Comment on "Trouble with the Lorentz Law of Force: Incompatibility with Special Relativity and Momentum Conservation" Reply

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Abstract and Figures

The Lorentz law of force is the fifth pillar of classical electrodynamics, the other four being Maxwell's macroscopic equations. The Lorentz law is the universal expression of the force exerted by electromagnetic fields on a volume containing a distribution of electrical charges and currents. If electric and magnetic dipoles also happen to be present in a material medium, they are traditionally treated by expressing the corresponding polarization and magnetization distributions in terms of bound-charge and bound-current densities, which are subsequently added to free-charge and free-current densities, respectively. In this way, Maxwell's macroscopic equations are reduced to his microscopic equations, and the Lorentz law is expected to provide a precise expression of the electromagnetic force density on material bodies at all points in space and time. This paper presents incontrovertible theoretical evidence of the incompatibility of the Lorentz law with the fundamental tenets of special relativity. We argue that the Lorentz law must be abandoned in favor of a more general expression of the electromagnetic force density, such as the one discovered by A. Einstein and J. Laub in 1908. Not only is the Einstein-Laub formula consistent with special relativity, it also solves the long-standing problem of "hidden momentum" in classical electrodynamics.

In the inertial x'y'z' frame, the point-...

Trouble with the Lorentz law of force: Incompatibility with special relativity and momentum conservation

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Abstract. The Lorentz law of force is the fifth pillar of classical electrodynamics, the other four being Maxwell's macroscopic equations. The Lorentz law is the universal expression of the force exerted by electromagnetic fields on a volume containing a distribution of electrical charges and currents. If electric and magnetic dipoles also happen to be present in a material medium, they are traditionally treated by expressing the corresponding polarization and magnetization distributions in terms of bound-charge and bound-current densities, which are subsequently added to free-charge and free-current densities, respectively. In this way, Maxwell's macroscopic equations are reduced to his microscopic equations, and the Lorentz law is expected to provide a precise expression of the electromagnetic force density on material bodies at all points in space and time. This paper presents incontrovertible theoretical evidence of the incompatibility of the Lorentz law with the fundamental tenets of special relativity. We argue that the Lorentz law must be abandoned in favor of a more general expression of the electromagnetic force density, such as the one discovered by A. Einstein and J. Laub in 1908. Not only is the Einstein-Laub formula consistent with special relativity, it also solves the long-standing problem of "hidden momentum" in classical electrodynamics.

1. Introduction. The classical theory of electrodynamics is based on Maxwell's macroscopic equations [1-3], which, in the MKSA system of units, may be written as follows:

$$\nabla \cdot \mathbf{D}(\mathbf{r},t) = \rho_{\text{free}}(\mathbf{r},t), \tag{1}$$

$$\nabla \times \mathbf{H}(\mathbf{r},t) = \mathbf{J}_{\text{free}}(\mathbf{r},t) + \partial \mathbf{D}(\mathbf{r},t)/\partial t, \tag{2}$$

$$\nabla \times \mathbf{E}(\mathbf{r},t) = -\partial \mathbf{B}(\mathbf{r},t)/\partial t, \tag{3}$$

$$\nabla \cdot \mathbf{B}(\mathbf{r},t) = 0.$$

Here $(\mathbf{r},t)$ represents position in space-time, ρ_{free} and $\mathbf{J}_{\text{free}}$ ar
free current, $\mathbf{E}$ is the electric field, $\mathbf{H}$ the magnetic field, $\mathbf{D}$ the d
induction. By definition, $\mathbf{D}(\mathbf{r},t) = \varepsilon_0 \mathbf{E}(\mathbf{r},t) + \mathbf{P}(\mathbf{r},t)$, where ε_0
and $\mathbf{P}$ the electric polarization. Similarly, $\mathbf{B}(\mathbf{r},t) = \mu_0 \mathbf{H}(\mathbf{r}$
permeability of free space and $\mathbf{M}$ the magnetization. The speed
these equations to be related to the electromagnetic (EM) proper

