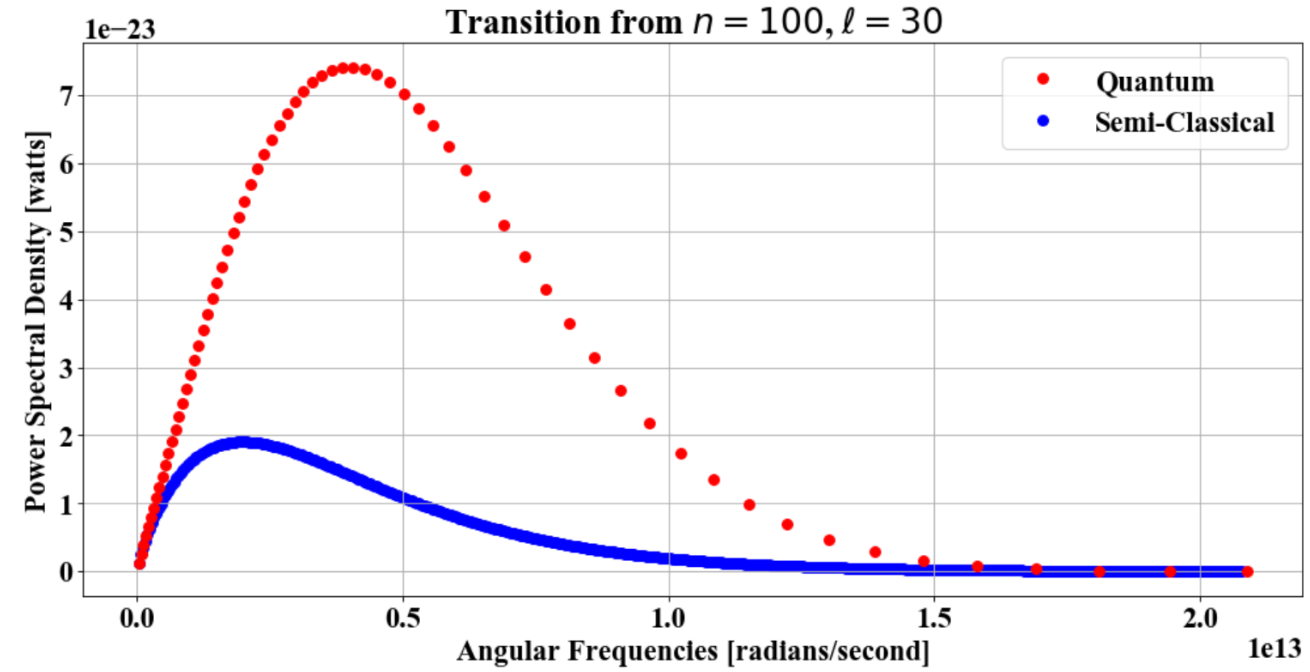
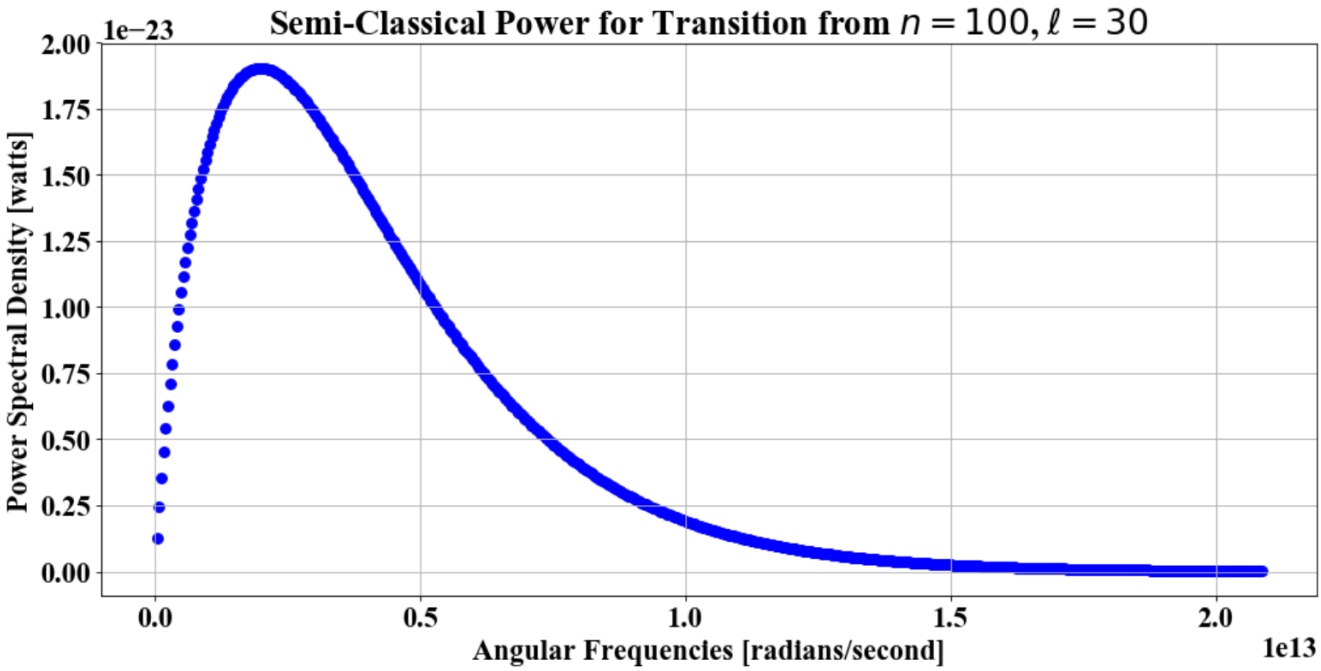
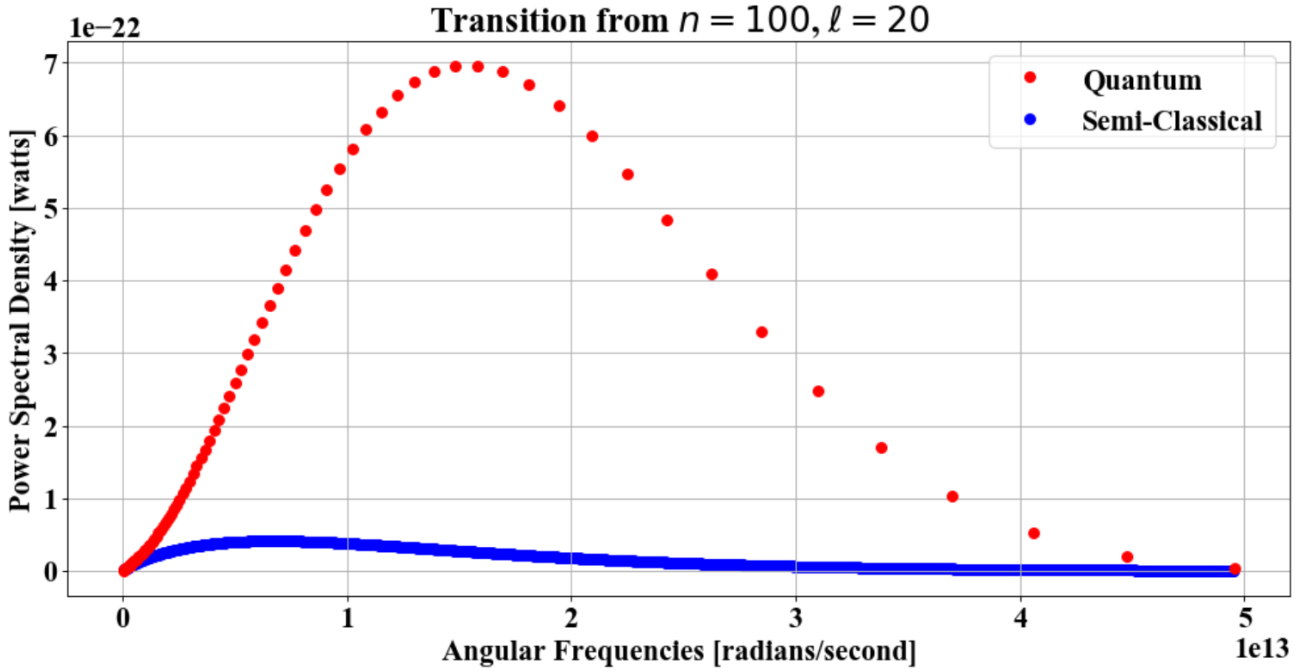
3.0
3.5

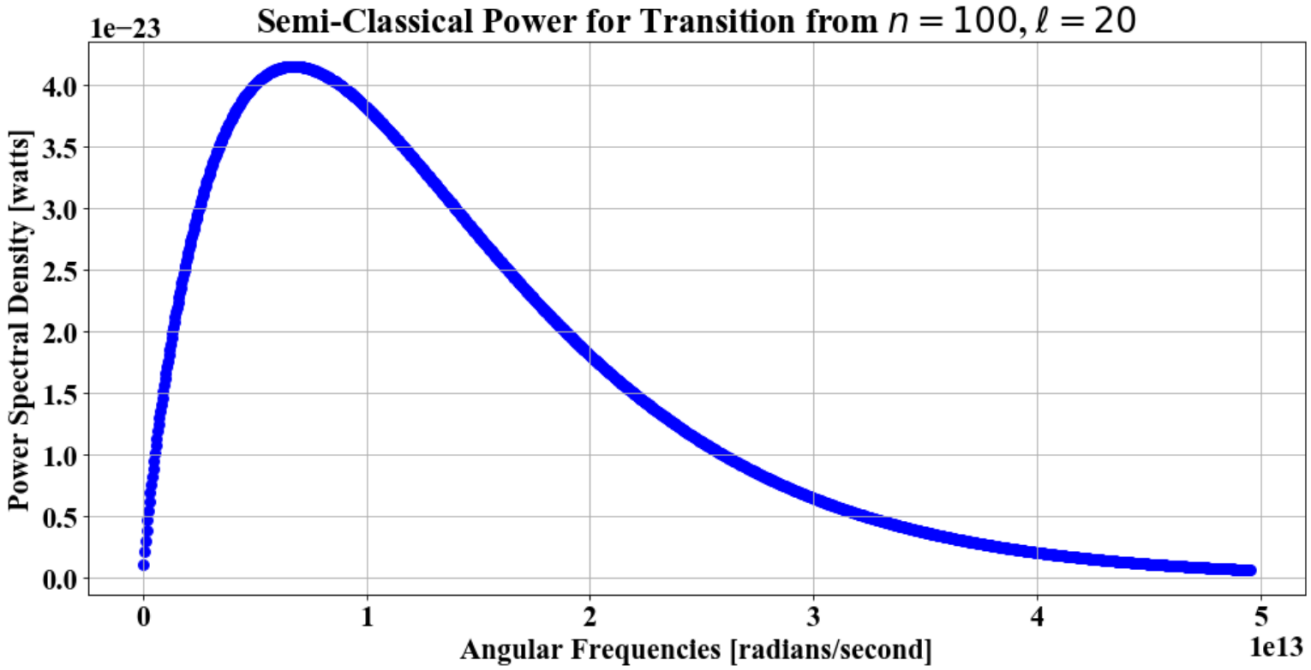

Transition from $n = 100, \ell = 50$
1e−23
Power Spectral Density [watts]
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
6
Angular Frequencies [radians/second]
1e12
Quantum
Semi-Classical

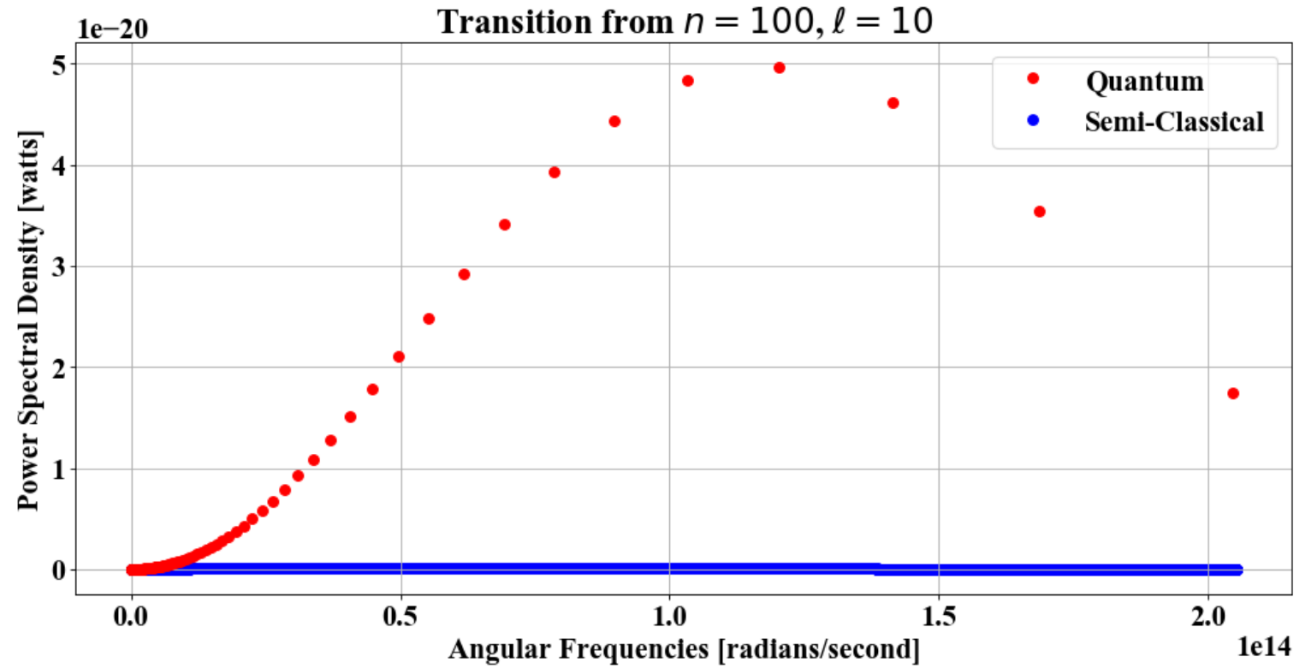

Transition from $n = 100, \ell = 40$
1e−23
Power Spectral Density [watts]
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Angular Frequencies [radians/second]
1e13
Quantum
Semi-Classical

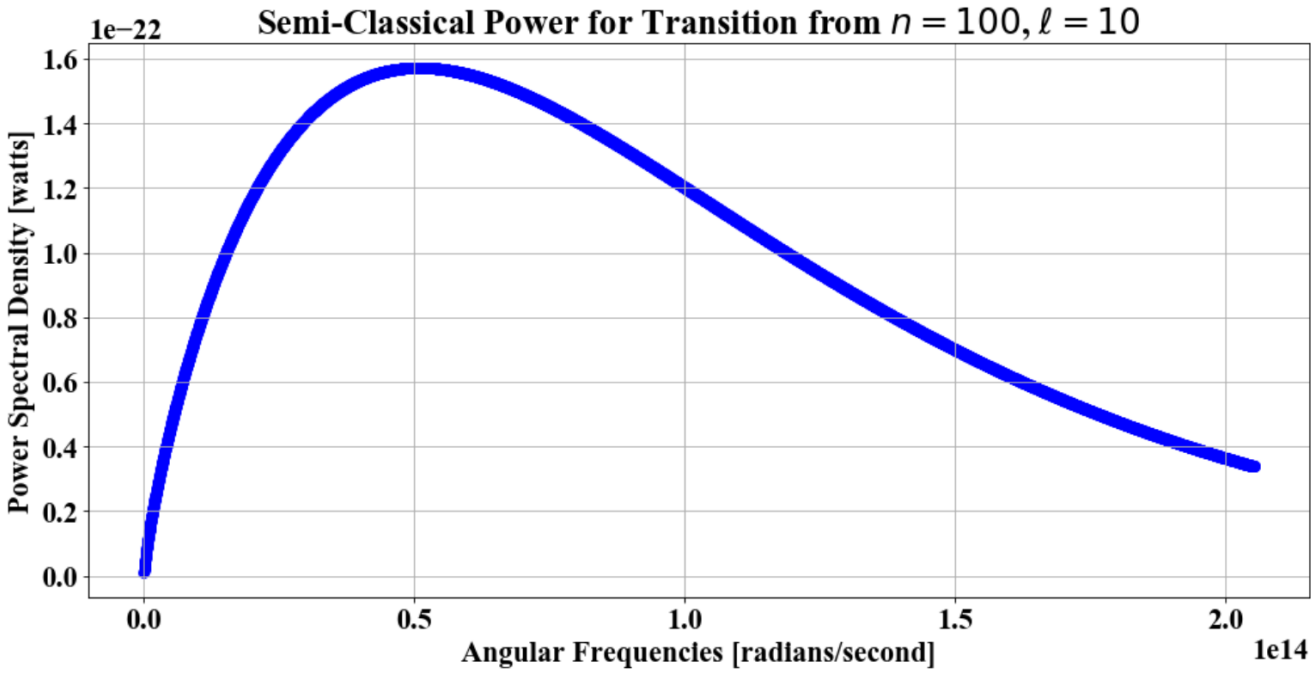

Transition from $n = 100, \ell = 30$
1e−23
Power Spectral Density [watts]
Angular Frequencies [radians/second]
1e13
Quantum
Semi-Classical


Semi-Classical Power for Transition from $n = 100, \ell = 30$
1e−23
Power Spectral Density [watts]
Angular Frequencies [radians/second]
1e13


