Tracking of the Motor Speed with PI Control
MAE 433 Spring 2012
Lab 4

Prof. Rowley and Prof. Littman
AIs: Brandt Belson and Jonathan Tu
Technical staff: Jonathan Prévost
Princeton University

March 6–9, 2012

1 Overview

In this lab we will use feedback control for the motor-tachometer system with the goal of *improved tracking* and *transient response*. As observed in earlier labs, proportional control (P) offers the advantage of faster transient response and reducing steady state error. While higher gains yield better performance, they come at the expense of increased sensitivity to noise. Next, we will learn that integral control (I) offers the advantage of zero steady state error for a step input to this system. Unfortunately, however, as the gain of I is increased to improve rise time, overshoot will also increase. Finally we will see that combining P and I (that is, P plus I) is better than either P or I alone by simultaneously offering the advantages of quick transient response, zero steady state error and decreased sensitivity to noise.

2 Goals

Our hands-on goals for today are to:

- Learn how to work as a designer. Use a *model* of the actual motor-tachometer system to design a suitable controller which is implemented in an *experiment.*
- Design a feedback controller for reference tracking using P, I, and PI approaches
- Compare *measured* responses to those calculated for a *model* system
- Use graphical analysis and Matlab to quickly compare gains and find choices that meet desired time specifications.

There is a lab report that you will hand in before leaving the lab. Check the deliverables in § 6 for the contents of the report.

3 Parameter identification

We will identify the parameters of the motor-tachometer system again, with the same method we used for Lab 3, that is with a step response. Recall from lab 3 that the transfer function of the plant is given by

$$P(s) = \frac{K}{\tau s + 1}. \tag{1}$$

Also, recall that the response to a step (of magnitude 1) for this first-order system is the function

$$y(t) = K(1 - e^{-t/\tau}). \tag{2}$$

For more detailed instructions, refer to Lab 3. The general procedure is below.

- Find the constant K by considering the DC gain (steady-state value) of the plant.
- Find the time constant τ by considering the growth during the initial transient response.

4 Tracking a reference signal with P, I, and PI control

Goals for control design

We aim to track the input reference signal r by using feedback control. This is a different design goal then last week, where we rejected input disturbances. The block-diagram we use for tracking a reference is shown in Fig. 1, notice the similarities and differences from last week's.

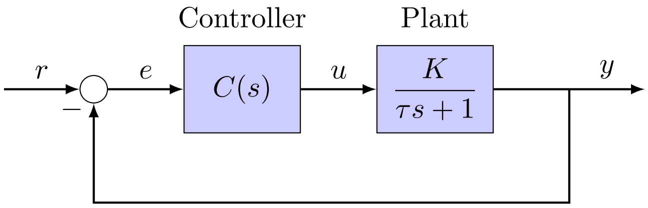

Figure 1: Block diagram for error based feedback control (reference tracking).

The proportional-integral controller has the transfer function

$$C(s) = k_p + \frac{k_i}{s}. \tag{3}$$

In the following, we consider three cases of control for the motor-tachometer system: *proportional*, *integral* and *proportional-integral*. For all three types of control we want to design a controller that satisfies the following specifications.

- rise time less than 0.2 s
- steady state error less than 1%
- overshoot less than 20%
- low sensitivity to noise.

Proportional control

First we consider simple *proportional* control, i.e. $C(s) = k_p$ and $k_i = 0$.

1. Find what gains k_p are necessary for the steady state error to be less than 30, 10 and 1%.

2. Are the above gains realistic for this experiment? Why or why not?

3. Use Matlab to plot the root locus for the system. What trends do you see, and do these pole locations make sense? **Before proceeding, check in with an AI to discuss your results.**
   By clicking on the grid-point button and then on the plot, identify the poles that correspond to the gains you used above.

4. Try to design a P-controller with rise time $t_r \leq 0.2$ s (where $t_r = 2.2\,\tau$ for a first-order system), less than 1% steady state error, and overshoot less than 20%. Can you meet all design specifications? Why or why not?

5. Try your gains in experiments! Save the results for your lab report. **If your motor makes weird sounds, turn it off!** Do the controllers operate as you expect? Why or why not?

Integral control

Next we consider *integral* control, i.e. $C(s) = \frac{k_i}{s}$.

1. Try to design an I-controller with rise time $t_r \leq 0.2$ s, less than 1% steady state error, and overshoot less than 20%.
   Do this by first drawing the region poles can be located using the standard 2nd-order guidelines from lecture. Then create a root locus plot in Matlab (manually set the frequencies, see the rlocus command help), and see how the two compare. If unsure, ask an AI.

2. Can you meet all design specifications? Why or why not? Make a sketch of the pole locations in the complex plane to support your answer. **Discuss with an AI before proceeding**

3. Verify your findings in experiments. Save the results for your lab report.

Proportional-Integral control

In the *proportional-integral* controller we can combine the favorable features of both P and I control, namely quick response, zero steady state error, and low sensitivity to noise. Unfortunately, since there are two independent gains, we cannot simply look at a single root locus plot.

1. Of time specifications rise time, overshoot, and settling time, which depend on only k_p, only k_i, and both k_i and k_p?
   Compare the closed-loop transfer function to the general 2nd order transfer function from lecture.

2. How does the rise time compare to the pure integral control case?

3. Try to design a PI-controller with rise time $t_r \leq 0.2$ s, less than 1% steady state error, and overshoot less than 20%.
   First find the required k_i, as you have been working on in previous steps. Then try to find a k_p that satisfies the remaining time specifications. If confused or algebra gets messy, ask an AI.

4. Can you meet all design specifications, and are these realistic gains for this experiment? Why or why not?

5. Verify your findings in experiments! Do the controllers operate as you expect? Why or why not? Save the results for your lab report. **Discuss with an AI.**

5 Take home messages

- Why do we need a model of the motor-tachometer system (plant)?
- What are the advantages and disadvantages of a P-controller, I-controller, PI-controller?
- Why are the choices for the gains limited in practice?

6 Deliverables

- Plots of P, I, and PI control, including the values of the gains.