In general, ρ

ρ_{free} and $\mathbf{J}_{free}$ are arbitrary functions of space-time that satisfy the continuity equation $\nabla\cdot\mathbf{J}_{free}+\partial\rho_{free}/\partial t=0$. Moreover, the 4-vector $(\mathbf{J}_{free}, c\rho_{free})$ is readily Lorentz transformed between different inertial frames. The remaining sources of the EM field, **P** and **M**, are also arbitrary functions of space-time which collectively form a 2nd rank tensor that can be Lorentz transformed from one inertial frame to another. Two other 2nd rank tensors that obey the Lorentz transformation rules are the field tensors, one formed by **E** and **B**, the other by **D** and **H** [1,3,4].

One could define bound-charge and bound-current densities $\rho_{bound}(\mathbf{r},t)=-\nabla\cdot\mathbf{P}$ and $\mathbf{J}_{bound}(\mathbf{r},t)=(\partial\mathbf{P}/\partial t)+\mu_o^{-1}\nabla\times\mathbf{M}$, and use them to eliminate **D** and **H** from Eqs.(1-4). Maxwell's equations then reduce to the so-called microscopic equations, relating **E** and **B** fields to the total charge- and current-densities $\rho_{total}=\rho_{free}+\rho_{bound}$ and $\mathbf{J}_{total}=\mathbf{J}_{free}+\mathbf{J}_{bound}$. It is thus clear that, as far as Maxwell's equations are concerned, $\mathbf{P}(\mathbf{r},t)$ and $\mathbf{M}(\mathbf{r},t)$ can be treated as distributions of charge and current, and that the knowledge of $\rho_{total}(\mathbf{r},t)$ and $\mathbf{J}_{total}(\mathbf{r},t)$ is all that is needed to determine the corresponding EM fields throughout the entire space-time.

$\mathbf{P}(\mathbf{r},t)$ and $\mathbf{M}(\mathbf{r},t)$ cease to behave as mere distributions of charge and current, however, as soon as one takes a close look at their interactions with EM fields involving energy and momentum exchange [5]. Since we have discussed these issues at great length elsewhere [6-12], we confine our remarks here to a summary of the conclusions reached in earlier studies, namely,

i) If the magnetic dipoles of Maxwell's equations were ordinary current loops, the rate of flow of EM energy (per unit area per unit time), instead of being the commonly accepted Poynting vector $\mathbf{S}(\mathbf{r},t)=\mathbf{E}\times\mathbf{H}$ [2,3,5], would be given by $\mu_o^{-1}\mathbf{E}\times\mathbf{B}$ [1].

ii) If the force-density exerted by EM fields on material media obeyed the conventional Lorentz law, namely,

$$\mathbf{F}(\mathbf{r},t)=\rho_{total}\mathbf{E}+\mathbf{J}_{total}\times\mathbf{B}, \tag{5}$$

there would arise situations where the momentum of a closed system would not be conserved. This problem has been known since the 1960s, when W. Shockley [13,14] pointed out the existence of "hidden momentum" in certain electromagnetic systems [15,16].

iii) A generalized version of the Lorentz law, originally proposed in 1908 by A. Einstein and J. Laub [17,18] and independently rediscovered by several authors afterward [5,16,19-21], not only justifies the definition of the Poynting vector as $\mathbf{S}(\mathbf{r},t)=\mathbf{E}\times\mathbf{H}$, but also eliminates the problem of hidden momentum, thus bringing classical electrodynamics into compliance with momentum conservation laws [10,15,20]. The Einstein-Laub formula for force density is

$$\mathbf{F}(\mathbf{r},t)=\rho_{free}\mathbf{E}+\mathbf{J}_{free}\times\mu_o\mathbf{H}+(\mathbf{P}\cdot\nabla)\mathbf{E}+(\partial\mathbf{P}/\partial t)\times\mu_o\mathbf{H}+(\mathbf{M}\cdot\nabla)\mathbf{H}-(\partial\mathbf{M}/\partial t)\times\varepsilon_o\mathbf{E}. \tag{6}$$