Transition from $n = 100, \ell = 20$
1e−22
Power Spectral Density [watts]
Angular Frequencies [radians/second]
1e13
Quantum
Semi-Classical


Semi-Classical Power for Transition from $n = 100, \ell = 20$
1e−23
Power Spectral Density [watts]
Angular Frequencies [radians/second]
1e13


Transition from $n = 100, \ell = 10$
1e−20
Power Spectral Density [watts]
Angular Frequencies [radians/second]
1e14
Quantum
Semi-Classical
0
1
2
3
4
5
0.0
0.5
1.0
1.5
2.0


Semi-Classical Power for Transition from $n = 100, \ell = 10$
1e−22
Power Spectral Density [watts]
Angular Frequencies [radians/second]
1e14
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
0.0
0.5
1.0
1.5
2.0

B Circular Frequency Table

Table 2: Angular frequency values for circular decay, up to $n = 50$ [radians/second]

n Value	Quantum	Semi-Classical	n Value	Quantum	Semi-Classical
2	1.549e+16	5.165e+15	27	2.222e+12	2.099e+12
3	2.869e+15	1.530e+15	28	1.988e+12	1.882e+12
4	1.004e+15	6.456e+14	29	1.786e+12	1.694e+12
5	4.648e+14	3.306e+14	30	1.610e+12	1.530e+12
6	2.525e+14	1.913e+14	31	1.457e+12	1.387e+12
7	1.523e+14	1.205e+14	32	1.323e+12	1.261e+12
8	9.882e+13	8.070e+13	33	1.204e+12	1.150e+12
9	6.775e+13	5.668e+13	34	1.100e+12	1.051e+12
10	4.846e+13	4.132e+13	35	1.007e+12	9.637e+11
11	3.586e+13	3.104e+13	36	9.239e+11	8.856e+11
12	2.727e+13	2.391e+13	37	8.500e+11	8.157e+11
13	2.122e+13	1.881e+13	38	7.838e+11	7.530e+11
14	1.684e+13	1.506e+13	39	7.243e+11	6.966e+11
15	1.359e+13	1.224e+13	40	6.707e+11	6.456e+11
16	1.112e+13	1.009e+13	41	6.222e+11	5.995e+11
17	9.215e+12	8.410e+12	42	5.783e+11	5.577e+11
18	7.722e+12	7.085e+12	43	5.384e+11	5.197e+11
19	6.535e+12	6.024e+12	44	5.021e+11	4.851e+11
20	5.580e+12	5.165e+12	45	4.690e+11	4.534e+11
21	4.802e+12	4.462e+12	46	4.388e+11	4.245e+11
22	4.162e+12	3.881e+12	47	4.111e+11	3.980e+11
23	3.631e+12	3.396e+12	48	3.856e+11	3.736e+11
24	3.187e+12	2.989e+12	49	3.623e+11	3.512e+11
25	2.812e+12	2.644e+12	50	3.407e+11	3.306e+11
26	2.494e+12	2.351e+12	—	—	—

C Circular Power Tables

Table 3: Spectral power density values for circular decay, up to $n = 26$ [watts]

n Value	Quantum	Semi-Classical (Nearly Circular)	Semi-Classical (Circular)
2	1.019e-09	1.012e-10	1.821e-10
3	1.947e-11	4.915e-12	7.104e-12
4	1.453e-12	5.436e-13	7.112e-13
5	2.075e-13	9.655e-14	1.193e-13
6	4.358e-14	2.330e-14	2.775e-14
7	1.183e-14	6.968e-15	8.085e-15
8	3.862e-15	2.441e-15	2.778e-15
9	1.447e-15	9.656e-16	1.083e-15
10	6.039e-16	4.206e-16	4.661e-16
11	2.748e-16	1.981e-16	2.174e-16
12	1.342e-16	9.954e-17	1.084e-16
13	6.950e-17	5.282e-17	5.714e-17
14	3.785e-17	2.936e-17	3.158e-17
15	2.152e-17	1.699e-17	1.819e-17
16	1.270e-17	1.018e-17	1.085e-17
17	7.742e-18	6.294e-18	6.682e-18
18	4.858e-18	3.998e-18	4.230e-18
19	3.128e-18	2.602e-18	2.744e-18
20	2.061e-18	1.731e-18	1.821e-18
21	1.386e-18	1.174e-18	1.232e-18
22	9.500e-19	8.112e-19	8.494e-19
23	6.623e-19	5.696e-19	5.952e-19
24	4.690e-19	4.060e-19	4.234e-19
25	3.368e-19	2.934e-19	3.055e-19
26	2.451e-19	2.147e-19	2.232e-19

Table 4: Spectral power density values for circular decay, $n = 27$ to $n = 50$ [watts]

n Value	Quantum	Semi-Classical (Nearly Circular)	Semi-Classical (Circular)
27	1.806e-19	1.590e-19	1.650e-19
28	1.345e-19	1.190e-19	1.234e-19
29	1.013e-19	8.999e-20	9.317e-20
30	7.699e-20	6.869e-20	7.104e-20
31	5.906e-20	5.290e-20	5.465e-20
32	4.570e-20	4.108e-20	4.239e-20
33	3.564e-20	3.214e-20	3.314e-20
34	2.800e-20	2.534e-20	2.610e-20
35	2.216e-20	2.011e-20	2.070e-20
36	1.765e-20	1.607e-20	1.652e-20
37	1.415e-20	1.291e-20	1.327e-20
38	1.141e-20	1.044e-20	1.072e-20
39	9.253e-21	8.487e-21	8.709e-21
40	7.544e-21	6.935e-21	7.112e-21
41	6.182e-21	5.696e-21	5.837e-21
42	5.091e-21	4.700e-21	4.814e-21
43	4.211e-21	3.896e-21	3.988e-21
44	3.499e-21	3.243e-21	3.318e-21
45	2.920e-21	2.711e-21	2.772e-21
46	2.446e-21	2.275e-21	2.325e-21
47	2.057e-21	1.916e-21	1.957e-21
48	1.736e-21	1.620e-21	1.654e-21
49	1.470e-21	1.374e-21	1.403e-21
50	1.250e-21	1.169e-21	1.193e-21

D Peak Angular Frequency Table

Table 5: Angular frequencies at which the maximum value of power occurs for $n = 100$ [radians/second]

l Value	Quantum	Semi-Classical
5	2.530e+14	3.596e+14
10	1.202e+14	5.095e+13
15	3.699e+13	1.570e+13
20	1.581e+13	6.735e+12
25	7.698e+12	3.471e+12
30	4.075e+12	1.983e+12
35	2.273e+12	1.240e+12
40	1.330e+12	8.264e+11
45	7.935e+11	5.785e+11
50	5.423e+11	4.132e+11
55	3.227e+11	2.892e+11
60	2.232e+11	2.066e+11
65	1.758e+11	1.653e+11
70	1.298e+11	1.240e+11
75	8.519e+10	8.264e+10
80	8.519e+10	8.264e+10
85	4.195e+10	4.132e+10
90	4.195e+10	4.132e+10
95	4.195e+10	4.132e+10

E Hydrogen Transition Tables

Equations Used:

$$d_{if} = \frac{(-1)^{n'-1}}{4(2l-1)!}\sqrt{\frac{(n+l)!(n'+l-1)!}{(n-l-1)!(n'-l)!}}\frac{(4nn')^{l+1}(n-n')^{n+n'-2l-2}}{(n+n')^{n+n'}} * \tag{147}$$

$$\left[F\left(-n_r, -n'_r, 2l, -\frac{4nn'}{(n-n')^2}\right) - \left(\frac{n-n'}{n+n'}\right)^2 F\left(-n_r-2, -n'_r, 2l, -\frac{4nn'}{(n-n')^2}\right)\right] \tag{148}$$

$$A_{if} = 8 * 10^9\left(\frac{\omega}{2\pi Ry}\right)^3\frac{\max(l,l')d_{if}^2}{3(2l+1)} \tag{149}$$

$$\Delta E = 10^{-2} * \frac{me^4}{4\pi(4\pi\epsilon_0)^2\hbar^3 c}\left(\frac{1}{n'^2} - \frac{1}{n^2}\right) \tag{150}$$

$$\omega = \frac{me^4}{2(4\pi\epsilon_0)^2\hbar^3}\left(\frac{1}{n'^2} - \frac{1}{n^2}\right) \tag{151}$$

$$\text{Power} = \hbar\omega A_{if} \tag{152}$$

Table 6: Values for allowed transitions of hydrogen, up to $n = 15$

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$2p-1s$	1.6648e+00	6.2334e+00	82,258.50	1.5495e+02	1.0186e+00
$3p-1s$	2.6697e-01	1.6641e+00	97,491.56	1.8364e+02	3.2229e-01
$4p-1s$	9.2771e-02	6.7844e-01	102,823.13	1.9369e+02	1.3858e-01
$5p-1s$	4.3557e-02	3.4203e-01	105,290.88	1.9834e+02	7.1540e-02
$6p-1s$	2.4067e-02	1.9629e-01	106,631.39	2.0086e+02	4.1580e-02
$7p-1s$	1.4743e-02	1.2300e-01	107,439.67	2.0238e+02	2.6252e-02
$8p-1s$	9.7019e-03	8.2134e-02	107,964.28	2.0337e+02	1.7616e-02
$9p-1s$	6.7314e-03	5.7558e-02	108,323.95	2.0405e+02	1.2386e-02
$10p-1s$	4.8647e-03	4.1894e-02	108,581.22	2.0453e+02	9.0366e-03
$11p-1s$	3.6316e-03	3.1439e-02	108,771.57	2.0489e+02	6.7933e-03
$12p-1s$	2.7836e-03	2.4195e-02	108,916.35	2.0516e+02	5.2349e-03
$13p-1s$	2.1811e-03	1.9017e-02	109,029.02	2.0538e+02	4.1188e-03
$14p-1s$	1.7411e-03	1.5217e-02	109,118.42	2.0555e+02	3.2987e-03
$15p-1s$	1.4121e-03	1.2367e-02	109,190.54	2.0568e+02	2.6825e-03
$3p-2s$	9.3931e+00	2.2336e-01	15,233.06	2.8694e+01	6.7590e-03
$4p-2s$	1.6442e+00	9.6195e-02	20,564.63	3.8737e+01	3.9298e-03
$5p-2s$	5.9900e-01	4.9235e-02	23,032.38	4.3386e+01	2.2527e-03
$6p-2s$	2.9200e-01	2.8440e-02	24,372.89	4.5911e+01	1.3770e-03
$7p-2s$	1.6647e-01	1.7881e-02	25,181.17	4.7433e+01	8.9449e-04

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$8p-2s$	1.0471e-01	1.1965e-02	25,705.78	4.8422e+01	6.1100e-04
$9p-2s$	7.0477e-02	8.3960e-03	26,065.45	4.9099e+01	4.3474e-04
$10p-2s$	4.9853e-02	6.1166e-03	26,322.72	4.9584e+01	3.1984e-04
$11p-2s$	3.6636e-02	4.5932e-03	26,513.07	4.9942e+01	2.4192e-04
$12p-2s$	2.7751e-02	3.5365e-03	26,657.85	5.0215e+01	1.8728e-04
$13p-2s$	2.1545e-02	2.7807e-03	26,770.52	5.0427e+01	1.4788e-04
$14p-2s$	1.7075e-02	2.2258e-03	26,859.92	5.0596e+01	1.1877e-04
$15p-2s$	1.3768e-02	1.8093e-03	26,932.04	5.0732e+01	9.6800e-05
$4p-3s$	2.9914e+01	3.0497e-02	5,331.57	1.0043e+01	3.2301e-04
$5p-3s$	5.1057e+00	1.6295e-02	7,799.32	1.4692e+01	2.5247e-04
$6p-3s$	1.8502e+00	9.5029e-03	9,139.83	1.7217e+01	1.7254e-04
$7p-3s$	9.0521e-01	5.9951e-03	9,948.12	1.8739e+01	1.1848e-04
$8p-3s$	5.1995e-01	4.0176e-03	10,472.73	1.9727e+01	8.3584e-05
$9p-3s$	3.2996e-01	2.8214e-03	10,832.40	2.0405e+01	6.0714e-05
$10p-3s$	2.2414e-01	2.0564e-03	11,089.66	2.0889e+01	4.5302e-05
$11p-3s$	1.5998e-01	1.5446e-03	11,280.01	2.1248e+01	3.4612e-05
$12p-3s$	1.1857e-01	1.1895e-03	11,424.79	2.1521e+01	2.6997e-05
$13p-3s$	9.0539e-02	9.3540e-04	11,537.46	2.1733e+01	2.1439e-05
$14p-3s$	7.0820e-02	7.4881e-04	11,626.86	2.1901e+01	1.7295e-05
$15p-3s$	5.6513e-02	6.0873e-04	11,698.99	2.2037e+01	1.4147e-05
$5p-4s$	7.2553e+01	7.3348e-03	2,467.76	4.6485e+00	3.5958e-05
$6p-4s$	1.1934e+01	4.4338e-03	3,808.26	7.1736e+00	3.3543e-05
$7p-4s$	4.2513e+00	2.8138e-03	4,616.55	8.6961e+00	2.5805e-05
$8p-4s$	2.0653e+00	1.8879e-03	5,141.16	9.6843e+00	1.9282e-05
$9p-4s$	1.1842e+00	1.3259e-03	5,500.83	1.0362e+01	1.4489e-05
$10p-4s$	7.5233e-01	9.6621e-04	5,758.10	1.0846e+01	1.1052e-05
$11p-4s$	5.1244e-01	7.2557e-04	5,948.45	1.1205e+01	8.5739e-06
$12p-4s$	3.6706e-01	5.5861e-04	6,093.22	1.1478e+01	6.7617e-06
$13p-4s$	2.7315e-01	4.3918e-04	6,205.89	1.1690e+01	5.4143e-06
$14p-4s$	2.0944e-01	3.5150e-04	6,295.29	1.1858e+01	4.3959e-06
$15p-4s$	1.6451e-01	2.8570e-04	6,367.42	1.1994e+01	3.6138e-06
$6p-5s$	1.4918e+02	2.4174e-03	1,340.51	2.5251e+00	6.4374e-06
$7p-5s$	2.3716e+01	1.5829e-03	2,148.79	4.0477e+00	6.7568e-06
$8p-5s$	8.2915e+00	1.0657e-03	2,673.40	5.0359e+00	5.6600e-06
$9p-5s$	3.9865e+00	7.4828e-04	3,033.07	5.7134e+00	4.5086e-06
$10p-5s$	2.2733e+00	5.4476e-04	3,290.34	6.1980e+00	3.5607e-06
$11p-5s$	1.4407e+00	4.0870e-04	3,480.69	6.5565e+00	2.8260e-06
$12p-5s$	9.8076e-01	3.1440e-04	3,625.47	6.8293e+00	2.2643e-06
$13p-5s$	7.0295e-01	2.4701e-04	3,738.14	7.0415e+00	1.8343e-06

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$14p - 5s$	5.2382e-01	1.9759e-04	3,827.54	7.2099e+00	1.5024e-06
$15p - 5s$	4.0237e-01	1.6052e-04	3,899.66	7.3458e+00	1.2435e-06
$7p - 6s$	2.7419e+02	9.7403e-04	808.28	1.5226e+00	1.5640e-06
$8p - 6s$	4.2342e+01	6.7451e-04	1,332.89	2.5108e+00	1.7860e-06
$9p - 6s$	1.4545e+01	4.7442e-04	1,692.56	3.1883e+00	1.5951e-06
$10p - 6s$	6.9163e+00	3.4490e-04	1,949.83	3.6729e+00	1.3359e-06
$11p - 6s$	3.9173e+00	2.5832e-04	2,140.18	4.0314e+00	1.0983e-06
$12p - 6s$	2.4725e+00	1.9842e-04	2,284.96	4.3042e+00	9.0067e-07
$13p - 6s$	1.6793e+00	1.5570e-04	2,397.63	4.5164e+00	7.4161e-07
$14p - 6s$	1.2024e+00	1.2442e-04	2,487.03	4.6848e+00	6.1472e-07
$15p - 6s$	8.9578e-01	1.0100e-04	2,559.15	4.8206e+00	5.1346e-07
$8p - 7s$	4.6453e+02	4.5118e-04	524.61	9.8820e-01	4.7020e-07
$9p - 7s$	7.0005e+01	3.2563e-04	884.28	1.6657e+00	5.7201e-07
$10p - 7s$	2.3668e+01	2.3684e-04	1,141.55	2.1503e+00	5.3710e-07
$11p - 7s$	1.1136e+01	1.7700e-04	1,331.90	2.5089e+00	4.6831e-07
$12p - 7s$	6.2623e+00	1.3565e-04	1,476.67	2.7816e+00	3.9792e-07
$13p - 7s$	3.9337e+00	1.0624e-04	1,589.34	2.9938e+00	3.3543e-07
$14p - 7s$	2.6634e+00	8.4764e-05	1,678.74	3.1622e+00	2.8268e-07
$15p - 7s$	1.9032e+00	6.8719e-05	1,750.87	3.2981e+00	2.3902e-07
$9p - 8s$	7.3968e+02	2.3152e-04	359.67	6.7751e-01	1.6542e-07
$10p - 8s$	1.0920e+02	1.7249e-04	616.94	1.1621e+00	2.1140e-07
$11p - 8s$	3.6402e+01	1.2884e-04	807.29	1.5207e+00	2.0662e-07
$12p - 8s$	1.6960e+01	9.8457e-05	952.07	1.7934e+00	1.8621e-07
$13p - 8s$	9.4707e+00	7.6902e-05	1,064.74	2.0056e+00	1.6266e-07
$14p - 8s$	5.9195e+00	6.1219e-05	1,154.14	2.1740e+00	1.4036e-07
$15p - 8s$	3.9938e+00	4.9540e-05	1,226.26	2.3099e+00	1.2068e-07
$10p - 9s$	1.1217e+03	1.2848e-04	257.27	4.8462e-01	6.5665e-08
$11p - 9s$	1.6271e+02	9.8166e-05	447.62	8.4318e-01	8.7290e-08
$12p - 9s$	5.3573e+01	7.4922e-05	592.40	1.1159e+00	8.8170e-08
$13p - 9s$	2.4735e+01	5.8323e-05	705.07	1.3281e+00	8.1689e-08
$14p - 9s$	1.3721e+01	4.6286e-05	794.47	1.4965e+00	7.3051e-08
$15p - 9s$	8.5340e+00	3.7361e-05	866.59	1.6324e+00	6.4318e-08
$11p - 10s$	1.6350e+03	7.5859e-05	190.35	3.5856e-01	2.8685e-08
$12p - 10s$	2.3363e+02	5.9155e-05	335.13	6.3128e-01	3.9382e-08
$13p - 10s$	7.6089e+01	4.5961e-05	447.80	8.4351e-01	4.0886e-08
$14p - 10s$	3.4844e+01	3.6338e-05	537.20	1.0119e+00	3.8779e-08
$15p - 10s$	1.9209e+01	2.9233e-05	609.32	1.1478e+00	3.5385e-08
$12p - 11s$	2.3068e+03	4.7091e-05	144.78	2.7271e-01	1.3544e-08
$13p - 11s$	3.2537e+02	3.7349e-05	257.45	4.8495e-01	1.9101e-08

