be the extent of the illuminating surface of the photosphere, the exterior parts of the corona will only receive an excess of light over that received by the interior part equal to the amount of photospheric light received by those parts during totality, or, as in the case above taken, the excess will be equal to that given by a ring of light from the photosphere $0''\cdot75$ wide (or GE in the figure), so that, when a few seconds of photosphere are visible to the observer, the difference between the spectra of the exterior and interior parts of the corona would be inappreciable.

5. What spectrum ought the corona to give before totality on the following side of the moon? In this case, when the angular distance of the limits of the sun and moon is some seconds, the difference between the spectra of the exterior and interior parts of the corona is small, since no part of the atmosphere in this case will be illuminated by the photosphere; so we ought to obtain a chromospheric spectrum, together with a faint photospheric one caused by a small amount of photospheric light reflected from the photosphere by the chromosphere.

6. On the foregoing hypothesis, during totality the parts of the corona nearest the centre should give a different spectrum from the more distant portions, since the portions nearer the centre receive less photospheric light than the more distant parts, and the same amount of light from the chromosphere.

In order to test the correctness of this theory, advantage may be taken of the following facts:—1st. At that period of the eclipse when the limb of the sun and moon are in line with the observer, there will be a difference between the central and distant parts of the corona; and this difference will decrease as the moon passes on, whereas, by the other theory, there should be the same difference as long as the corona is visible. 2nd. If the corona be terrestrial, the spectrum of any portion of it ought to be continually changing during the passage of the moon; but if solar, the spectrum should remain unchanged.

XVI. *On a Mechanical Theorem applicable to Heat.* *By* R. Clausius*.

In a treatise which appeared in 1862, on the mechanical theory of heat†, I advanced a theorem which, in its simplest form, may be thus expressed:—*The effective force of heat is*

* Translated from a separate impression communicated by the Author, having been read before the Niederrheinischen Gesellschaft für Natur- und Heilkunde, on June 13, 1870.

† Phil. Mag. S. 4. vol. xxiv. pp. 81, 201; The Mechanical Theory of Heat, p. 215.

proportional to the absolute temperature. From this theorem, in conjunction with that of the equivalence of heat and work, I have, in the subsequent portion of that treatise, deduced various conclusions concerning the deportment of bodies towards heat. As the theorem of the equivalence of heat and work may be reduced to a simple mechanical one, namely that of the equivalence of *vis viva* and mechanical work, I was convinced *à priori* that there must be a mechanical theorem which would explain that of the increase of the effective force of heat with the temperature. This theorem I think I shall be able to communicate in what follows.

Let there be any system whatever of material points in stationary motion. By stationary motion I mean one in which the points do not continually remove further and further from their original position, and the velocities do not alter continuously in the same direction, but the points move within a limited space, and the velocities only fluctuate within certain limits. Of this nature are all periodic motions—such as those of the planets about the sun, and the vibrations of elastic bodies,—further, such irregular motions as are attributed to the atoms and molecules of a body in order to explain its heat.

Now let m, m', m'', &c. be the given material points, $x, y, z, x', y', z', x'', y'', z''$, &c. their rectangular coordinates at the time t, and $X, Y, Z, X', Y', Z', X'', Y'', Z''$, &c. the components, taken in the directions of the coordinates, of the forces acting upon them. Then we form first the sum

$$\Sigma\frac{m}{2}\left[\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2\right],$$

for which, v, v', v'', &c. being the velocities of the points, we may write, more briefly,

$$\Sigma\frac{m}{2}v^2,$$

which sum is known under the name of the *vis viva* of the system. Further, we will form the following expression:—

$$-\tfrac{1}{2}\Sigma(Xx+Yy+Zz).$$

The magnitude represented by this expression depends, as is evident, essentially upon the forces acting in the system, and, if with given coordinates all the forces varied in equal ratio, would be proportional to the forces. We will therefore give to the mean value which this magnitude has during the stationary motion of the system the name of *Virial* of the system, from the Latin word *vis* (force).

In relation to these two magnitudes the following theorem may now be advanced:—

The mean vis viva *of the system is equal to its* virial.

