Control of the Motor Shaft Angle
MAE 433 Spring 2012
Lab 5

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1 Overview

In this lab we will use feedback to control the *shaft angle* of the motor-encoder system. In other words, we will design a *servo control system.* Even though the physical system is the same, the transfer function is different than it was in the previous labs because we will measure the motor shaft angle rather than the motor speed. Since the motor angle is the integral of the angular speed, there is an additional factor of s in the denominator of the plant transfer function. The input to the motor is still, of course, voltage.

For the motor-position system we will see that with proportional control we can track a step input with zero steady-state error (at least in the absence of static friction). On the other hand, as we increase the bandwidth in an effort to speed up the transient response, the overshoot and settling times will increase too. To correct this situation we will implement proportional-derivative (PD) control. PD control allows us to adjust both bandwidth and phase margin, so that we can obtain both a fast transient and a limited overshoot. Finally, we will implement a proportional-integral-derivative (PID) controller to overcome the nonlinear static friction of the motor (due to the brushes).

2 Goals

Our hands-on goals for today are to:

- Learn to design in the frequency-domain using Bode and Nyquist plots.
- Understand how frequency-domain quantities such as bandwidth and phase margin relate to the closed-loop step response.
- Design a servo controller for setting the shaft angle of the motor using P, PD, and PID approaches.
- Compare measured responses to those observed in simulation.

System parameter identification

As in the previous lab, we will use a step input to identify the plant parameters. The transfer function of the motor-position system is given by

$$P(s) = \frac{\Theta(s)}{U(s)} = \frac{K}{s(\tau s + 1)}, \tag{1}$$

where Θ and U denote the motor shaft angle (radians) and the applied motor voltage (volts) respectively. Compare this plant transfer function to the one in the previous lab and observe the additional "s" in the denominator.

1. Understand why there is an extra s in the denominator.
   Discuss with an AI if unsure.

2. What will response be to a step input to the motor?

3. Run the open-loop system with a 6 volt step and save both the step and the response to a mat file.
   We are beginning to estimate the two parameters, K and τ.

4. Convert the angular positions to angular velocities within Matlab.
   We will do this by taking the derivative of the signal.

   (a) In Matlab, create a transfer function $D(s) = \dfrac{s}{s/200 + 1}$ with:
   ```
   D = tf([1 0],[1./200 1]);
   ```
   You should recognize this as a time-derivative and a low-pass filter.

   (b) Now use the experimental position data as input to this transfer function, simulate it to take the derivative with:
   ```
   speed = lsim(D, position, time);
   ```
   The output is the angular speed in radians/sec.

5. As in previous weeks, find K and τ from angular speed.

6. Compare your fit with the measured data and make sure that the fit is in good agreement.
   Discuss with an AI before proceeding.

3 Control of the motor shaft angle

Proportional-Integral-Derivative (PID) control

A block-diagram for tracking the shaft angle is shown in Fig. 1. Recall that a proportional-integral-derivative controller has transfer function

$$C(s) = k_p + \frac{k_i}{s} + k_d s. \tag{2}$$

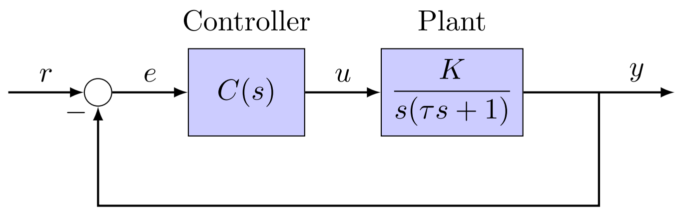

Figure 1: Block diagram for error based feedback control (reference tracking).

In the following, we consider *proportional*, *proportional-derivative* and *proportional-integral-derivative* control for the motor-position system. For all three types of control we would like to design a controller such that:

- Bandwidth of closed-loop system $\omega_b \geq 16$.
  Bandwidth is defined as the frequency range over which the gain has decreased by no more than a factor of $1/\sqrt{2} \approx -3$ *dB from its reference value (DC gain or 0 dB). For our purposes, you can approximate this as the crossover frequency (0 dB) if you wish.*
- Phase margin $\phi \geq 60°$.

You, the designer, need to find out if you can meet all specifications.