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$14p - 11s$	1.0494e+02	2.9457e-05	346.85	6.5335e-01	2.0297e-08
$15p - 11s$	4.7701e+01	2.3600e-05	418.97	7.8921e-01	1.9642e-08
$13p - 12s$	3.1667e+03	3.0470e-05	112.67	2.1224e-01	6.8199e-09
$14p - 12s$	4.4162e+02	2.4512e-05	202.07	3.8064e-01	9.8398e-09
$15p - 12s$	1.4121e+02	1.9582e-05	274.20	5.1650e-01	1.0666e-08
$14p - 13s$	4.2469e+03	2.0414e-05	89.40	1.6840e-01	3.6254e-09
$15p - 13s$	5.8636e+02	1.6623e-05	161.52	3.0426e-01	5.3339e-09
$15p - 14s$	5.5820e+03	1.4088e-05	72.12	1.3586e-01	2.0185e-09
$3s - 2p$	8.8060e-01	6.2819e-02	15,233.06	2.8694e+01	1.9010e-03
$4s - 2p$	1.4615e-01	2.5652e-02	20,564.63	3.8737e+01	1.0479e-03
$5s - 2p$	5.1997e-02	1.2822e-02	23,032.38	4.3386e+01	5.8665e-04
$6s - 2p$	2.5028e-02	7.3131e-03	24,372.89	4.5911e+01	3.5408e-04
$7s - 2p$	1.4162e-02	4.5635e-03	25,181.17	4.7433e+01	2.2828e-04
$8s - 2p$	8.8644e-03	3.0387e-03	25,705.78	4.8422e+01	1.5517e-04
$9s - 2p$	5.9465e-03	2.1252e-03	26,065.45	4.9099e+01	1.1004e-04
$10s - 2p$	4.1964e-03	1.5446e-03	26,322.72	4.9584e+01	8.0769e-05
$11s - 2p$	3.0784e-03	1.1579e-03	26,513.07	4.9942e+01	6.0984e-05
$12s - 2p$	2.3287e-03	8.9031e-04	26,657.85	5.0215e+01	4.7148e-05
$13s - 2p$	1.8061e-03	6.9931e-04	26,770.52	5.0427e+01	3.7190e-05
$14s - 2p$	1.4302e-03	5.5931e-04	26,859.92	5.0596e+01	2.9844e-05
$15s - 2p$	1.1525e-03	4.5434e-04	26,932.04	5.0732e+01	2.4308e-05
$4s - 3p$	5.9709e+00	1.8262e-02	5,331.57	1.0043e+01	1.9342e-04
$5s - 3p$	9.4014e-01	9.0015e-03	7,799.32	1.4692e+01	1.3947e-04
$6s - 3p$	3.2749e-01	5.0462e-03	9,139.83	1.7217e+01	9.1622e-05
$7s - 3p$	1.5665e-01	3.1125e-03	9,948.12	1.8739e+01	6.1510e-05
$8s - 3p$	8.8712e-02	2.0564e-03	10,472.73	1.9727e+01	4.2783e-05
$9s - 3p$	5.5764e-02	1.4304e-03	10,832.40	2.0405e+01	3.0782e-05
$10s - 3p$	3.7626e-02	1.0356e-03	11,089.66	2.0889e+01	2.2814e-05
$11s - 3p$	2.6724e-02	7.7406e-04	11,280.01	2.1248e+01	1.7345e-05
$12s - 3p$	1.9734e-02	5.9388e-04	11,424.79	2.1521e+01	1.3479e-05
$13s - 3p$	1.5025e-02	4.6568e-04	11,537.46	2.1733e+01	1.0673e-05
$14s - 3p$	1.1726e-02	3.7194e-04	11,626.86	2.1901e+01	8.5908e-06
$15s - 3p$	9.3398e-03	3.0181e-04	11,698.99	2.2037e+01	7.0142e-06
$5s - 4p$	2.1163e+01	6.4183e-03	2,467.76	4.6485e+00	3.1465e-05
$6s - 4p$	3.1978e+00	3.5644e-03	3,808.26	7.1736e+00	2.6966e-05
$7s - 4p$	1.0893e+00	2.1629e-03	4,616.55	8.6961e+00	1.9836e-05
$8s - 4p$	5.1503e-01	1.4124e-03	5,141.16	9.6843e+00	1.4425e-05
$9s - 4p$	2.9011e-01	9.7451e-04	5,500.83	1.0362e+01	1.0649e-05
$10s - 4p$	1.8205e-01	7.0143e-04	5,758.10	1.0846e+01	8.0234e-06

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$11s - 4p$	1.2290e-01	5.2205e-04	5,948.45	1.1205e+01	6.1690e-06
$12s - 4p$	8.7448e-02	3.9924e-04	6,093.22	1.1478e+01	4.8326e-06
$13s - 4p$	6.4740e-02	3.1227e-04	6,205.89	1.1690e+01	3.8498e-06
$14s - 4p$	4.9439e-02	2.4892e-04	6,295.29	1.1858e+01	3.1130e-06
$15s - 4p$	3.8708e-02	2.0167e-04	6,367.42	1.1994e+01	2.5509e-06
$6s - 5p$	5.4887e+01	2.6683e-03	1,340.51	2.5251e+00	7.1055e-06
$7s - 5p$	8.0363e+00	1.6091e-03	2,148.79	4.0477e+00	6.8688e-06
$8s - 5p$	2.6834e+00	1.0347e-03	2,673.40	5.0359e+00	5.4952e-06
$9s - 5p$	1.2533e+00	7.0576e-04	3,033.07	5.7134e+00	4.2524e-06
$10s - 5p$	7.0084e-01	5.0383e-04	3,290.34	6.1980e+00	3.2933e-06
$11s - 5p$	4.3800e-01	3.7275e-04	3,480.69	6.5565e+00	2.5774e-06
$12s - 5p$	2.9510e-01	2.8380e-04	3,625.47	6.8293e+00	2.0439e-06
$13s - 5p$	2.0985e-01	2.2122e-04	3,738.14	7.0415e+00	1.6428e-06
$14s - 5p$	1.5541e-01	1.7587e-04	3,827.54	7.2099e+00	1.3372e-06
$15s - 5p$	1.1880e-01	1.4218e-04	3,899.66	7.3458e+00	1.1014e-06
$7s - 6p$	1.1811e+02	1.2588e-03	808.28	1.5226e+00	2.0212e-06
$8s - 6p$	1.6873e+01	8.0636e-04	1,332.89	2.5108e+00	2.1351e-06
$9s - 6p$	5.5390e+00	5.4202e-04	1,692.56	3.1883e+00	1.8225e-06
$10s - 6p$	2.5575e+00	3.8261e-04	1,949.83	3.6729e+00	1.4820e-06
$11s - 6p$	1.4192e+00	2.8076e-04	2,140.18	4.0314e+00	1.1937e-06
$12s - 6p$	8.8252e-01	2.1247e-04	2,284.96	4.3042e+00	9.6445e-07
$13s - 6p$	5.9271e-01	1.6487e-04	2,397.63	4.5164e+00	7.8526e-07
$14s - 6p$	4.2070e-01	1.3061e-04	2,487.03	4.6848e+00	6.4527e-07
$15s - 6p$	3.1128e-01	1.0529e-04	2,559.15	4.8206e+00	5.3529e-07
$8s - 7p$	2.2435e+02	6.5370e-04	524.61	9.8820e-01	6.8125e-07
$9s - 7p$	3.1426e+01	4.3854e-04	884.28	1.6657e+00	7.7036e-07
$10s - 7p$	1.0169e+01	3.0528e-04	1,141.55	2.1503e+00	6.9228e-07
$11s - 7p$	4.6467e+00	2.2156e-04	1,331.90	2.5089e+00	5.8623e-07
$12s - 7p$	2.5594e+00	1.6632e-04	1,476.67	2.7816e+00	4.8789e-07
$13s - 7p$	1.5832e+00	1.2827e-04	1,589.34	2.9938e+00	4.0500e-07
$14s - 7p$	1.0594e+00	1.0115e-04	1,678.74	3.1622e+00	3.3733e-07
$15s - 7p$	7.5010e-01	8.1251e-05	1,750.87	3.2981e+00	2.8261e-07
$9s - 8p$	3.8964e+02	3.6586e-04	359.67	6.7751e-01	2.6141e-07
$10s - 8p$	5.3715e+01	2.5455e-04	616.94	1.1621e+00	3.1197e-07
$11s - 8p$	1.7168e+01	1.8229e-04	807.29	1.5207e+00	2.9234e-07
$12s - 8p$	7.7728e+00	1.3537e-04	952.07	1.7934e+00	2.5603e-07
$13s - 8p$	4.2518e+00	1.0357e-04	1,064.74	2.0056e+00	2.1907e-07
$14s - 8p$	2.6165e+00	8.1180e-05	1,154.14	2.1740e+00	1.8612e-07
$15s - 8p$	1.7442e+00	6.4907e-05	1,226.26	2.3099e+00	1.5812e-07

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$10s - 9p$	6.3256e+02	2.1738e-04	257.27	4.8462e-01	1.1110e-07
$11s - 9p$	8.6059e+01	1.5577e-04	447.62	8.4318e-01	1.3851e-07
$12s - 9p$	2.7217e+01	1.1419e-04	592.40	1.1159e+00	1.3438e-07
$13s - 9p$	1.2221e+01	8.6449e-05	705.07	1.3281e+00	1.2108e-07
$14s - 9p$	6.6426e+00	6.7223e-05	794.47	1.4965e+00	1.0609e-07
$15s - 9p$	4.0678e+00	5.3426e-05	866.59	1.6324e+00	9.1974e-08
$11s - 10p$	9.7424e+02	1.3560e-04	190.35	3.5856e-01	5.1277e-08
$12s - 10p$	1.3108e+02	9.9565e-05	335.13	6.3128e-01	6.6285e-08
$13s - 10p$	4.1078e+01	7.4439e-05	447.80	8.4351e-01	6.6219e-08
$14s - 10p$	1.8311e+01	5.7288e-05	537.20	1.0119e+00	6.1135e-08
$15s - 10p$	9.8946e+00	4.5174e-05	609.32	1.1478e+00	5.4681e-08
$12s - 11p$	1.4383e+03	8.8085e-05	144.78	2.7271e-01	2.5334e-08
$13s - 11p$	1.9169e+02	6.6012e-05	257.45	4.8495e-01	3.3760e-08
$14s - 11p$	5.9598e+01	5.0187e-05	346.85	6.5335e-01	3.4580e-08
$15s - 11p$	2.6393e+01	3.9174e-05	418.97	7.8921e-01	3.2604e-08
$13s - 12p$	2.0510e+03	5.9203e-05	112.67	2.1224e-01	1.3251e-08
$14s - 12p$	2.7113e+02	4.5148e-05	202.07	3.8064e-01	1.8123e-08
$15s - 12p$	8.3706e+01	3.4824e-05	274.20	5.1650e-01	1.8969e-08
$14s - 13p$	2.8410e+03	4.0968e-05	89.40	1.6840e-01	7.2758e-09
$15s - 13p$	3.7290e+02	3.1715e-05	161.52	3.0426e-01	1.0176e-08
$15s - 14p$	3.8396e+03	2.9072e-05	72.12	1.3586e-01	4.1654e-09
$3d - 2p$	2.2543e+01	6.4326e-01	15,233.06	2.8694e+01	1.9466e-02
$4d - 2p$	2.9231e+00	2.0522e-01	20,564.63	3.8737e+01	8.3836e-03
$5d - 2p$	9.5080e-01	9.3780e-02	23,032.38	4.3386e+01	4.2909e-03
$6d - 2p$	4.3799e-01	5.1191e-02	24,372.89	4.5911e+01	2.4786e-03
$7d - 2p$	2.4170e-01	3.1153e-02	25,181.17	4.7433e+01	1.5584e-03
$8d - 2p$	1.4892e-01	2.0420e-02	25,705.78	4.8422e+01	1.0428e-03
$9d - 2p$	9.8851e-02	1.4131e-02	26,065.45	4.9099e+01	7.3173e-04
$10d - 2p$	6.9240e-02	1.0194e-02	26,322.72	4.9584e+01	5.3307e-04
$11d - 2p$	5.0518e-02	7.6003e-03	26,513.07	4.9942e+01	4.0030e-04
$12d - 2p$	3.8058e-02	5.8201e-03	26,657.85	5.0215e+01	3.0821e-04
$13d - 2p$	2.9424e-02	4.5570e-03	26,770.52	5.0427e+01	2.4234e-04
$14d - 2p$	2.3240e-02	3.6355e-03	26,859.92	5.0596e+01	1.9398e-04
$15d - 2p$	1.8690e-02	2.9473e-03	26,932.04	5.0732e+01	1.5768e-04
$4d - 3p$	5.7235e+01	7.0023e-02	5,331.57	1.0043e+01	7.4164e-04
$5d - 3p$	8.8109e+00	3.3744e-02	7,799.32	1.4692e+01	5.2282e-04
$6d - 3p$	3.0313e+00	1.8684e-02	9,139.83	1.7217e+01	3.3923e-04
$7d - 3p$	1.4392e+00	1.1438e-02	9,948.12	1.8739e+01	2.2604e-04
$8d - 3p$	8.1107e-01	7.5205e-03	10,472.73	1.9727e+01	1.5646e-04

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$9d-3p$	5.0814e-01	5.2139e-03	10,832.40	2.0405e+01	1.1220e-04
$10d-3p$	3.4205e-01	3.7658e-03	11,089.66	2.0889e+01	8.2960e-05
$11d-3p$	2.4252e-01	2.8098e-03	11,280.01	2.1248e+01	6.2963e-05
$12d-3p$	1.7884e-01	2.1529e-03	11,424.79	2.1521e+01	4.8862e-05
$13d-3p$	1.3603e-01	1.6864e-03	11,537.46	2.1733e+01	3.8652e-05
$14d-3p$	1.0607e-01	1.3459e-03	11,626.86	2.1901e+01	3.1085e-05
$15d-3p$	8.4434e-02	1.0914e-03	11,698.99	2.2037e+01	2.5364e-05
$5d-4p$	1.2186e+02	1.4783e-02	2,467.76	4.6485e+00	7.2472e-05
$6d-4p$	1.9241e+01	8.5786e-03	3,808.26	7.1736e+00	6.4899e-05
$7d-4p$	6.6868e+00	5.3110e-03	4,616.55	8.6961e+00	4.8707e-05
$8d-4p$	3.1968e+00	3.5068e-03	5,141.16	9.6843e+00	3.5815e-05
$9d-4p$	1.8130e+00	2.4361e-03	5,500.83	1.0362e+01	2.6621e-05
$10d-4p$	1.1429e+00	1.7614e-03	5,758.10	1.0846e+01	2.0148e-05
$11d-4p$	7.7401e-01	1.3151e-03	5,948.45	1.1205e+01	1.5541e-05
$12d-4p$	5.5202e-01	1.0081e-03	6,093.22	1.1478e+01	1.2202e-05
$13d-4p$	4.0939e-01	7.8988e-04	6,205.89	1.1690e+01	9.7378e-06
$14d-4p$	3.1306e-01	6.3050e-04	6,295.29	1.1858e+01	7.8849e-06
$15d-4p$	2.4537e-01	5.1135e-04	6,367.42	1.1994e+01	6.4681e-06
$6d-5p$	2.2999e+02	4.4723e-03	1,340.51	2.5251e+00	1.1910e-05
$7d-5p$	3.6056e+01	2.8878e-03	2,148.79	4.0477e+00	1.2327e-05
$8d-5p$	1.2471e+01	1.9236e-03	2,673.40	5.0359e+00	1.0216e-05
$9d-5p$	5.9492e+00	1.3400e-03	3,033.07	5.7134e+00	8.0741e-06
$10d-5p$	3.3729e+00	9.6991e-04	3,290.34	6.1980e+00	6.3397e-06
$11d-5p$	2.1283e+00	7.2448e-04	3,480.69	6.5565e+00	5.0094e-06
$12d-5p$	1.4439e+00	5.5544e-04	3,625.47	6.8293e+00	4.0003e-06
$13d-5p$	1.0322e+00	4.3523e-04	3,738.14	7.0415e+00	3.2320e-06
$14d-5p$	7.6750e-01	3.4741e-04	3,827.54	7.2099e+00	2.6416e-06
$15d-5p$	5.8855e-01	2.8176e-04	3,899.66	7.3458e+00	2.1827e-06
$7d-6p$	3.9775e+02	1.6956e-03	808.28	1.5226e+00	2.7226e-06
$8d-6p$	6.1390e+01	1.1735e-03	1,332.89	2.5108e+00	3.1073e-06
$9d-6p$	2.1029e+01	8.2313e-04	1,692.56	3.1883e+00	2.7676e-06
$10d-6p$	9.9721e+00	5.9674e-04	1,949.83	3.6729e+00	2.3114e-06
$11d-6p$	5.6343e+00	4.4586e-04	2,140.18	4.0314e+00	1.8956e-06
$12d-6p$	3.5491e+00	3.4179e-04	2,284.96	4.3042e+00	1.5514e-06
$13d-6p$	2.4065e+00	2.6776e-04	2,397.63	4.5164e+00	1.2753e-06
$14d-6p$	1.7207e+00	2.1367e-04	2,487.03	4.6848e+00	1.0557e-06
$15d-6p$	1.2805e+00	1.7325e-04	2,559.15	4.8206e+00	8.8078e-07
$8d-7p$	6.4380e+02	7.5035e-04	524.61	9.8820e-01	7.8198e-07
$9d-7p$	9.7676e+01	5.4520e-04	884.28	1.6657e+00	9.5773e-07