Distinguishing the mean value of a magnitude from its variable value by drawing a horizontal line over the formula which represents the latter, we can express our theorem by the following equation:—

$$\Sigma \frac{m}{2} \overline{v^2} = -\tfrac{1}{2} \Sigma \overline{(Xx + Yy + Zz)}.$$

As regards the value of the virial, in the most important of the cases occurring in nature it takes a very simple form. For example, the forces which act upon the points of the mass may be attractions or repulsions which those points exert upon one another, and which are governed by some law of the distance. Let us denote, then, the reciprocal force between two points of the mass, m and m', at the distance r from each other, by $\phi(r)$, in which an attraction will reckon as a positive, and a repulsion as a negative force; we thus have, for the reciprocal action:—

$$Xx + X'x' = \phi(r) \frac{x' - x}{r} x + \phi(r) \frac{x - x'}{r} x' = -\phi(r) \frac{(x' - x)^2}{r}.$$

And since for the two other coordinates corresponding equations may be formed, there results

$$-\tfrac{1}{2}(Xx + Yy + Zz + X'x' + Y'y' + Z'z') = \tfrac{1}{2} r\phi(r).$$

Extending this result to the whole system of points, we obtain

$$-\tfrac{1}{2}\Sigma(Xx + Yy + Zz) = \tfrac{1}{2}\Sigma r\phi(r),$$

in which the sign of summation on the right-hand side of the equation relates to all combinations of the points of the mass in pairs. Thence comes for the virial the expression

$$\tfrac{1}{2}\overline{\Sigma r\phi(r)};$$

and we immediately recognize the analogy between this expression and that which serves to determine the work accomplished in the motion. Introducing the function $\Phi(r)$ with the signification

$$\Phi(r) = \int \phi(r) dr,$$

we obtain the familiar equation

$$-\Sigma(Xdx + Ydy + Zdz) = d\Sigma\Phi(r).$$

The sum $\Sigma\Phi(r)$ is that which, in the case of attractions and repulsions, which act inversely as the square of the distance, is named, irrespective of the sign, the reciprocal *potential* of the system of points. As it is advisable to have a convenient name* for the case in which the attractions and repulsions are governed by any law whatever, or, more generally still, for every case in which the work accomplished in an infinitely small motion of the system may be represented by the differential of any magnitude dependent only on the space-coordinates of the points, I propose to name the magnitude whose differential represents the negative value of the work, from the Greek word ἔργον (work), the *ergal* of the system. The theorem of the equivalence of *vis viva* and work can then be expressed very simply; and in order to exhibit distinctly the analogy between this theorem and that respecting the virial, I will place the two in juxtaposition:—

(1) The sum of the *vis viva* and the ergal is constant.

(2) The mean *vis viva* is equal to the virial.

In order to apply our theorem to heat, let us consider a body as a system of material points in motion. With respect to the forces which act upon these points we have a distinction to make: in the first place, the elements of the body exert upon one another attractive or repulsive forces; and, secondly, forces may act upon the body from without. Accordingly we can divide the virial into two parts, which refer respectively to the internal and the external forces, and which we will call the *internal* and the *external virial.*

Provided that the whole of the internal forces can be reduced to central forces, the internal virial is represented by the formula above given for a system of points acting by way of attraction or repulsion upon one another. It is further to be remarked that, with a body in which innumerable atoms move irregularly but in essentially like circumstances, so that all possible phases of motion occur simultaneously, it is not necessary to take the mean value of $r\phi(r)$ for each pair of atoms, but the values of $r\phi(r)$ may be taken for the precise position of the atoms at a certain moment, as the sum formed therefrom does not importantly differ from their total value throughout the course of the individual motions. Consequently we have for the internal virial the expression

$$\tfrac{1}{2}\Sigma r\phi(r).$$

As to the external forces, the case most frequently to be considered is where the body is acted upon by a uniform pressure normal to the surface. The virial relative to this can be expressed

* The term *force-function*, besides some inconvenience, has the disadvantage of having been already used for another magnitude, which stands to the one in question in a relation similar to that in which the potential-function stands to the potential.

very simply; for, p signifying the pressure, and v the volume of the body, it is represented by

$$\tfrac{3}{2}pv.$$

Denoting, further, by h the *vis viva* of the internal motions (which we call heat), we can form the following equation:—

$$h=\tfrac{1}{2}\Sigma r\phi(r)+\tfrac{3}{2}pv.$$

We have still to adduce the proof of our theorem of the relation between the *vis viva* and the virial, which can be done very easily.