Proportional control

First we consider *proportional* control, i.e.

$$C(s) = k_p.$$

1. Write the closed-loop transfer function and enter it into Matlab.
2. Make a root-locus plot in Matlab to get an idea of what P-control does to the closed-loop system.
3. Find a gain $k_p = k_w$ that meets the given specification on bandwidth (see first item above) using a Bode plot.
4. Find a gain $k_p = k_{pm}$ that meets the given specification on phase margin (see second item above) using a Bode plot.
5. Can you find a gain that meets the specifications on both the bandwidth and phase margin?
   Try to answer this by looking at range of gains and how they effect the Bode plot.
6. Simulate the step response of the closed-loop system with each k_p. What differences do you observe? **Discuss with an AI before proceeding.**
7. Implement your controllers in experiment! Try a reference step input of 4 rad and verify that the flywheel rotates 2/3 of a revolution. Do the controllers operate as you expect, and why?
8. Push the flywheel, what physical system does this remind you of?
   Hint: There are two poles. **Discuss with an AI before proceeding.**

Proportional-Derivative control

Next we consider *proportional-derivative* control, i.e.

$$C(s) = k_d s + k_p.$$

1. Write the closed-loop transfer function.

2. Begin by factoring the proportional gain from $C(s)$. What does the controller form look like? *It is a product of two transfer functions.* **Discuss with an AI before proceeding.**.

3. Begin with the k_p value that met the bandwidth requirement from P control.

4. Try to place the zero in such a way that both specifications are met. Show your result in a Bode and Nyquist diagram. **Discuss with an AI before proceeding.**

5. Simulate the step response. How is the step response different from the P-controller?

6. Consider the Nyquist plots in Fig. 2. Which corresponds to the system with the best performance? Do the corresponding Bode plots tell you anything that can help you predict the performance?
   Consider the order of the poles and zeros and the performance at low frequencies. **Discuss with an AI before proceeding.**

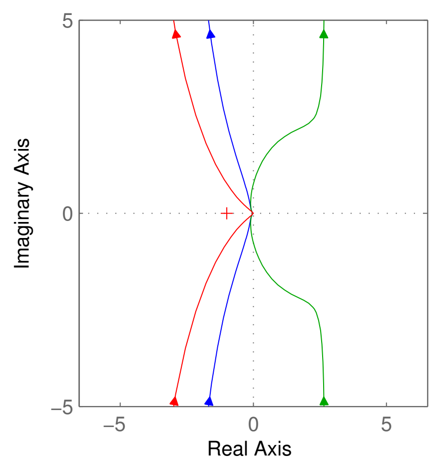

Figure 2: "Good" and "bad" Nyquist diagrams.

7. Design three controllers that generate Nyquist plots similar to those shown below.

8. Simulate the step responses for all three closed-loop systems. Were your predictions about performance correct?

9. How do bandwidth and phase margin relate to time-domain performance specifications such as rise time and overshoot of the closed-loop system?

10. Implement your controllers in experiment! To filter noise use your gains but

$$C(s) = k_p + k_d\left(\frac{s}{s/\omega_c + 1}\right),$$

where $\omega_c = 200$ is the cutoff frequency for our low-pass filter. Set the step input to 4 radians and verity the flywheel rotates about two thirds of a revolution. Do the controllers operate as you expect and why/why not?
Note that at low frequencies, this controller behaves the same as the original.

Proportional-Integral-Derivative control

In the *proportional-integral-derivative* controller we combine the favorable features of PD control, namely quick response and zero steady state error, and add integral control to overcome the stick-slip friction.

$$C(s) = k_p + \frac{k_i}{s} + k_d s.$$

1. What effect do you expect integral control to have on the steady-state error?

2. What happens for small k_i? Using the same k_d and k_i as before, look at the Bode plot of the new loop gain.

3. What happens for larger k_i? Is there a limit to how big k_i can be?
Try looking at Nyquist or root locus plots. **Discuss with an AI before proceeding.**

4. Implement at least one PID controller in experiment! Set the step input to 4 radians and verify that the flywheel rotates about two thirds of a revolution.

4 Questions you should be able to answer

You don't have to write these down, but if you don't know the answer then ask an AI!

- Why do we need a model of the motor-position system (plant)?
- What are the advantages of a P-controller? What are its disadvantages?
- Similarly, for a PD-controller?
- Which time-domain performance specifications are influenced by the bandwidth and phase margin of the system?
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5 Deliverables

1. Plots of experimental P control for the two gains.

2. Plots of experimental PD control for one pair of gains.

3. Plots of experimental PID control for one set of three gains .