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$10d - 7p$	3.3090e+01	3.9736e-04	1,141.55	2.1503e+00	9.0110e-07
$11d - 7p$	1.5576e+01	2.9709e-04	1,331.90	2.5089e+00	7.8605e-07
$12d - 7p$	8.7583e+00	2.2766e-04	1,476.67	2.7816e+00	6.6783e-07
$13d - 7p$	5.4999e+00	1.7824e-04	1,589.34	2.9938e+00	5.6277e-07
$14d - 7p$	3.7223e+00	1.4216e-04	1,678.74	3.1622e+00	4.7408e-07
$15d - 7p$	2.6589e+00	1.1520e-04	1,750.87	3.2981e+00	4.0070e-07
$9d - 8p$	9.8934e+02	3.7159e-04	359.67	6.7751e-01	2.6550e-07
$10d - 8p$	1.4764e+02	2.7986e-04	616.94	1.1621e+00	3.4299e-07
$11d - 8p$	4.9466e+01	2.1009e-04	807.29	1.5207e+00	3.3692e-07
$12d - 8p$	2.3107e+01	1.6097e-04	952.07	1.7934e+00	3.0445e-07
$13d - 8p$	1.2923e+01	1.2592e-04	1,064.74	2.0056e+00	2.6634e-07
$14d - 8p$	8.0845e+00	1.0033e-04	1,154.14	2.1740e+00	2.3003e-07
$15d - 8p$	5.4575e+00	8.1236e-05	1,226.26	2.3099e+00	1.9789e-07
$10d - 9p$	1.4581e+03	2.0043e-04	257.27	4.8462e-01	1.0243e-07
$11d - 9p$	2.1433e+02	1.5517e-04	447.62	8.4318e-01	1.3798e-07
$12d - 9p$	7.1057e+01	1.1925e-04	592.40	1.1159e+00	1.4033e-07
$13d - 9p$	3.2943e+01	9.3209e-05	705.07	1.3281e+00	1.3055e-07
$14d - 9p$	1.8322e+01	7.4168e-05	794.47	1.4965e+00	1.1706e-07
$15d - 9p$	1.1416e+01	5.9975e-05	866.59	1.6324e+00	1.0325e-07
$11d - 10p$	2.0763e+03	1.1560e-04	190.35	3.5856e-01	4.3713e-08
$12d - 10p$	3.0105e+02	9.1470e-05	335.13	6.3128e-01	6.0896e-08
$13d - 10p$	9.8843e+01	7.1647e-05	447.80	8.4351e-01	6.3735e-08
$14d - 10p$	4.5496e+01	5.6936e-05	537.20	1.0119e+00	6.0760e-08
$15d - 10p$	2.5168e+01	4.5961e-05	609.32	1.1478e+00	5.5634e-08
$12d - 11p$	2.8728e+03	7.0375e-05	144.78	2.7271e-01	2.0240e-08
$13d - 11p$	4.1146e+02	5.6676e-05	257.45	4.8495e-01	2.8986e-08
$14d - 11p$	1.3389e+02	4.5100e-05	346.85	6.5335e-01	3.1075e-08
$15d - 11p$	6.1213e+01	3.6342e-05	418.97	7.8921e-01	3.0247e-08
$13d - 12p$	3.8790e+03	4.4787e-05	112.67	2.1224e-01	1.0025e-08
$14d - 12p$	5.4947e+02	3.6599e-05	202.07	3.8064e-01	1.4692e-08
$15d - 12p$	1.7735e+02	2.9513e-05	274.20	5.1650e-01	1.6075e-08
$14d - 13p$	5.1286e+03	2.9582e-05	89.40	1.6840e-01	5.2538e-09
$15d - 13p$	7.1934e+02	2.4471e-05	161.52	3.0426e-01	7.8522e-09
$15d - 14p$	6.6582e+03	2.0165e-05	72.12	1.3586e-01	2.8892e-09
$4p - 3d$	1.6959e+00	3.4579e-03	5,331.57	1.0043e+01	3.6624e-05
$5p - 3d$	2.3309e-01	1.4878e-03	7,799.32	1.4692e+01	2.3052e-05
$6p - 3d$	7.5784e-02	7.7848e-04	9,139.83	1.7217e+01	1.4135e-05
$7p - 3d$	3.4825e-02	4.6128e-04	9,948.12	1.8739e+01	9.1160e-06
$8p - 3d$	1.9225e-02	2.9711e-04	10,472.73	1.9727e+01	6.1811e-06

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$9p - 3d$	1.1879e-02	2.0314e-04	10,832.40	2.0405e+01	4.3714e-06
$10p - 3d$	7.9178e-03	1.4528e-04	11,089.66	2.0889e+01	3.2006e-06
$11p - 3d$	5.5735e-03	1.0763e-04	11,280.01	2.1248e+01	2.4117e-06
$12p - 3d$	4.0879e-03	8.2016e-05	11,424.79	2.1521e+01	1.8614e-06
$13p - 3d$	3.0961e-03	6.3974e-05	11,537.46	2.1733e+01	1.4663e-06
$14p - 3d$	2.4063e-03	5.0885e-05	11,626.86	2.1901e+01	1.1753e-06
$15p - 3d$	1.9103e-03	4.1152e-05	11,698.99	2.2037e+01	9.5640e-07
$5p - 4d$	9.2740e+00	1.8751e-03	2,467.76	4.6485e+00	9.1924e-06
$6p - 4d$	1.2609e+00	9.3695e-04	3,808.26	7.1736e+00	7.0883e-06
$7p - 4d$	4.0518e-01	5.3637e-04	4,616.55	8.6961e+00	4.9190e-06
$8p - 4d$	1.8481e-01	3.3788e-04	5,141.16	9.6843e+00	3.4508e-06
$9p - 4d$	1.0164e-01	2.2762e-04	5,500.83	1.0362e+01	2.4873e-06
$10p - 4d$	6.2723e-02	1.6111e-04	5,758.10	1.0846e+01	1.8429e-06
$11p - 4d$	4.1829e-02	1.1845e-04	5,948.45	1.1205e+01	1.3997e-06
$12p - 4d$	2.9490e-02	8.9759e-05	6,093.22	1.1478e+01	1.0865e-06
$13p - 4d$	2.1678e-02	6.9710e-05	6,205.89	1.1690e+01	8.5940e-07
$14p - 4d$	1.6462e-02	5.5258e-05	6,295.29	1.1858e+01	6.9105e-07
$15p - 4d$	1.2831e-02	4.4567e-05	6,367.42	1.1994e+01	5.6373e-07
$6p - 5d$	2.9452e+01	9.5451e-04	1,340.51	2.5251e+00	2.5418e-06
$7p - 5d$	3.9559e+00	5.2806e-04	2,148.79	4.0477e+00	2.2541e-06
$8p - 5d$	1.2562e+00	3.2292e-04	2,673.40	5.0359e+00	1.7150e-06
$9p - 5d$	5.6801e-01	2.1324e-04	3,033.07	5.7134e+00	1.2848e-06
$10p - 5d$	3.1062e-01	1.4887e-04	3,290.34	6.1980e+00	9.7308e-07
$11p - 5d$	1.9104e-01	1.0839e-04	3,480.69	6.5565e+00	7.4944e-07
$12p - 5d$	1.2718e-01	8.1539e-05	3,625.47	6.8293e+00	5.8725e-07
$13p - 5d$	8.9611e-02	6.2978e-05	3,738.14	7.0415e+00	4.6767e-07
$14p - 5d$	6.5888e-02	4.9708e-05	3,827.54	7.2099e+00	3.7796e-07
$15p - 5d$	5.0075e-02	3.9954e-05	3,899.66	7.3458e+00	3.0952e-07
$7p - 6d$	7.1484e+01	5.0787e-04	808.28	1.5226e+00	8.1549e-07
$8p - 6d$	9.4950e+00	3.0251e-04	1,332.89	2.5108e+00	8.0099e-07
$9p - 6d$	2.9825e+00	1.9457e-04	1,692.56	3.1883e+00	6.5420e-07
$10p - 6d$	1.3373e+00	1.3337e-04	1,949.83	3.6729e+00	5.1660e-07
$11p - 6d$	7.2683e-01	9.5861e-05	2,140.18	4.0314e+00	4.0756e-07
$12p - 6d$	4.4513e-01	7.1446e-05	2,284.96	4.3042e+00	3.2431e-07
$13p - 6d$	2.9551e-01	5.4799e-05	2,397.63	4.5164e+00	2.6101e-07
$14p - 6d$	2.0786e-01	4.3021e-05	2,487.03	4.6848e+00	2.1255e-07
$15p - 6d$	1.5270e-01	3.4434e-05	2,559.15	4.8206e+00	1.7506e-07
$8p - 7d$	1.4716e+02	2.8586e-04	524.61	9.8820e-01	2.9791e-07
$9p - 7d$	1.9357e+01	1.8008e-04	884.28	1.6657e+00	3.1634e-07

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$10p - 7d$	6.0224e+00	1.2053e-04	1,141.55	2.1503e+00	2.7334e-07
$11p - 7d$	2.6792e+00	8.5169e-05	1,331.90	2.5089e+00	2.2535e-07
$12p - 7d$	1.4475e+00	6.2710e-05	1,476.67	2.7816e+00	1.8396e-07
$13p - 7d$	8.8256e-01	4.7671e-05	1,589.34	2.9938e+00	1.5051e-07
$14p - 7d$	5.8402e-01	3.7174e-05	1,678.74	3.1622e+00	1.2397e-07
$15p - 7d$	4.0987e-01	2.9598e-05	1,750.87	3.2981e+00	1.0295e-07
$9p - 8d$	2.7081e+02	1.6952e-04	359.67	6.7751e-01	1.2112e-07
$10p - 8d$	3.5324e+01	1.1160e-04	616.94	1.1621e+00	1.3677e-07
$11p - 8d$	1.0898e+01	7.7143e-05	807.29	1.5207e+00	1.2371e-07
$12p - 8d$	4.8142e+00	5.5896e-05	952.07	1.7934e+00	1.0572e-07
$13p - 8d$	2.5863e+00	4.2002e-05	1,064.74	2.0056e+00	8.8840e-08
$14p - 8d$	1.5700e+00	3.2473e-05	1,154.14	2.1740e+00	7.4452e-08
$15p - 8d$	1.0354e+00	2.5687e-05	1,226.26	2.3099e+00	6.2574e-08
$10p - 9d$	4.5929e+02	1.0522e-04	257.27	4.8462e-01	5.3776e-08
$11p - 9d$	5.9476e+01	7.1767e-05	447.62	8.4318e-01	6.3817e-08
$12p - 9d$	1.8216e+01	5.0949e-05	592.40	1.1159e+00	5.9958e-08
$13p - 9d$	7.9958e+00	3.7706e-05	705.07	1.3281e+00	5.2812e-08
$14p - 9d$	4.2732e+00	2.8829e-05	794.47	1.4965e+00	4.5500e-08
$15p - 9d$	2.5830e+00	2.2617e-05	866.59	1.6324e+00	3.8935e-08
$11p - 10d$	7.3201e+02	6.7926e-05	190.35	3.5856e-01	2.5685e-08
$12p - 10d$	9.4197e+01	4.7701e-05	335.13	6.3128e-01	3.1756e-08
$13p - 10d$	2.8665e+01	3.4630e-05	447.80	8.4351e-01	3.0805e-08
$14p - 10d$	1.2511e+01	2.6095e-05	537.20	1.0119e+00	2.7847e-08
$15p - 10d$	6.6542e+00	2.0253e-05	609.32	1.1478e+00	2.4515e-08
$12p - 11d$	1.1109e+03	4.5356e-05	144.78	2.7271e-01	1.3045e-08
$13p - 11d$	1.4217e+02	3.2639e-05	257.45	4.8495e-01	1.6692e-08
$14p - 11d$	4.3018e+01	2.4151e-05	346.85	6.5335e-01	1.6640e-08
$15p - 11d$	1.8680e+01	1.8484e-05	418.97	7.8921e-01	1.5384e-08
$13p - 12d$	1.6205e+03	3.1183e-05	112.67	2.1224e-01	6.9796e-09
$14p - 12d$	2.0638e+02	2.2911e-05	202.07	3.8064e-01	9.1969e-09
$15p - 12d$	6.2133e+01	1.7233e-05	274.20	5.1650e-01	9.3868e-09
$14p - 13d$	2.2877e+03	2.1992e-05	89.40	1.6840e-01	3.9058e-09
$15p - 13d$	2.9011e+02	1.6449e-05	161.52	3.0426e-01	5.2781e-09
$15p - 14d$	3.1421e+03	1.5860e-05	72.12	1.3586e-01	2.2724e-09
$4f - 3d$	1.0466e+02	1.3719e-01	5,331.57	1.0043e+01	1.4530e-03
$5f - 3d$	1.1014e+01	4.5193e-02	7,799.32	1.4692e+01	7.0021e-04
$6f - 3d$	3.2334e+00	2.1353e-02	9,139.83	1.7217e+01	3.8769e-04
$7f - 3d$	1.4104e+00	1.2010e-02	9,948.12	1.8739e+01	2.3734e-04
$8f - 3d$	7.5503e-01	7.5010e-03	10,472.73	1.9727e+01	1.5605e-04

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$9f - 3d$	4.5733e-01	5.0277e-03	10,832.40	2.0405e+01	1.0819e-04
$10f - 3d$	3.0070e-01	3.5470e-03	11,089.66	2.0889e+01	7.8141e-05
$11f - 3d$	2.0960e-01	2.6020e-03	11,280.01	2.1248e+01	5.8305e-05
$12f - 3d$	1.5261e-01	1.9684e-03	11,424.79	2.1521e+01	4.4674e-05
$13f - 3d$	1.1494e-01	1.5268e-03	11,537.46	2.1733e+01	3.4994e-05
$14f - 3d$	8.8942e-02	1.2091e-03	11,626.86	2.1901e+01	2.7927e-05
$15f - 3d$	7.0361e-02	9.7443e-04	11,698.99	2.2037e+01	2.2646e-05
$5f - 4d$	1.9783e+02	2.5714e-02	2,467.76	4.6485e+00	1.2606e-04
$6f - 4d$	2.6806e+01	1.2805e-02	3,808.26	7.1736e+00	9.6874e-05
$7f - 4d$	8.5818e+00	7.3030e-03	4,616.55	8.6961e+00	6.6976e-05
$8f - 4d$	3.9032e+00	4.5875e-03	5,141.16	9.6843e+00	4.6852e-05
$9f - 4d$	2.1422e+00	3.0840e-03	5,500.83	1.0362e+01	3.3700e-05
$10f - 4d$	1.3199e+00	2.1795e-03	5,758.10	1.0846e+01	2.4930e-05
$11f - 4d$	8.7920e-01	1.6006e-03	5,948.45	1.1205e+01	1.8913e-05
$12f - 4d$	6.1930e-01	1.2117e-03	6,093.22	1.1478e+01	1.4667e-05
$13f - 4d$	4.5492e-01	9.4042e-04	6,205.89	1.1690e+01	1.1594e-05
$14f - 4d$	3.4527e-01	7.4504e-04	6,295.29	1.1858e+01	9.3173e-06
$15f - 4d$	2.6899e-01	6.0061e-04	6,367.42	1.1994e+01	7.5972e-06
$6f - 5d$	3.4540e+02	7.1962e-03	1,340.51	2.5251e+00	1.9163e-05
$7f - 5d$	5.0218e+01	4.3094e-03	2,148.79	4.0477e+00	1.8395e-05
$8f - 5d$	1.6579e+01	2.7397e-03	2,673.40	5.0359e+00	1.4550e-05
$9f - 5d$	7.6673e+00	1.8504e-03	3,033.07	5.7134e+00	1.1149e-05
$10f - 5d$	4.2539e+00	1.3106e-03	3,290.34	6.1980e+00	8.5669e-06
$11f - 5d$	2.6422e+00	9.6370e-04	3,480.69	6.5565e+00	6.6635e-06
$12f - 5d$	1.7715e+00	7.3015e-04	3,625.47	6.8293e+00	5.2586e-06
$13f - 5d$	1.2549e+00	5.6694e-04	3,738.14	7.0415e+00	4.2101e-06
$14f - 5d$	9.2643e-01	4.4930e-04	3,827.54	7.2099e+00	3.4163e-06
$15f - 5d$	7.0634e-01	3.6230e-04	3,899.66	7.3458e+00	2.8067e-06
$7f - 6d$	5.6512e+02	2.5811e-03	808.28	1.5226e+00	4.1444e-06
$8f - 6d$	8.3835e+01	1.7170e-03	1,332.89	2.5108e+00	4.5465e-06
$9f - 6d$	2.7939e+01	1.1717e-03	1,692.56	3.1883e+00	3.9396e-06
$10f - 6d$	1.2991e+01	8.3294e-04	1,949.83	3.6729e+00	3.2263e-06
$11f - 6d$	7.2349e+00	6.1342e-04	2,140.18	4.0314e+00	2.6080e-06
$12f - 6d$	4.5078e+00	4.6512e-04	2,284.96	4.3042e+00	2.1113e-06
$13f - 6d$	3.0308e+00	3.6130e-04	2,397.63	4.5164e+00	1.7209e-06
$14f - 6d$	2.1526e+00	2.8641e-04	2,487.03	4.6848e+00	1.4150e-06
$15f - 6d$	1.5933e+00	2.3098e-04	2,559.15	4.8206e+00	1.1742e-06
$8f - 7d$	8.7732e+02	1.0956e-03	524.61	9.8820e-01	1.1417e-06
$9f - 7d$	1.3041e+02	7.7990e-04	884.28	1.6657e+00	1.3700e-06