To guarantee the conservation of angular momentum, Eq.(6) must be supplemented with the following expression for the torque-density exerted by EM fields

$$\mathbf{T}(\mathbf{r},t)=\mathbf{r}\times\mathbf{F}(\mathbf{r},t)+\mathbf{P}(\mathbf{r},t)\times\mathbf{E}(\mathbf{r},t)+\mathbf{M}(\mathbf{r},t)$$

Equations (6) and (7) guarantee momentum conservation u that the Abraham momentum-density, $\mathbf{p}(\mathbf{r},t)=\mathbf{S}(\mathbf{r},t)/c^2$, is assun the EM angular-momentum-density is accordingly assumed to b

iv) Conservation of energy is guaranteed by the Poynting th Eqs.(2) and (3), in conjunction with the postulate that the rate are given by the Poynting vector $\mathbf{S}(\mathbf{r},t)=\mathbf{E}\times\mathbf{H}$. The energy continuity equation may thus be written as

$$\nabla\cdot\mathbf{S}(\mathbf{r},t)+\frac{\partial}{\partial t}\left(\tfrac{1}{2}\varepsilon_o\mathbf{E}\cdot\mathbf{E}+\tfrac{1}{2}\mu_o\mathbf{H}\cdot\mathbf{H}\right)+\mathbf{E}\cdot\mathbf{J}_{free}+\mathbf{E}\cdot\frac{\partial\mathbf{P}}{\partial t}+\mathbf{H}\cdot\frac{\partial\mathbf{M}}{\partial t}=0. \qquad (8)$$

Note in the above equation that energy exchange between EM fields and electric dipoles is governed by the term $\mathbf{E}\cdot\partial\mathbf{P}/\partial t$, which is sensible considering the force term $(\mathbf{P}\cdot\nabla)\mathbf{E}$ of Eq.(6) and the torque term $\mathbf{P}\times\mathbf{E}$ of Eq.(7). Similarly, the exchange of energy with magnetic dipoles is governed by the term $\mathbf{H}\cdot\partial\mathbf{M}/\partial t$ of Eq.(8), which is compatible with the force term $(\mathbf{M}\cdot\nabla)\mathbf{H}$ and the torque term $\mathbf{M}\times\mathbf{H}$ if one imagines a magnetic dipole as a pair of north and south magnetic poles attached to the opposite ends of a short spring.

The goal of the present paper is to demonstrate that the conventional Lorentz law of Eq.(5) violates a basic requirement of special relativity. The elementary example presented in the next section establishes the inadequacy of the Lorentz law; several other examples could be cited in support of the argument, but a single instance of incongruity is all it takes to prove the point. In addition, we show that the Einstein-Laub force-density expression of Eq.(6), taken together with the torque-density expression of Eq.(7), does not suffer from the aforesaid shortcoming. The totality of evidence against the Lorentz law and in support of the Einstein-Laub formula thus compels us to abandon the former in favor of the latter. The ultimate test of any physical theory, of course, is whether or not its predictions agree with experimental observations. Thus, our argument in favor of the Einstein-Laub formulation should not be construed as an argument against any other law of force that has been proposed in the past or might be proposed in the future, provided that the proposed law is also universal, is consistent with Maxwell's equations and with conservation laws, and complies with special relativity.

2. Lorentz law and the principle of relativity. Consider a point-charge q at a fixed distance d from a magnetic point-dipole $m_o\hat{\mathbf{x}}'$, as shown in Fig.1. The magnetic dipole which, in the Lorentz picture, is essentially a small, charge-neutral loop of current, experiences neither a force nor a torque from the point-charge. The situation is very different, however, for a stationary observer in the xyz frame, who watches the point-charge and the magnetic dipole move together with constant velocity V along the z-axis. It is not difficult to Lorentz transform the current loop and also the electric field of the point-charge from x'y'z' to the xyz frame. The stationary observer will see an electric dipole $p_o\hat{\mathbf{y}}=V\varepsilon_o m_o\hat{\mathbf{y}}$, traveling along with the magnetic dipole $m_o\hat{\mathbf{x}}$, both experiencing the electric as well as the magnetic field produced by the moving point-charge q. The net Lorentz force on the pair of dipoles ($m_o\hat{\mathbf{x}}$ and $p_o\hat{\mathbf{y}}$) turns out to be zero, but a net torque $\mathbf{T}=(Vqm_o/4\pi d^2)\hat{\mathbf{x}}$ acts on the dipole pair. The appearance of this torque in the xyz frame in the absence of a corresponding torque in the x'y'z' frame is sufficient proof of the inadequacy of the Lorentz law. In contrast, the force- and torque-density expressions in Eqs.(6) and (7) yield zero force and zero torque on the dipole(s) in both reference frames.