The equations of the motion of a material point are:—

$$m\frac{d^2x}{dt^2}=X;\quad m\frac{d^2y}{dt^2}=Y;\quad m\frac{d^2z}{dt^2}=Z.$$

But we have

$$\frac{d^2(x^2)}{dt^2}=2\frac{d}{dt}\left(x\frac{dx}{dt}\right)=2\left(\frac{dx}{dt}\right)^2+2x\frac{d^2x}{dt^2},$$

or, differently arranged,

$$2\left(\frac{dx}{dt}\right)^2=-2x\frac{d^2x}{dt^2}+\frac{d^2(x^2)}{dt^2}.$$

Multiplying this equation by $\frac{m}{4}$, and putting the magnitude X for $m\frac{d^2x}{dt^2}$, we obtain

$$\frac{m}{2}\left(\frac{dx}{dt}\right)^2=-\tfrac{1}{2}Xx+\frac{m}{4}\cdot\frac{d^2(x^2)}{dt^2}.$$

The terms of this equation may now be integrated for the time from 0 to t, and the integral divided by t; we thereby obtain

$$\frac{m}{2t}\int_0^t\left(\frac{dx}{dt}\right)^2dt=-\frac{1}{2t}\int_0^t Xx\,dt+\frac{m}{4t}\left[\frac{d(x^2)}{dt}-\left(\frac{d(x^2)}{dt}\right)_0\right],$$

in which $\left(\frac{d(x^2)}{dt}\right)_0$ denotes the initial value of $\frac{d(x^2)}{dt}$.

The formulæ

$$\frac{1}{t}\int_0^t\left(\frac{dx}{dt}\right)^2dt \text{ and } \frac{1}{t}\int_0^t Xx\,dt,$$

occurring in the above equation, represent, if the duration of time t is properly chosen, the mean values of $\left(\frac{dx}{dt}\right)^2$ and Xx, which were denoted above by $\overline{\left(\frac{dx}{dt}\right)^2}$ and $\overline{Xx}$. For a periodic motion the duration of a period may be taken as the time t; but for irregular motions (and, if we please, also for periodic ones) we have only to consider that the time t, in proportion to the times during which the point moves in the same direction in respect of any one of the directions of coordinates is very great, so that in the course of the time t many changes of motion have taken place, and the above expressions of the mean values have become sufficiently constant.

The last term of the equation, which has its factor included in the square brackets, becomes, when the motion is periodic, $=0$ at the end of each period, as at the end of the period $\frac{d(x^2)}{dt}$ resumes the initial value $\left(\frac{d(x^2)}{dt}\right)_0$. When the motion is not periodic, but irregularly varying, the factor in brackets does not so regularly become $=0$; yet its value cannot continually increase with the time, but can only fluctuate within certain limits; and the divisor t, by which the term is affected, must accordingly cause the term to become vanishingly small with very great values of t. Hence, omitting it, we may write

$$\frac{m}{2}\overline{\left(\frac{dx}{dt}\right)^2}=-\tfrac{1}{2}\overline{Xx}.$$

As the same equation is valid also for the remaining coordinates, we have

$$\frac{m}{2}\overline{\left[\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2\right]}=-\tfrac{1}{2}\overline{(Xx+Yy+Zz)},$$

or, more briefly,

$$\frac{m}{2}\overline{v^2}=-\tfrac{1}{2}\overline{(Xx+Yy+Zz)},$$

and for a system of any number of points we have the perfectly corresponding one

$$\Sigma\frac{m}{2}\overline{v^2}=-\tfrac{1}{2}\Sigma\overline{(Xx+Yy+Zz)}.$$

Hence our theorem is demonstrated; and at the same time it is evident that it is not merely valid for the whole system of material points, and for the three directions of coordinates together, but also for each material point and for each direction separately.