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$10f - 7d$	4.3472e+01	5.5932e-04	1,141.55	2.1503e+00	1.2684e-06
$11f - 7d$	2.0212e+01	4.1304e-04	1,331.90	2.5089e+00	1.0928e-06
$12f - 7d$	1.1256e+01	3.1349e-04	1,476.67	2.7816e+00	9.1962e-07
$13f - 7d$	7.0156e+00	2.4361e-04	1,589.34	2.9938e+00	7.6914e-07
$14f - 7d$	4.7198e+00	1.9313e-04	1,678.74	3.1622e+00	6.4406e-07
$15f - 7d$	3.3551e+00	1.5575e-04	1,750.87	3.2981e+00	5.4173e-07
$9f - 8d$	1.3049e+03	5.2512e-04	359.67	6.7751e-01	3.7520e-07
$10f - 8d$	1.9294e+02	3.9185e-04	616.94	1.1621e+00	4.8024e-07
$11f - 8d$	6.4068e+01	2.9154e-04	807.29	1.5207e+00	4.6755e-07
$12f - 8d$	2.9704e+01	2.2171e-04	952.07	1.7934e+00	4.1932e-07
$13f - 8d$	1.6510e+01	1.7236e-04	1,064.74	2.0056e+00	3.6458e-07
$14f - 8d$	1.0277e+01	1.3665e-04	1,154.14	2.1740e+00	3.1329e-07
$15f - 8d$	6.9086e+00	1.1018e-04	1,226.26	2.3099e+00	2.6841e-07
$10f - 9d$	1.8733e+03	2.7589e-04	257.27	4.8462e-01	1.4100e-07
$11f - 9d$	2.7472e+02	2.1310e-04	447.62	8.4318e-01	1.8949e-07
$12f - 9d$	9.0703e+01	1.6309e-04	592.40	1.1159e+00	1.9193e-07
$13f - 9d$	4.1877e+01	1.2695e-04	705.07	1.3281e+00	1.7781e-07
$14f - 9d$	2.3205e+01	1.0064e-04	794.47	1.4965e+00	1.5884e-07
$15f - 9d$	1.4412e+01	8.1125e-05	866.59	1.6324e+00	1.3966e-07
$11f - 10d$	2.6105e+03	1.5572e-04	190.35	3.5856e-01	5.8885e-08
$12f - 10d$	3.7933e+02	1.2349e-04	335.13	6.3128e-01	8.2210e-08
$13f - 10d$	1.2443e+02	9.6639e-05	447.80	8.4351e-01	8.5967e-08
$14f - 10d$	5.7175e+01	7.6663e-05	537.20	1.0119e+00	8.1812e-08
$15f - 10d$	3.1569e+01	6.1769e-05	609.32	1.1478e+00	7.4768e-08
$12f - 11d$	3.5470e+03	9.3097e-05	144.78	2.7271e-01	2.6775e-08
$13f - 11d$	5.1064e+02	7.5363e-05	257.45	4.8495e-01	3.8543e-08
$14f - 11d$	1.6640e+02	6.0053e-05	346.85	6.5335e-01	4.1378e-08
$15f - 11d$	7.6076e+01	4.8392e-05	418.97	7.8921e-01	4.0277e-08
$13f - 12d$	4.7159e+03	5.8340e-05	112.67	2.1224e-01	1.3058e-08
$14f - 12d$	6.7284e+02	4.8017e-05	202.07	3.8064e-01	1.9275e-08
$15f - 12d$	2.1782e+02	3.8837e-05	274.20	5.1650e-01	2.1155e-08
$14f - 13d$	6.1528e+03	3.8025e-05	89.40	1.6840e-01	6.7531e-09
$15f - 13d$	8.7039e+02	3.1725e-05	161.52	3.0426e-01	1.0180e-08
$15f - 14d$	7.8958e+03	2.5621e-05	72.12	1.3586e-01	3.6710e-09
$5d - 4f$	2.7599e+00	5.0223e-04	2,467.76	4.6485e+00	2.4621e-06
$6d - 4f$	3.1912e-01	2.1342e-04	3,808.26	7.1736e+00	1.6146e-06
$7d - 4f$	9.3864e-02	1.1183e-04	4,616.55	8.6961e+00	1.0256e-06
$8d - 4f$	4.0553e-02	6.6727e-05	5,141.16	9.6843e+00	6.8149e-07
$9d - 4f$	2.1517e-02	4.3368e-05	5,500.83	1.0362e+01	4.7391e-07

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$10d-4f$	1.2951e-02	2.9938e-05	5,758.10	1.0846e+01	3.4245e-07
$11d-4f$	8.4808e-03	2.1615e-05	5,948.45	1.1205e+01	2.5542e-07
$12d-4f$	5.8981e-03	1.6157e-05	6,093.22	1.1478e+01	1.9557e-07
$13d-4f$	4.2903e-03	1.2416e-05	6,205.89	1.1690e+01	1.5307e-07
$14d-4f$	3.2311e-03	9.7612e-06	6,295.29	1.1858e+01	1.2207e-07
$15d-4f$	2.5018e-03	7.8205e-06	6,367.42	1.1994e+01	9.8923e-08
$6d-5f$	1.3331e+01	3.8883e-04	1,340.51	2.5251e+00	1.0355e-06
$7d-5f$	1.5835e+00	1.9024e-04	2,148.79	4.0477e+00	8.1207e-07
$8d-5f$	4.6807e-01	1.0829e-04	2,673.40	5.0359e+00	5.7512e-07
$9d-5f$	2.0211e-01	6.8287e-05	3,033.07	5.7134e+00	4.1145e-07
$10d-5f$	1.0708e-01	4.6190e-05	3,290.34	6.1980e+00	3.0191e-07
$11d-5f$	6.4376e-02	3.2871e-05	3,480.69	6.5565e+00	2.2729e-07
$12d-5f$	4.2135e-02	2.4313e-05	3,625.47	6.8293e+00	1.7511e-07
$13d-5f$	2.9306e-02	1.8536e-05	3,738.14	7.0415e+00	1.3765e-07
$14d-5f$	2.1329e-02	1.4482e-05	3,827.54	7.2099e+00	1.1011e-07
$15d-5f$	1.6078e-02	1.1546e-05	3,899.66	7.3458e+00	8.9443e-08
$7d-6f$	3.9272e+01	2.5112e-04	808.28	1.5226e+00	4.0322e-07
$8d-6f$	4.7259e+00	1.3551e-04	1,332.89	2.5108e+00	3.5881e-07
$9d-6f$	1.3978e+00	8.2067e-05	1,692.56	3.1883e+00	2.7594e-07
$10d-6f$	6.0207e-01	5.4043e-05	1,949.83	3.6729e+00	2.0933e-07
$11d-6f$	3.1810e-01	3.7758e-05	2,140.18	4.0314e+00	1.6053e-07
$12d-6f$	1.9079e-01	2.7560e-05	2,284.96	4.3042e+00	1.2510e-07
$13d-6f$	1.2467e-01	2.0806e-05	2,397.63	4.5164e+00	9.9100e-08
$14d-6f$	8.6615e-02	1.6134e-05	2,487.03	4.6848e+00	7.9710e-08
$15d-6f$	6.3006e-02	1.2787e-05	2,559.15	4.8206e+00	6.5008e-08
$8d-7f$	9.0669e+01	1.5851e-04	524.61	9.8820e-01	1.6519e-07
$9d-7f$	1.0983e+01	9.1961e-05	884.28	1.6657e+00	1.6154e-07
$10d-7f$	3.2445e+00	5.8443e-05	1,141.55	2.1503e+00	1.3253e-07
$11d-7f$	1.3931e+00	3.9856e-05	1,331.90	2.5089e+00	1.0545e-07
$12d-7f$	7.3361e-01	2.8603e-05	1,476.67	2.7816e+00	8.3907e-08
$13d-7f$	4.3876e-01	2.1330e-05	1,589.34	2.9938e+00	6.7344e-08
$14d-7f$	2.8606e-01	1.6387e-05	1,678.74	3.1622e+00	5.4650e-08
$15d-7f$	1.9842e-01	1.2896e-05	1,750.87	3.2981e+00	4.4854e-08
$9d-8f$	1.8014e+02	1.0149e-04	359.67	6.7751e-01	7.2513e-08
$10d-8f$	2.1896e+01	6.2258e-05	616.94	1.1621e+00	7.6302e-08
$11d-8f$	6.4551e+00	4.1123e-05	807.29	1.5207e+00	6.5950e-08
$12d-8f$	2.7623e+00	2.8865e-05	952.07	1.7934e+00	5.4592e-08
$13d-8f$	1.4497e+00	2.1188e-05	1,064.74	2.0056e+00	4.4816e-08
$14d-8f$	8.6445e-01	1.6092e-05	1,154.14	2.1740e+00	3.6895e-08

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$15d - 8f$	5.6221e-01	1.2553e-05	1,226.26	2.3099e+00	3.0579e-08
$10d - 9f$	3.2283e+02	6.6565e-05	257.27	4.8462e-01	3.4020e-08
$11d - 9f$	3.9307e+01	4.2687e-05	447.62	8.4318e-01	3.7958e-08
$12d - 9f$	1.1561e+01	2.9102e-05	592.40	1.1159e+00	3.4248e-08
$13d - 9f$	4.9306e+00	2.0926e-05	705.07	1.3281e+00	2.9310e-08
$14d - 9f$	2.5790e+00	1.5660e-05	794.47	1.4965e+00	2.4715e-08
$15d - 9f$	1.5333e+00	1.2083e-05	866.59	1.6324e+00	2.0801e-08
$11d - 10f$	5.3644e+02	4.4800e-05	190.35	3.5856e-01	1.6941e-08
$12d - 10f$	6.5360e+01	2.9788e-05	335.13	6.3128e-01	1.9831e-08
$13d - 10f$	1.9177e+01	2.0851e-05	447.80	8.4351e-01	1.8548e-08
$14d - 10f$	8.1529e+00	1.5304e-05	537.20	1.0119e+00	1.6332e-08
$15d - 10f$	4.2510e+00	1.1645e-05	609.32	1.1478e+00	1.4095e-08
$12d - 11f$	8.4119e+02	3.0909e-05	144.78	2.7271e-01	8.8897e-09
$13d - 11f$	1.0250e+02	2.1178e-05	257.45	4.8495e-01	1.0831e-08
$14d - 11f$	3.0003e+01	1.5160e-05	346.85	6.5335e-01	1.0445e-08
$15d - 11f$	1.2717e+01	1.1325e-05	418.97	7.8921e-01	9.4261e-09
$13d - 12f$	1.2598e+03	2.1819e-05	112.67	2.1224e-01	4.8837e-09
$14d - 12f$	1.5347e+02	1.5334e-05	202.07	3.8064e-01	6.1553e-09
$15d - 12f$	4.4823e+01	1.1189e-05	274.20	5.1650e-01	6.0945e-09
$14d - 13f$	1.8177e+03	1.5727e-05	89.40	1.6840e-01	2.7930e-09
$15d - 13f$	2.2133e+02	1.1294e-05	161.52	3.0426e-01	3.6240e-09
$15d - 14f$	2.5425e+03	1.1551e-05	72.12	1.3586e-01	1.6549e-09
$5g - 4f$	3.1402e+02	4.2328e-02	2,467.76	4.6485e+00	2.0750e-04
$6g - 4f$	2.7572e+01	1.3659e-02	3,808.26	7.1736e+00	1.0333e-04
$7g - 4f$	7.2816e+00	6.4260e-03	4,616.55	8.6961e+00	5.8933e-05
$8g - 4f$	2.9739e+00	3.6247e-03	5,141.16	9.6843e+00	3.7019e-05
$9g - 4f$	1.5254e+00	2.2774e-03	5,500.83	1.0362e+01	2.4886e-05
$10g - 4f$	8.9790e-01	1.5376e-03	5,758.10	1.0846e+01	1.7588e-05
$11g - 4f$	5.7896e-01	1.0930e-03	5,948.45	1.1205e+01	1.2916e-05
$12g - 4f$	3.9812e-01	8.0783e-04	6,093.22	1.1478e+01	9.7783e-06
$13g - 4f$	2.8714e-01	6.1556e-04	6,205.89	1.1690e+01	7.5888e-06
$14g - 4f$	2.1483e-01	4.8075e-04	6,295.29	1.1858e+01	6.0122e-06
$15g - 4f$	1.6548e-01	3.8317e-04	6,367.42	1.1994e+01	4.8467e-06
$6g - 5f$	5.0918e+02	1.1001e-02	1,340.51	2.5251e+00	2.9296e-05
$7g - 5f$	6.1287e+01	5.4540e-03	2,148.79	4.0477e+00	2.3281e-05
$8g - 5f$	1.8199e+01	3.1190e-03	2,673.40	5.0359e+00	1.6564e-05
$9g - 5f$	7.8744e+00	1.9707e-03	3,033.07	5.7134e+00	1.1874e-05
$10g - 5f$	4.1762e+00	1.3344e-03	3,290.34	6.1980e+00	8.7219e-06
$11g - 5f$	2.5121e+00	9.5015e-04	3,480.69	6.5565e+00	6.5699e-06

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$12g - 5f$	1.6447e+00	7.0300e-04	3,625.47	6.8293e+00	5.0631e-06
$13g - 5f$	1.1442e+00	5.3607e-04	3,738.14	7.0415e+00	3.9809e-06
$14g - 5f$	8.3286e-01	4.1888e-04	3,827.54	7.2099e+00	3.1850e-06
$15g - 5f$	6.2789e-01	3.3399e-04	3,899.66	7.3458e+00	2.5874e-06
$7g - 6f$	7.9061e+02	3.7447e-03	808.28	1.5226e+00	6.0129e-06
$8g - 6f$	1.0597e+02	2.2509e-03	1,332.89	2.5108e+00	5.9599e-06
$9g - 6f$	3.3112e+01	1.4401e-03	1,692.56	3.1883e+00	4.8420e-06
$10g - 6f$	1.4736e+01	9.7982e-04	1,949.83	3.6729e+00	3.7953e-06
$11g - 6f$	7.9540e+00	6.9937e-04	2,140.18	4.0314e+00	2.9734e-06
$12g - 6f$	4.8423e+00	5.1814e-04	2,284.96	4.3042e+00	2.3519e-06
$13g - 6f$	3.1986e+00	3.9543e-04	2,397.63	4.5164e+00	1.8834e-06
$14g - 6f$	2.2406e+00	3.0916e-04	2,487.03	4.6848e+00	1.5274e-06
$15g - 6f$	1.6403e+00	2.4659e-04	2,559.15	4.8206e+00	1.2536e-06
$8g - 7f$	1.1801e+03	1.5283e-03	524.61	9.8820e-01	1.5927e-06
$9g - 7f$	1.6528e+02	1.0251e-03	884.28	1.6657e+00	1.8007e-06
$10g - 7f$	5.2861e+01	7.0531e-04	1,141.55	2.1503e+00	1.5994e-06
$11g - 7f$	2.3854e+01	5.0552e-04	1,331.90	2.5089e+00	1.3375e-06
$12g - 7f$	1.2992e+01	3.7523e-04	1,476.67	2.7816e+00	1.1007e-06
$13g - 7f$	7.9605e+00	2.8665e-04	1,589.34	2.9938e+00	9.0505e-07
$14g - 7f$	5.2844e+00	2.2424e-04	1,678.74	3.1622e+00	7.4781e-07
$15g - 7f$	3.7165e+00	1.7892e-04	1,750.87	3.2981e+00	6.2232e-07
$9g - 8f$	1.7023e+03	7.1039e-04	359.67	6.7751e-01	5.0757e-07
$10g - 8f$	2.4272e+02	5.1120e-04	616.94	1.1621e+00	6.2651e-07
$11g - 8f$	7.8423e+01	3.7008e-04	807.29	1.5207e+00	5.9350e-07
$12g - 8f$	3.5612e+01	2.7565e-04	952.07	1.7934e+00	5.2134e-07
$13g - 8f$	1.9478e+01	2.1088e-04	1,064.74	2.0056e+00	4.4604e-07
$14g - 8f$	1.1971e+01	1.6507e-04	1,154.14	2.1740e+00	3.7847e-07
$15g - 8f$	7.9664e+00	1.3176e-04	1,226.26	2.3099e+00	3.2096e-07
$10g - 9f$	2.3841e+03	3.6412e-04	257.27	4.8462e-01	1.8610e-07
$11g - 9f$	3.4195e+02	2.7508e-04	447.62	8.4318e-01	2.4460e-07
$12g - 9f$	1.1086e+02	2.0672e-04	592.40	1.1159e+00	2.4328e-07
$13g - 9f$	5.0445e+01	1.5859e-04	705.07	1.3281e+00	2.2213e-07
$14g - 9f$	2.7629e+01	1.2427e-04	794.47	1.4965e+00	1.9612e-07
$15g - 9f$	1.6998e+01	9.9223e-05	866.59	1.6324e+00	1.7081e-07
$11g - 10f$	3.2553e+03	2.0138e-04	190.35	3.5856e-01	7.6149e-08
$12g - 10f$	4.6686e+02	1.5761e-04	335.13	6.3128e-01	1.0493e-07
$13g - 10f$	1.5132e+02	1.2187e-04	447.80	8.4351e-01	1.0842e-07
$14g - 10f$	6.8829e+01	9.5707e-05	537.20	1.0119e+00	1.0214e-07
$15g - 10f$	3.7684e+01	7.6465e-05	609.32	1.1478e+00	9.2557e-08