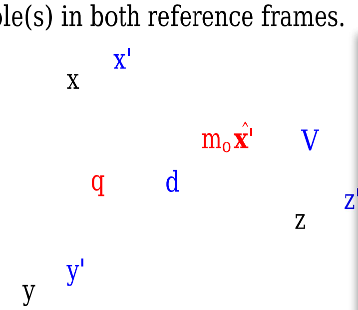

Fig. 1. In the inertial x'y'z' frame, the point-charge q and the point-dipole $m_0\mathbf{x}'$ are stationary. The x'y'z' system moves with constant velocity V along the z-axis, as seen by a stationary observer in the xyz frame. The origins of the two coordinate systems coincide at t = t' = 0.

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... In the 1960s, Shockley pointed out that the LO law contradicts the universal momentum conservation in certain systems involving magnetic media [8][9][10]. More recently, the LO law was found to be incompatible with the special relativity, as it predicts different results in different reference frames [11] . These problems of the LO law could be avoided by introducing an additional hidden momentum of electromagnetic field in magnetic media [8,11]. ...
... More recently, the LO law was found to be incompatible with the special relativity, as it predicts different results in different reference frames [11]. These problems of the LO law could be avoided by introducing an additional hidden momentum of electromagnetic field in magnetic media [8, 11] . However, some people have concerns because the hidden momentum is experimentally unobservable with current techniques. ...
... However, some people have concerns because the hidden momentum is experimentally unobservable with current techniques. In the meantime, another formulation originally proposed by Einstein and Laub (EL) has been widely used as an alternative of the electromagnetic force formulation [4, [11] [12][13][14][15][16][17][18][19][20][21], as it complies with both the special relativity and universal conservation laws without the need for a hidden momentum [11,22,23]. The EL formulation is also consistent with the Maxwell's equations, and it agrees with the existing measurement results of the total force or torque that support the LO formulation [19,24]. ...

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... When using the Lagrangian of Eq. (8), one should take care to amend the equation of motion by removing the hidden momentum contribution from Eq. (15). [25] [26][27][28][29][30][31][32] It would have been desirable, of course, if a Lagrangian existed that yielded an equation of motion based solely on the Einstein-Laub force. However, in the absence of such a Lagrangian, one must always remember to apply the aforementioned correction. ...
... This is in contradistinction to the Einstein-Laub formulation 18 of the EM force acting on electric and magnetic dipoles, which automatically discounts such hidden momentum contributions. [22][23][24] [25] [26][27][28][29][30][31] [32] Unfortunately, a corresponding Lagrangian for the Einstein-Laub formalism does not seem to exist and, as such, one must continue to be aware of the presence of the hidden momentum × 0 in theoretical studies that are based on the • + • interaction Lagrangian. ...

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... erefore, it is possible that the increase in springs' density compensates for the decrease in their constants, and vice versa, so that the upper and lower resultant forces would finally balance each other, which prohibits the plate from additional rotation. It is also possible that this inclination is somehow related to the disputatious arguments about Mansuripur's article where a similar nonuniformity in the distribution of some point-like electrical charges causes the moving observer to detect a possible torque on a current-carrying loop of wire, whereas the lab observer does not, due to the uniform distribution of the charges [13] [14][15][16]. A comprehensive discussion is beyond the scope of this article. ...

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March 2009 · Physical Review E

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