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$12g - 11f$	4.3481e+03	1.1835e-04	144.78	2.7271e-01	3.4037e-08
$13g - 11f$	6.2161e+02	9.5136e-05	257.45	4.8495e-01	4.8655e-08
$14g - 11f$	2.0102e+02	7.5236e-05	346.85	6.5335e-01	5.1840e-08
$15g - 11f$	9.1273e+01	6.0209e-05	418.97	7.8921e-01	5.0112e-08
$13g - 12f$	5.6972e+03	7.3090e-05	112.67	2.1224e-01	1.6359e-08
$14g - 12f$	8.1062e+02	5.9993e-05	202.07	3.8064e-01	2.4082e-08
$15g - 12f$	2.6127e+02	4.8309e-05	274.20	5.1650e-01	2.6314e-08
$14g - 13f$	7.3400e+03	4.7042e-05	89.40	1.6840e-01	8.3545e-09
$15g - 13f$	1.0386e+03	3.9260e-05	161.52	3.0426e-01	1.2597e-08
$15g - 14f$	9.3163e+03	3.1351e-05	72.12	1.3586e-01	4.4918e-09
$6f - 5g$	4.0735e+00	1.1316e-04	1,340.51	2.5251e+00	3.0133e-07
$7f - 5g$	4.0445e-01	4.6277e-05	2,148.79	4.0477e+00	1.9754e-07
$8f - 5g$	1.0785e-01	2.3764e-05	2,673.40	5.0359e+00	1.2620e-07
$9f - 5g$	4.3612e-02	1.4033e-05	3,033.07	5.7134e+00	8.4556e-08
$10f - 5g$	2.2097e-02	9.0773e-06	3,290.34	6.1980e+00	5.9333e-08
$11f - 5g$	1.2866e-02	6.2568e-06	3,480.69	6.5565e+00	4.3263e-08
$12f - 5g$	8.2237e-03	4.5193e-06	3,625.47	6.8293e+00	3.2549e-08
$13f - 5g$	5.6170e-03	3.3836e-06	3,738.14	7.0415e+00	2.5127e-08
$14f - 5g$	4.0306e-03	2.6064e-06	3,827.54	7.2099e+00	1.9818e-08
$15f - 5g$	3.0043e-03	2.0546e-06	3,899.66	7.3458e+00	1.5917e-08
$7f - 6g$	1.8139e+01	1.1047e-04	808.28	1.5226e+00	1.7737e-07
$8f - 6g$	1.9080e+00	5.2105e-05	1,332.89	2.5108e+00	1.3797e-07
$9f - 6g$	5.2003e-01	2.9079e-05	1,692.56	3.1883e+00	9.7773e-08
$10f - 6g$	2.1216e-01	1.8137e-05	1,949.83	3.6729e+00	7.0252e-08
$11f - 6g$	1.0789e-01	1.2197e-05	2,140.18	4.0314e+00	5.1854e-08
$12f - 6g$	6.2918e-02	8.6560e-06	2,284.96	4.3042e+00	3.9291e-08
$13f - 6g$	4.0246e-02	6.3971e-06	2,397.63	4.5164e+00	3.0469e-08
$14f - 6g$	2.7503e-02	4.8790e-06	2,487.03	4.6848e+00	2.4105e-08
$15f - 6g$	1.9745e-02	3.8164e-06	2,559.15	4.8206e+00	1.9402e-08
$8f - 7g$	5.0609e+01	8.4263e-05	524.61	9.8820e-01	8.7815e-08
$9f - 7g$	5.5065e+00	4.3909e-05	884.28	1.6657e+00	7.7133e-08
$10f - 7g$	1.5202e+00	2.6078e-05	1,141.55	2.1503e+00	5.9138e-08
$11f - 7g$	6.2315e-01	1.6979e-05	1,331.90	2.5089e+00	4.4924e-08
$12f - 7g$	3.1735e-01	1.1784e-05	1,476.67	2.7816e+00	3.4569e-08
$13f - 7g$	1.8511e-01	8.5701e-06	1,589.34	2.9938e+00	2.7058e-08
$14f - 7g$	1.1837e-01	6.4582e-06	1,678.74	3.1622e+00	2.1538e-08
$15f - 7g$	8.0863e-02	5.0052e-06	1,750.87	3.2981e+00	1.7409e-08
$9f - 8g$	1.1241e+02	6.0313e-05	359.67	6.7751e-01	4.3094e-08
$10f - 8g$	1.2499e+01	3.3845e-05	616.94	1.1621e+00	4.1480e-08

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$11f - 8g$	3.4780e+00	2.1102e-05	807.29	1.5207e+00	3.3841e-08
$12f - 8g$	1.4293e+00	1.4225e-05	952.07	1.7934e+00	2.6903e-08
$13f - 8g$	7.2814e-01	1.0136e-05	1,064.74	2.0056e+00	2.1438e-08
$14f - 8g$	4.2446e-01	7.5252e-06	1,154.14	2.1740e+00	1.7253e-08
$15f - 8g$	2.7120e-01	5.7670e-06	1,226.26	2.3099e+00	1.4049e-08
$10f - 9g$	2.1698e+02	4.2609e-05	257.27	4.8462e-01	2.1777e-08
$11f - 9g$	2.4486e+01	2.5325e-05	447.62	8.4318e-01	2.2519e-08
$12f - 9g$	6.8486e+00	1.6419e-05	592.40	1.1159e+00	1.9322e-08
$13f - 9g$	2.8181e+00	1.1391e-05	705.07	1.3281e+00	1.5954e-08
$14f - 9g$	1.4351e+00	8.2987e-06	794.47	1.4965e+00	1.3097e-08
$15f - 9g$	8.3572e-01	6.2721e-06	866.59	1.6324e+00	1.0798e-08
$11f - 10g$	3.8032e+02	3.0250e-05	190.35	3.5856e-01	1.1439e-08
$12f - 10g$	4.3371e+01	1.8825e-05	335.13	6.3128e-01	1.2533e-08
$13f - 10g$	1.2172e+01	1.2604e-05	447.80	8.4351e-01	1.1212e-08
$14f - 10g$	5.0110e+00	8.9585e-06	537.20	1.0119e+00	9.5602e-09
$15f - 10g$	2.5499e+00	6.6523e-06	609.32	1.1478e+00	8.0522e-09
$12f - 11g$	6.2094e+02	2.1730e-05	144.78	2.7271e-01	6.2496e-09
$13f - 11g$	7.1362e+01	1.4042e-05	257.45	4.8495e-01	7.1817e-09
$14f - 11g$	2.0073e+01	9.6593e-06	346.85	6.5335e-01	6.6555e-09
$15f - 11g$	8.2636e+00	7.0087e-06	418.97	7.8921e-01	5.8333e-09
$13f - 12g$	9.5988e+02	1.5833e-05	112.67	2.1224e-01	3.5438e-09
$14f - 12g$	1.1096e+02	1.0559e-05	202.07	3.8064e-01	4.2385e-09
$15f - 12g$	3.1260e+01	7.4316e-06	274.20	5.1650e-01	4.0480e-09
$14f - 13g$	1.4207e+03	1.1707e-05	89.40	1.6840e-01	2.0791e-09
$15f - 13g$	1.6499e+02	8.0182e-06	161.52	3.0426e-01	2.5728e-09
$15f - 14g$	2.0296e+03	8.7813e-06	72.12	1.3586e-01	1.2582e-09
$7g - 6h$	5.6367e+00	3.3373e-05	808.28	1.5226e+00	5.3586e-08
$8g - 6h$	4.8931e-01	1.2991e-05	1,332.89	2.5108e+00	3.4398e-08
$9g - 6h$	1.1892e-01	6.4649e-06	1,692.56	3.1883e+00	2.1737e-08
$10g - 6h$	4.5004e-02	3.7404e-06	1,949.83	3.6729e+00	1.4488e-08
$11g - 6h$	2.1717e-02	2.3868e-06	2,140.18	4.0314e+00	1.0148e-08
$12g - 6h$	1.2190e-02	1.6304e-06	2,284.96	4.3042e+00	7.4007e-09
$13g - 6h$	7.5756e-03	1.1707e-06	2,397.63	4.5164e+00	5.5759e-09
$14g - 6h$	5.0625e-03	8.7315e-07	2,487.03	4.6848e+00	4.3138e-09
$15g - 6h$	3.5708e-03	6.7101e-07	2,559.15	4.8206e+00	3.4113e-09
$8g - 7h$	2.3699e+01	3.8363e-05	524.61	9.8820e-01	3.9980e-08
$9g - 7h$	2.2341e+00	1.7320e-05	884.28	1.6657e+00	3.0425e-08
$10g - 7h$	5.6368e-01	9.4014e-06	1,141.55	2.1503e+00	2.1320e-08
$11g - 7h$	2.1733e-01	5.7572e-06	1,331.90	2.5089e+00	1.5233e-08

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$12g - 7h$	1.0591e-01	3.8237e-06	1,476.67	2.7816e+00	1.1217e-08
$13g - 7h$	5.9778e-02	2.6907e-06	1,589.34	2.9938e+00	8.4954e-09
$14g - 7h$	3.7272e-02	1.9770e-06	1,678.74	3.1622e+00	6.5931e-09
$15g - 7h$	2.4959e-02	1.5020e-06	1,750.87	3.2981e+00	5.2241e-09
$9g - 8h$	6.3455e+01	3.3102e-05	359.67	6.7751e-01	2.3651e-08
$10g - 8h$	6.2957e+00	1.6575e-05	616.94	1.1621e+00	2.0314e-08
$11g - 8h$	1.6271e+00	9.5981e-06	807.29	1.5207e+00	1.5393e-08
$12g - 8h$	6.3500e-01	6.1440e-06	952.07	1.7934e+00	1.1620e-08
$13g - 8h$	3.1143e-01	4.2147e-06	1,064.74	2.0056e+00	8.9147e-09
$14g - 8h$	1.7637e-01	3.0401e-06	1,154.14	2.1740e+00	6.9701e-09
$15g - 8h$	1.1017e-01	2.2778e-06	1,226.26	2.3099e+00	5.5486e-09
$10g - 9h$	1.3668e+02	2.6094e-05	257.27	4.8462e-01	1.3336e-08
$11g - 9h$	1.4036e+01	1.4114e-05	447.62	8.4318e-01	1.2550e-08
$12g - 9h$	3.6877e+00	8.5955e-06	592.40	1.1159e+00	1.0115e-08
$13g - 9h$	1.4511e+00	5.7025e-06	705.07	1.3281e+00	7.9872e-09
$14g - 9h$	7.1469e-01	4.0181e-06	794.47	1.4965e+00	6.3416e-09
$15g - 9h$	4.0561e-01	2.9595e-06	866.59	1.6324e+00	5.0949e-09
$11g - 10h$	2.5766e+02	1.9924e-05	190.35	3.5856e-01	7.5341e-09
$12g - 10h$	2.7118e+01	1.1444e-05	335.13	6.3128e-01	7.6184e-09
$13g - 10h$	7.2091e+00	7.2578e-06	447.80	8.4351e-01	6.4563e-09
$14g - 10h$	2.8534e+00	4.9595e-06	537.20	1.0119e+00	5.2927e-09
$15g - 10h$	1.4093e+00	3.5746e-06	609.32	1.1478e+00	4.3269e-09
$12g - 11h$	4.4321e+02	1.5079e-05	144.78	2.7271e-01	4.3369e-09
$13g - 11h$	4.7507e+01	9.0887e-06	257.45	4.8495e-01	4.6482e-09
$14g - 11h$	1.2740e+01	5.9603e-06	346.85	6.5335e-01	4.1068e-09
$15g - 11h$	5.0640e+00	4.1756e-06	418.97	7.8921e-01	3.4754e-09
$13g - 12h$	7.1268e+02	1.1429e-05	112.67	2.1224e-01	2.5580e-09
$14g - 12h$	7.7471e+01	7.1669e-06	202.07	3.8064e-01	2.8769e-09
$15g - 12h$	2.0914e+01	4.8338e-06	274.20	5.1650e-01	2.6330e-09
$14g - 13h$	1.0879e+03	8.7157e-06	89.40	1.6840e-01	1.5479e-09
$15g - 13h$	1.1958e+02	5.6500e-06	161.52	3.0426e-01	1.8129e-09
$15g - 14h$	1.5934e+03	6.7026e-06	72.12	1.3586e-01	9.6032e-10
$6h - 5g$	7.4063e+02	1.6366e-02	1,340.51	2.5251e+00	4.3581e-05
$7h - 5g$	5.5612e+01	5.0615e-03	2,148.79	4.0477e+00	2.1606e-05
$8h - 5g$	1.3274e+01	2.3265e-03	2,673.40	5.0359e+00	1.2356e-05
$9h - 5g$	5.0621e+00	1.2957e-03	3,033.07	5.7134e+00	7.8070e-06
$10h - 5g$	2.4748e+00	8.0871e-04	3,290.34	6.1980e+00	5.2860e-06
$11h - 5g$	1.4072e+00	5.4436e-04	3,480.69	6.5565e+00	3.7640e-06
$12h - 5g$	8.8457e-01	3.8668e-04	3,625.47	6.8293e+00	2.7849e-06

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$13h - 5g$	5.9681e-01	2.8597e-04	3,738.14	7.0415e+00	2.1236e-06
$14h - 5g$	4.2427e-01	2.1824e-04	3,827.54	7.2099e+00	1.6594e-06
$15h - 5g$	3.1395e-01	1.7079e-04	3,899.66	7.3458e+00	1.3231e-06
$7h - 6g$	1.0933e+03	5.2960e-03	808.28	1.5226e+00	8.5038e-06
$8h - 6g$	1.1808e+02	2.5649e-03	1,332.89	2.5108e+00	6.7915e-06
$9h - 6g$	3.2558e+01	1.4481e-03	1,692.56	3.1883e+00	4.8692e-06
$10h - 6g$	1.3366e+01	9.0891e-04	1,949.83	3.6729e+00	3.5206e-06
$11h - 6g$	6.8229e+00	6.1355e-04	2,140.18	4.0314e+00	2.6085e-06
$12h - 6g$	3.9890e+00	4.3654e-04	2,284.96	4.3042e+00	1.9815e-06
$13h - 6g$	2.5562e+00	3.2319e-04	2,397.63	4.5164e+00	1.5393e-06
$14h - 6g$	1.7491e+00	2.4682e-04	2,487.03	4.6848e+00	1.2194e-06
$15h - 6g$	1.2569e+00	1.9325e-04	2,559.15	4.8206e+00	9.8246e-07
$8h - 7g$	1.5715e+03	2.0814e-03	524.61	9.8820e-01	2.1691e-06
$9h - 7g$	1.9484e+02	1.2359e-03	884.28	1.6657e+00	2.1710e-06
$10h - 7g$	5.7573e+01	7.8565e-04	1,141.55	2.1503e+00	1.7816e-06
$11h - 7g$	2.4593e+01	5.3302e-04	1,331.90	2.5089e+00	1.4103e-06
$12h - 7g$	1.2871e+01	3.8019e-04	1,476.67	2.7816e+00	1.1153e-06
$13h - 7g$	7.6534e+00	2.8186e-04	1,589.34	2.9938e+00	8.8991e-07
$14h - 7g$	4.9641e+00	2.1544e-04	1,678.74	3.1622e+00	7.1846e-07
$15h - 7g$	3.4279e+00	1.6878e-04	1,750.87	3.2981e+00	5.8704e-07
$9h - 8g$	2.2011e+03	9.3947e-04	359.67	6.7751e-01	6.7125e-07
$10h - 8g$	2.9101e+02	6.2685e-04	616.94	1.1621e+00	7.6825e-07
$11h - 8g$	8.9163e+01	4.3032e-04	807.29	1.5207e+00	6.9012e-07
$12h - 8g$	3.8946e+01	3.0831e-04	952.07	1.7934e+00	5.8312e-07
$13h - 8g$	2.0687e+01	2.2906e-04	1,064.74	2.0056e+00	4.8449e-07
$14h - 8g$	1.2429e+01	1.7528e-04	1,154.14	2.1740e+00	4.0186e-07
$15h - 8g$	8.1233e+00	1.3741e-04	1,226.26	2.3099e+00	3.3472e-07
$10h - 9g$	3.0108e+03	4.7030e-04	257.27	4.8462e-01	2.4036e-07
$11h - 9g$	4.1108e+02	3.3821e-04	447.62	8.4318e-01	3.0074e-07
$12h - 9g$	1.2844e+02	2.4495e-04	592.40	1.1159e+00	2.8826e-07
$13h - 9g$	5.6821e+01	1.8269e-04	705.07	1.3281e+00	2.5589e-07
$14h - 9g$	3.0444e+01	1.4004e-04	794.47	1.4965e+00	2.2102e-07
$15h - 9g$	1.8406e+01	1.0988e-04	866.59	1.6324e+00	1.8916e-07
$11h - 10g$	4.0317e+03	2.5508e-04	190.35	3.5856e-01	9.6456e-08
$12h - 10g$	5.5948e+02	1.9317e-04	335.13	6.3128e-01	1.2860e-07
$13h - 10g$	1.7665e+02	1.4551e-04	447.80	8.4351e-01	1.2944e-07
$14h - 10g$	7.8696e+01	1.1191e-04	537.20	1.0119e+00	1.1943e-07
$15h - 10g$	4.2373e+01	8.7933e-05	609.32	1.1478e+00	1.0644e-07
$12h - 11g$	5.2979e+03	1.4748e-04	144.78	2.7271e-01	4.2415e-08

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$13h - 11g$	7.4078e+02	1.1595e-04	257.45	4.8495e-01	5.9302e-08
$14h - 11g$	2.3510e+02	8.9992e-05	346.85	6.5335e-01	6.2007e-08
$15h - 11g$	1.0511e+02	7.0910e-05	418.97	7.8921e-01	5.9019e-08
$13h - 12g$	6.8456e+03	8.9819e-05	112.67	2.1224e-01	2.0104e-08
$14h - 12g$	9.5975e+02	7.2643e-05	202.07	3.8064e-01	2.9161e-08
$15h - 12g$	3.0519e+02	5.7714e-05	274.20	5.1650e-01	3.1437e-08
$14h - 13g$	8.7139e+03	5.7116e-05	89.40	1.6840e-01	1.0144e-08
$15h - 13g$	1.2214e+03	4.7218e-05	161.52	3.0426e-01	1.5151e-08
$15h - 14g$	1.0944e+04	3.7666e-05	72.12	1.3586e-01	5.3967e-09
$7i - 6h$	1.4985e+03	7.3706e-03	808.28	1.5226e+00	1.1835e-05
$8i - 6h$	9.8142e+01	2.1647e-03	1,332.89	2.5108e+00	5.7318e-06
$9i - 6h$	2.1310e+01	9.6246e-04	1,692.56	3.1883e+00	3.2361e-06
$10i - 6h$	7.5944e+00	5.2437e-04	1,949.83	3.6729e+00	2.0311e-06
$11i - 6h$	3.5319e+00	3.2249e-04	2,140.18	4.0314e+00	1.3711e-06
$12i - 6h$	1.9341e+00	2.1491e-04	2,284.96	4.3042e+00	9.7553e-07
$13i - 6h$	1.1811e+00	1.5163e-04	2,397.63	4.5164e+00	7.2222e-07
$14i - 6h$	7.7912e-01	1.1164e-04	2,487.03	4.6848e+00	5.5155e-07
$15i - 6h$	5.4412e-01	8.4945e-05	2,559.15	4.8206e+00	4.3185e-07
$8i - 7h$	2.0761e+03	2.7920e-03	524.61	9.8820e-01	2.9097e-06
$9i - 7h$	2.0308e+02	1.3079e-03	884.28	1.6657e+00	2.2975e-06
$10i - 7h$	5.2134e+01	7.2237e-04	1,141.55	2.1503e+00	1.6381e-06
$11i - 7h$	2.0299e+01	4.4673e-04	1,331.90	2.5089e+00	1.1820e-06
$12i - 7h$	9.9541e+00	2.9855e-04	1,476.67	2.7816e+00	8.7578e-07
$13i - 7h$	5.6420e+00	2.1098e-04	1,589.34	2.9938e+00	6.6613e-07
$14i - 7h$	3.5286e+00	1.5549e-04	1,678.74	3.1622e+00	5.1855e-07
$15i - 7h$	2.3683e+00	1.1840e-04	1,750.87	3.2981e+00	4.1181e-07
$9i - 8h$	2.8261e+03	1.2248e-03	359.67	6.7751e-01	8.7510e-07
$10i - 8h$	3.2540e+02	7.1171e-04	616.94	1.1621e+00	8.7225e-07
$11i - 8h$	9.0984e+01	4.4587e-04	807.29	1.5207e+00	7.1504e-07
$12i - 8h$	3.7261e+01	2.9951e-04	952.07	1.7934e+00	5.6646e-07
$13i - 8h$	1.8874e+01	2.1220e-04	1,064.74	2.0056e+00	4.4883e-07
$14i - 8h$	1.0937e+01	1.5661e-04	1,154.14	2.1740e+00	3.5907e-07
$15i - 8h$	6.9494e+00	1.1936e-04	1,226.26	2.3099e+00	2.9076e-07
$10i - 9h$	3.7782e+03	5.9925e-04	257.27	4.8462e-01	3.0626e-07
$11i - 9h$	4.7207e+02	3.9436e-04	447.62	8.4318e-01	3.5067e-07
$12i - 9h$	1.3848e+02	2.6815e-04	592.40	1.1159e+00	3.1557e-07
$13i - 9h$	5.8466e+01	1.9088e-04	705.07	1.3281e+00	2.6735e-07
$14i - 9h$	3.0229e+01	1.4119e-04	794.47	1.4965e+00	2.2284e-07
$15i - 9h$	1.7773e+01	1.0774e-04	866.59	1.6324e+00	1.8547e-07

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$11i - 10h$	4.9651e+03	3.1897e-04	190.35	3.5856e-01	1.2061e-07
$12i - 10h$	6.4883e+02	2.2747e-04	335.13	6.3128e-01	1.5143e-07
$13i - 10h$	1.9581e+02	1.6377e-04	447.80	8.4351e-01	1.4568e-07
$14i - 10h$	8.4249e+01	1.2165e-04	537.20	1.0119e+00	1.2983e-07
$15i - 10h$	4.4140e+01	9.3010e-05	609.32	1.1478e+00	1.1258e-07
$12i - 11h$	6.4222e+03	1.8153e-04	144.78	2.7271e-01	5.2208e-08
$13i - 11h$	8.6103e+02	1.3685e-04	257.45	4.8495e-01	6.9989e-08
$14i - 11h$	2.6435e+02	1.0274e-04	346.85	6.5335e-01	7.0792e-08
$15i - 11h$	1.1510e+02	7.8848e-05	418.97	7.8921e-01	6.5625e-08
$13i - 12h$	8.1875e+03	1.0908e-04	112.67	2.1224e-01	2.4414e-08
$14i - 12h$	1.1140e+03	8.5619e-05	202.07	3.8064e-01	3.4369e-08
$15i - 12h$	3.4559e+02	6.6358e-05	274.20	5.1650e-01	3.6145e-08
$14i - 13h$	1.0302e+04	6.8562e-05	89.40	1.6840e-01	1.2176e-08
$15i - 13h$	1.4133e+03	5.5476e-05	161.52	3.0426e-01	1.7801e-08
$15i - 14h$	1.2808e+04	4.4758e-05	72.12	1.3586e-01	6.4128e-09
$8h - 7i$	7.4497e+00	1.1840e-05	524.61	9.8820e-01	1.2339e-08
$9h - 7i$	5.7384e-01	4.3678e-06	884.28	1.6657e+00	7.6727e-09
$10h - 7i$	1.2788e-01	2.0941e-06	1,141.55	2.1503e+00	4.7487e-09
$11h - 7i$	4.5363e-02	1.1798e-06	1,331.90	2.5089e+00	3.1217e-09
$12h - 7i$	2.0835e-02	7.3852e-07	1,476.67	2.7816e+00	2.1664e-09
$13h - 7i$	1.1255e-02	4.9738e-07	1,589.34	2.9938e+00	1.5704e-09
$14h - 7i$	6.7857e-03	3.5339e-07	1,678.74	3.1622e+00	1.1785e-09
$15h - 7i$	4.4260e-03	2.6150e-07	1,750.87	3.2981e+00	9.0955e-10
$9h - 8i$	3.0010e+01	1.5370e-05	359.67	6.7751e-01	1.0982e-08
$10h - 8i$	2.5614e+00	6.6208e-06	616.94	1.1621e+00	8.1143e-09
$11h - 8i$	6.0084e-01	3.4798e-06	807.29	1.5207e+00	5.5806e-09
$12h - 8i$	2.1920e-01	2.0824e-06	952.07	1.7934e+00	3.9384e-09
$13h - 8i$	1.0234e-01	1.3598e-06	1,064.74	2.0056e+00	2.8762e-09
$14h - 8i$	5.5836e-02	9.4492e-07	1,154.14	2.1740e+00	2.1664e-09
$15h - 8i$	3.3880e-02	6.8770e-07	1,226.26	2.3099e+00	1.6752e-09
$10h - 9i$	7.7809e+01	1.4585e-05	257.27	4.8462e-01	7.4539e-09
$11h - 9i$	7.0918e+00	7.0015e-06	447.62	8.4318e-01	6.2258e-09
$12h - 9i$	1.7214e+00	3.9394e-06	592.40	1.1159e+00	4.6360e-09
$13h - 9i$	6.4022e-01	2.4702e-06	705.07	1.3281e+00	3.4598e-09
$14h - 9i$	3.0232e-01	1.6688e-06	794.47	1.4965e+00	2.6338e-09
$15h - 9i$	1.6610e-01	1.1899e-06	866.59	1.6324e+00	2.0485e-09
$11h - 10i$	1.6347e+02	1.2411e-05	190.35	3.5856e-01	4.6931e-09
$12h - 10i$	1.5591e+01	6.4597e-06	335.13	6.3128e-01	4.3005e-09
$13h - 10i$	3.8772e+00	3.8324e-06	447.80	8.4351e-01	3.4092e-09

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$14h - 10i$	1.4620e+00	2.4950e-06	537.20	1.0119e+00	2.6626e-09
$15h - 10i$	6.9609e-01	1.7335e-06	609.32	1.1478e+00	2.0983e-09
$12h - 11i$	3.0213e+02	1.0093e-05	144.78	2.7271e-01	2.9027e-09
$13h - 11i$	2.9786e+01	5.5948e-06	257.45	4.8495e-01	2.8613e-09
$14h - 11i$	7.5408e+00	3.4637e-06	346.85	6.5335e-01	2.3866e-09
$15h - 11i$	2.8729e+00	2.3258e-06	418.97	7.8921e-01	1.9358e-09
$13h - 12i$	5.1145e+02	8.0527e-06	112.67	2.1224e-01	1.8024e-09
$14h - 12i$	5.1704e+01	4.6961e-06	202.07	3.8064e-01	1.8851e-09
$15h - 12i$	1.3269e+01	3.0112e-06	274.20	5.1650e-01	1.6402e-09
$14h - 13i$	8.1160e+02	6.3837e-06	89.40	1.6840e-01	1.1337e-09
$15h - 13i$	8.3674e+01	3.8816e-06	161.52	3.0426e-01	1.2455e-09
$15h - 14i$	1.2253e+03	5.0604e-06	72.12	1.3586e-01	7.2504e-10
$8j - 7i$	2.7256e+03	3.7061e-03	524.61	9.8820e-01	3.8623e-06
$9j - 7i$	1.5816e+02	1.0300e-03	884.28	1.6657e+00	1.8093e-06
$10j - 7i$	3.1453e+01	4.4066e-04	1,141.55	2.1503e+00	9.9929e-07
$11j - 7i$	1.0496e+01	2.3357e-04	1,331.90	2.5089e+00	6.1798e-07
$12j - 7i$	4.6426e+00	1.4079e-04	1,476.67	2.7816e+00	4.1300e-07
$13j - 7i$	2.4449e+00	9.2442e-05	1,589.34	2.9938e+00	2.9186e-07
$14j - 7i$	1.4476e+00	6.4500e-05	1,678.74	3.1622e+00	2.1510e-07
$15j - 7i$	9.3157e-01	4.7090e-05	1,750.87	3.2981e+00	1.6379e-07
$9j - 8i$	3.6077e+03	1.5809e-03	359.67	6.7751e-01	1.1295e-06
$10j - 8i$	3.2222e+02	7.1258e-04	616.94	1.1621e+00	8.7332e-07
$11j - 8i$	7.7270e+01	3.8287e-04	807.29	1.5207e+00	6.1401e-07
$12j - 8i$	2.8553e+01	2.3207e-04	952.07	1.7934e+00	4.3891e-07
$13j - 8i$	1.3441e+01	1.5280e-04	1,064.74	2.0056e+00	3.2320e-07
$14j - 8i$	7.3756e+00	1.0679e-04	1,154.14	2.1740e+00	2.4484e-07
$15j - 8i$	4.4940e+00	7.8043e-05	1,226.26	2.3099e+00	1.9011e-07
$10j - 9i$	4.7165e+03	7.5637e-04	257.27	4.8462e-01	3.8656e-07
$11j - 9i$	5.0636e+02	4.2770e-04	447.62	8.4318e-01	3.8032e-07
$12j - 9i$	1.3415e+02	2.6265e-04	592.40	1.1159e+00	3.0909e-07
$13j - 9i$	5.2663e+01	1.7384e-04	705.07	1.3281e+00	2.4349e-07
$14j - 9i$	2.5792e+01	1.2181e-04	794.47	1.4965e+00	1.9224e-07
$15j - 9i$	1.4545e+01	8.9148e-05	866.59	1.6324e+00	1.5347e-07
$11j - 10i$	6.0854e+03	3.9528e-04	190.35	3.5856e-01	1.4947e-07
$12j - 10i$	7.1976e+02	2.5514e-04	335.13	6.3128e-01	1.6985e-07
$13j - 10i$	2.0219e+02	1.7098e-04	447.80	8.4351e-01	1.5210e-07
$14j - 10i$	8.2454e+01	1.2038e-04	537.20	1.0119e+00	1.2847e-07
$15j - 10i$	4.1451e+01	8.8315e-05	609.32	1.1478e+00	1.0690e-07
$12j - 11i$	7.7513e+03	2.2153e-04	144.78	2.7271e-01	6.3712e-08

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$13j - 11i$	9.6968e+02	1.5583e-04	257.45	4.8495e-01	7.9696e-08
$14j - 11i$	2.8250e+02	1.1102e-04	346.85	6.5335e-01	7.6494e-08
$15j - 11i$	1.1811e+02	8.1808e-05	418.97	7.8921e-01	6.8089e-08
$13j - 12i$	9.7536e+03	1.3139e-04	112.67	2.1224e-01	2.9408e-08
$14j - 12i$	1.2626e+03	9.8117e-05	202.07	3.8064e-01	3.9386e-08
$15j - 12i$	3.7657e+02	7.3111e-05	274.20	5.1650e-01	3.9823e-08
$14j - 13i$	1.2134e+04	8.1658e-05	89.40	1.6840e-01	1.4502e-08
$15j - 13i$	1.6049e+03	6.3695e-05	161.52	3.0426e-01	2.0438e-08
$15j - 14i$	1.4939e+04	5.2785e-05	72.12	1.3586e-01	7.5628e-09
$9i - 8j$	9.5126e+00	4.8096e-06	359.67	6.7751e-01	3.4365e-09
$10i - 8j$	6.5813e-01	1.6794e-06	616.94	1.1621e+00	2.0582e-09
$11i - 8j$	1.3527e-01	7.7336e-07	807.29	1.5207e+00	1.2402e-09
$12i - 8j$	4.5084e-02	4.2279e-07	952.07	1.7934e+00	7.9964e-10
$13i - 8j$	1.9719e-02	2.5865e-07	1,064.74	2.0056e+00	5.4708e-10
$14i - 8j$	1.0244e-02	1.7114e-07	1,154.14	2.1740e+00	3.9237e-10
$15i - 8j$	5.9845e-03	1.1992e-07	1,226.26	2.3099e+00	2.9212e-10
$10i - 9j$	3.7070e+01	6.8595e-06	257.27	4.8462e-01	3.5057e-09
$11i - 9j$	2.8897e+00	2.8163e-06	447.62	8.4318e-01	2.5043e-09
$12i - 9j$	6.3284e-01	1.4297e-06	592.40	1.1159e+00	1.6825e-09
$13i - 9j$	2.1884e-01	8.3353e-07	705.07	1.3281e+00	1.1675e-09
$14i - 9j$	9.7914e-02	5.3355e-07	794.47	1.4965e+00	8.4207e-10
$15i - 9j$	5.1615e-02	3.6503e-07	866.59	1.6324e+00	6.2840e-10
$11i - 10j$	9.3667e+01	7.0202e-06	190.35	3.5856e-01	2.6546e-09
$12i - 10j$	7.8933e+00	3.2284e-06	335.13	6.3128e-01	2.1493e-09
$13i - 10j$	1.8051e+00	1.7614e-06	447.80	8.4351e-01	1.5669e-09
$14i - 10j$	6.4063e-01	1.0792e-06	537.20	1.0119e+00	1.1517e-09
$15i - 10j$	2.9135e-01	7.1624e-07	609.32	1.1478e+00	8.6696e-10
$12i - 11j$	1.9278e+02	6.3572e-06	144.78	2.7271e-01	1.8284e-09
$13i - 11j$	1.7161e+01	3.1821e-06	257.45	4.8495e-01	1.6274e-09
$14i - 11j$	4.0492e+00	1.8361e-06	346.85	6.5335e-01	1.2651e-09
$15i - 11j$	1.4647e+00	1.1706e-06	418.97	7.8921e-01	9.7429e-10
$13i - 12j$	3.5040e+02	5.4462e-06	112.67	2.1224e-01	1.2190e-09
$14i - 12j$	3.2484e+01	2.9126e-06	202.07	3.8064e-01	1.1692e-09
$15i - 12j$	7.8466e+00	1.7578e-06	274.20	5.1650e-01	9.5746e-10
$14i - 13j$	5.8502e+02	4.5425e-06	89.40	1.6840e-01	8.0674e-10
$15i - 13j$	5.5952e+01	2.5623e-06	161.52	3.0426e-01	8.2218e-10
$15i - 14j$	9.1766e+02	3.7413e-06	72.12	1.3586e-01	5.3604e-10
$9k - 8j$	4.5839e+03	2.0255e-03	359.67	6.7751e-01	1.4472e-06
$10k - 8j$	2.3868e+02	5.3228e-04	616.94	1.1621e+00	6.5234e-07

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$11k - 8j$	4.3741e+01	2.1856e-04	807.29	1.5207e+00	3.5050e-07
$12k - 8j$	1.3706e+01	1.1233e-04	952.07	1.7934e+00	2.1245e-07
$13k - 8j$	5.7692e+00	6.6134e-05	1,064.74	2.0056e+00	1.3988e-07
$14k - 8j$	2.9204e+00	4.2639e-05	1,154.14	2.1740e+00	9.7760e-08
$15k - 8j$	1.6748e+00	2.9329e-05	1,226.26	2.3099e+00	7.1445e-08
$10k - 9j$	5.8621e+03	9.4799e-04	257.27	4.8462e-01	4.8450e-07
$11k - 9j$	4.8148e+02	4.1010e-04	447.62	8.4318e-01	3.6467e-07
$12k - 9j$	1.0822e+02	2.1366e-04	592.40	1.1159e+00	2.5144e-07
$13k - 9j$	3.7999e+01	1.2649e-04	705.07	1.3281e+00	1.7717e-07
$14k - 9j$	1.7172e+01	8.1779e-05	794.47	1.4965e+00	1.2907e-07
$15k - 9j$	9.1158e+00	5.6341e-05	866.59	1.6324e+00	9.6992e-08
$11k - 10j$	7.4285e+03	4.8658e-04	190.35	3.5856e-01	1.8399e-07
$12k - 10j$	7.4650e+02	2.6684e-04	335.13	6.3128e-01	1.7765e-07
$13k - 10j$	1.8770e+02	1.6007e-04	447.80	8.4351e-01	1.4239e-07
$14k - 10j$	7.0661e+01	1.0403e-04	537.20	1.0119e+00	1.1102e-07
$15k - 10j$	3.3447e+01	7.1860e-05	609.32	1.1478e+00	8.6983e-08
$12k - 11j$	9.3208e+03	2.6862e-04	144.78	2.7271e-01	7.7257e-08
$13k - 11j$	1.0456e+03	1.6943e-04	257.45	4.8495e-01	8.6653e-08
$14k - 11j$	2.8142e+02	1.1152e-04	346.85	6.5335e-01	7.6842e-08
$15k - 11j$	1.1084e+02	7.7417e-05	418.97	7.8921e-01	6.4434e-08
$13k - 12j$	1.1579e+04	1.5729e-04	112.67	2.1224e-01	3.5206e-08
$14k - 12j$	1.3877e+03	1.0874e-04	202.07	3.8064e-01	4.3650e-08
$15k - 12j$	3.9025e+02	7.6403e-05	274.20	5.1650e-01	4.1617e-08
$14k - 13j$	1.4248e+04	9.6687e-05	89.40	1.6840e-01	1.7171e-08
$15k - 13j$	1.7807e+03	7.1268e-05	161.52	3.0426e-01	2.2868e-08
$15k - 14j$	1.7373e+04	6.1902e-05	72.12	1.3586e-01	8.8691e-09
$10j - 9k$	1.1825e+01	2.1673e-06	257.27	4.8462e-01	1.1077e-09
$11j - 9k$	7.4225e-01	7.1651e-07	447.62	8.4318e-01	6.3713e-10
$12j - 9k$	1.4146e-01	3.1653e-07	592.40	1.1159e+00	3.7250e-10
$13j - 9k$	4.4416e-02	1.6756e-07	705.07	1.3281e+00	2.3469e-10
$14j - 9k$	1.8517e-02	9.9942e-08	794.47	1.4965e+00	1.5773e-10
$15j - 9k$	9.2516e-03	6.4805e-08	866.59	1.6324e+00	1.1156e-10
$11j - 10k$	4.4882e+01	3.3318e-06	190.35	3.5856e-01	1.2599e-09
$12j - 10k$	3.2187e+00	1.3039e-06	335.13	6.3128e-01	8.6808e-10
$13j - 10k$	6.6067e-01	6.3852e-07	447.80	8.4351e-01	5.6801e-10
$14j - 10k$	2.1697e-01	3.6203e-07	537.20	1.0119e+00	3.8635e-10
$15j - 10k$	9.3096e-02	2.2668e-07	609.32	1.1478e+00	2.7439e-10
$12j - 11k$	1.1103e+02	3.6264e-06	144.78	2.7271e-01	1.0430e-09
$13j - 11k$	8.6991e+00	1.5977e-06	257.45	4.8495e-01	8.1709e-10

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$14j - 11k$	1.8799e+00	8.4431e-07	346.85	6.5335e-01	5.8175e-10
$15j - 11k$	6.3756e-01	5.0469e-07	418.97	7.8921e-01	4.2005e-10
$13j - 12k$	2.2460e+02	3.4578e-06	112.67	2.1224e-01	7.7393e-10
$14j - 12k$	1.8743e+01	1.6646e-06	202.07	3.8064e-01	6.6820e-10
$15j - 12k$	4.2058e+00	9.3321e-07	274.20	5.1650e-01	5.0832e-10
$14j - 13k$	4.0244e+02	3.0951e-06	89.40	1.6840e-01	5.4968e-10
$15j - 13k$	3.5207e+01	1.5969e-06	161.52	3.0426e-01	5.1242e-10
$15j - 14k$	6.6390e+02	2.6809e-06	72.12	1.3586e-01	3.8411e-10
$10l - 9k$	7.2596e+03	1.1817e-03	257.27	4.8462e-01	6.0394e-07
$11l - 9k$	3.4270e+02	2.9382e-04	447.62	8.4318e-01	2.6126e-07
$12l - 9k$	5.8200e+01	1.1566e-04	592.40	1.1159e+00	1.3612e-07
$13l - 9k$	1.7171e+01	5.7533e-05	705.07	1.3281e+00	8.0583e-08
$14l - 9k$	6.8864e+00	3.3011e-05	794.47	1.4965e+00	5.2100e-08
$15l - 9k$	3.3514e+00	2.0850e-05	866.59	1.6324e+00	3.5894e-08
$11l - 10k$	9.0372e+03	5.9584e-04	190.35	3.5856e-01	2.2531e-07
$12l - 10k$	6.8681e+02	2.4712e-04	335.13	6.3128e-01	1.6452e-07
$13l - 10k$	1.4516e+02	1.2460e-04	447.80	8.4351e-01	1.1084e-07
$14l - 10k$	4.8508e+01	7.1888e-05	537.20	1.0119e+00	7.6717e-08
$15l - 10k$	2.1054e+01	4.5531e-05	609.32	1.1478e+00	5.5113e-08
$12l - 11k$	1.1172e+04	3.2410e-04	144.78	2.7271e-01	9.3213e-08
$13l - 11k$	1.0547e+03	1.7204e-04	257.45	4.8495e-01	8.7987e-08
$14l - 11k$	2.5215e+02	1.0058e-04	346.85	6.5335e-01	6.9302e-08
$15l - 11k$	9.1090e+01	6.4041e-05	418.97	7.8921e-01	5.3302e-08
$13l - 12k$	1.3706e+04	1.8741e-04	112.67	2.1224e-01	4.1946e-08
$14l - 12k$	1.4611e+03	1.1524e-04	202.07	3.8064e-01	4.6261e-08
$15l - 12k$	3.7714e+02	7.4323e-05	274.20	5.1650e-01	4.0484e-08
$14l - 13k$	1.6684e+04	1.1396e-04	89.40	1.6840e-01	2.0239e-08
$15l - 13k$	1.9170e+03	7.7229e-05	161.52	3.0426e-01	2.4781e-08
$15l - 14k$	2.0152e+04	7.2275e-05	72.12	1.3586e-01	1.0355e-08
$11k - 10l$	1.4388e+01	1.0602e-06	190.35	3.5856e-01	4.0092e-10
$12k - 10l$	8.2624e-01	3.3226e-07	335.13	6.3128e-01	2.2120e-10
$13k - 10l$	1.4672e-01	1.4076e-07	447.80	8.4351e-01	1.2521e-10
$14k - 10l$	4.3513e-02	7.2072e-08	537.20	1.0119e+00	7.6913e-11
$15k - 10l$	1.7314e-02	4.1848e-08	609.32	1.1478e+00	5.0655e-11
$12k - 11l$	5.3443e+01	1.7327e-06	144.78	2.7271e-01	4.9834e-10
$13k - 11l$	3.5483e+00	6.4689e-07	257.45	4.8495e-01	3.3084e-10
$14k - 11l$	6.8509e-01	3.0543e-07	346.85	6.5335e-01	2.1045e-10
$15k - 11l$	2.1409e-01	1.6822e-07	418.97	7.8921e-01	1.4001e-10
$13k - 12l$	1.2989e+02	1.9850e-06	112.67	2.1224e-01	4.4428e-10

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$14k - 12l$	9.5085e+00	8.3823e-07	202.07	3.8064e-01	3.3649e-10
$15k - 12l$	1.9470e+00	4.2884e-07	274.20	5.1650e-01	2.3359e-10
$14k - 13l$	2.5893e+02	1.9767e-06	89.40	1.6840e-01	3.5107e-10
$15k - 13l$	2.0335e+01	9.1561e-07	161.52	3.0426e-01	2.9379e-10
$15k - 14l$	4.5826e+02	1.8369e-06	72.12	1.3586e-01	2.6319e-10
$11m - 10l$	1.0962e+04	7.2660e-04	190.35	3.5856e-01	2.7475e-07
$12m - 10l$	4.7321e+02	1.7116e-04	335.13	6.3128e-01	1.1395e-07
$13m - 10l$	7.4848e+01	6.4589e-05	447.80	8.4351e-01	5.7456e-08
$14m - 10l$	2.0850e+01	3.1063e-05	537.20	1.0119e+00	3.3149e-08
$15m - 10l$	7.9783e+00	1.7345e-05	609.32	1.1478e+00	2.0995e-08
$12m - 11l$	1.3355e+04	3.8947e-04	144.78	2.7271e-01	1.1201e-07
$13m - 11l$	9.4421e+02	1.5483e-04	257.45	4.8495e-01	7.9186e-08
$14m - 11l$	1.8823e+02	7.5482e-05	346.85	6.5335e-01	5.2009e-08
$15m - 11l$	5.9964e+01	4.2381e-05	418.97	7.8921e-01	3.5274e-08
$13m - 12l$	1.6182e+04	2.2243e-04	112.67	2.1224e-01	4.9785e-08
$14m - 12l$	1.4398e+03	1.1417e-04	202.07	3.8064e-01	4.5830e-08
$15m - 12l$	3.2790e+02	6.4960e-05	274.20	5.1650e-01	3.5384e-08
$14m - 13l$	1.9488e+04	1.3382e-04	89.40	1.6840e-01	2.3766e-08
$15m - 13l$	1.9780e+03	8.0106e-05	161.52	3.0426e-01	2.5704e-08
$15m - 14l$	2.3322e+04	8.4087e-05	72.12	1.3586e-01	1.2048e-08
$12l - 11m$	1.7201e+01	5.5443e-07	144.78	2.7271e-01	1.5946e-10
$13l - 11m$	9.1012e-01	1.6495e-07	257.45	4.8495e-01	8.4362e-11
$14l - 11m$	1.5124e-01	6.7033e-08	346.85	6.5335e-01	4.6188e-11
$15l - 11m$	4.2477e-02	3.3182e-08	418.97	7.8921e-01	2.7618e-11
$13l - 12m$	6.2755e+01	9.5339e-07	112.67	2.1224e-01	2.1339e-10
$14l - 12m$	3.8784e+00	3.3991e-07	202.07	3.8064e-01	1.3645e-10
$15l - 12m$	7.0668e-01	1.5474e-07	274.20	5.1650e-01	8.4286e-11
$14l - 13m$	1.5026e+02	1.1404e-06	89.40	1.6840e-01	2.0253e-10
$15l - 13m$	1.0321e+01	4.6198e-07	161.52	3.0426e-01	1.4824e-10
$15l - 14m$	2.9577e+02	1.1787e-06	72.12	1.3586e-01	1.6887e-10
$12n - 11m$	1.5927e+04	4.6648e-04	144.78	2.7271e-01	1.3416e-07
$13n - 11m$	6.3322e+02	1.0429e-04	257.45	4.8495e-01	5.3336e-08
$14n - 11m$	9.3698e+01	3.7737e-05	346.85	6.5335e-01	2.6002e-08
$15n - 11m$	2.4710e+01	1.7541e-05	418.97	7.8921e-01	1.4599e-08
$13n - 12m$	1.9062e+04	2.6315e-04	112.67	2.1224e-01	5.8900e-08
$14n - 12m$	1.2597e+03	1.0032e-04	202.07	3.8064e-01	4.0270e-08
$15n - 12m$	2.3755e+02	4.7267e-05	274.20	5.1650e-01	2.5746e-08
$14n - 13m$	2.2715e+04	1.5666e-04	89.40	1.6840e-01	2.7822e-08
$15n - 13m$	1.9108e+03	7.7722e-05	161.52	3.0426e-01	2.4939e-08

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Hydrogen Transitions – *Continued*

Transition	d_{if}^2	A_{if} $[10^8 s^{-1}]$	ΔE $[cm^{-1}]$	ω $[10^{14} s^{-1}]$	Power $[10^{-9} W]$
$15n - 14m$	2.6936e+04	9.7540e-05	72.12	1.3586e-01	1.3975e-08
$13m - 12n$	2.0264e+01	3.0639e-07	112.67	2.1224e-01	6.8577e-11
$14m - 12n$	9.9393e-01	8.6694e-08	202.07	3.8064e-01	3.4801e-11
$15m - 12n$	1.5517e-01	3.3816e-08	274.20	5.1650e-01	1.8420e-11
$14m - 13n$	7.2816e+01	5.5001e-07	89.40	1.6840e-01	9.7681e-11
$15m - 13n$	4.2089e+00	1.8750e-07	161.52	3.0426e-01	6.0165e-11
$15m - 14n$	1.7212e+02	6.8265e-07	72.12	1.3586e-01	9.7808e-11
$13o - 12n$	2.2410e+04	3.1050e-04	112.67	2.1224e-01	6.9497e-08
$14o - 12n$	8.2573e+02	6.5999e-05	202.07	3.8064e-01	2.6494e-08
$15o - 12n$	1.1476e+02	2.2917e-05	274.20	5.1650e-01	1.2483e-08
$14o - 13n$	2.6427e+04	1.8292e-04	89.40	1.6840e-01	3.2486e-08
$15o - 13n$	1.6391e+03	6.6915e-05	161.52	3.0426e-01	2.1471e-08
$15o - 14n$	3.1054e+04	1.1286e-04	72.12	1.3586e-01	1.6170e-08
$14n - 13o$	2.3576e+01	1.7738e-07	89.40	1.6840e-01	3.1502e-11
$15n - 13o$	1.0777e+00	4.7819e-08	161.52	3.0426e-01	1.5344e-11
$15n - 14o$	8.3628e+01	3.3036e-07	72.12	1.3586e-01	4.7333e-11
$14q - 13o$	3.0694e+04	2.1311e-04	89.40	1.6840e-01	3.7848e-08
$15q - 13o$	1.0537e+03	4.3149e-05	161.52	3.0426e-01	1.3846e-08
$15q - 14o$	3.5745e+04	1.3031e-04	72.12	1.3586e-01	1.8671e-08
$15o - 14q$	2.7139e+01	1.0685e-07	72.12	1.3586e-01	1.5309e-11
$15r - 14q$	4.1086e+04	1.5018e-04	72.12	1.3586e-01	2.1517e-08