Solenoids and Magnetic Fields

This lecture is based on HRW, Section 30.4.

- A solenoid is a long coil of wire wrapped in many turns. When a current passes through it, it creates a nearly uniform magnetic field inside.
- Solenoids can convert electric current to mechanical action, and so are very commonly used as switches.
- The magnetic field within a solenoid depends upon the current and density of turns.
- In order to estimate roughly the force with which a solenoid pulls on ferromagnetic rods placed near it, one can use the change in magnetic field energy as the rod is inserted into the solenoid. The force is roughly

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                      change in magnetic field energy
force on rod   =   --------------------------------
                    distance rod moves into solenoid
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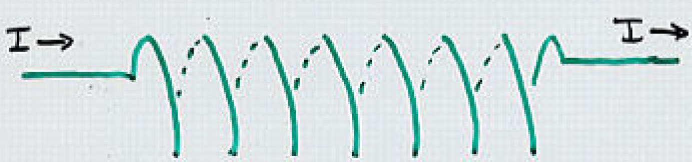
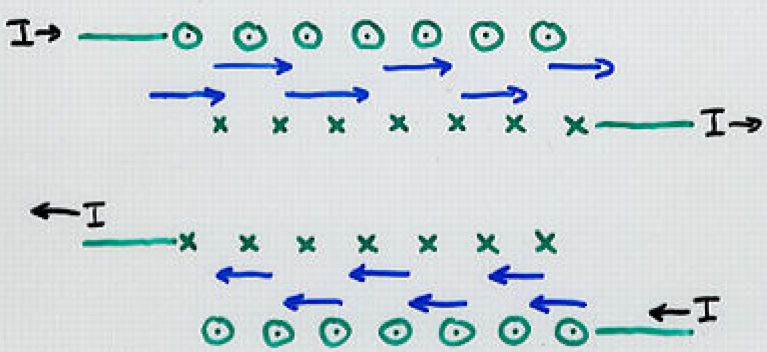
- The energy density of the magnetic field depends on the strength of the field, squared, and also upon the magnetic permeability of the material it fills. Iron has a much, much larger permeability than a vacuum.
- Even small solenoids can exert forces of a few newtons.

The Solenoid

One common and important magnetic tool is the **solenoid**: a coil of wire wrapped tightly around a cylinder.



Within the coils, a strong magnetic field arises whenever current is run through the wire. The direction of the magnetic field depends on the direction of the current



Viewgraph 1

Outside the coils, the magnetic field is small.

What's so great about the solenoid? Two things:

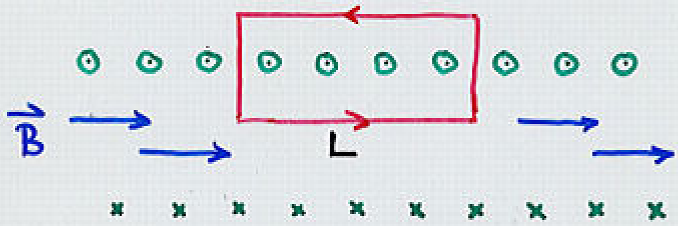
- it provides a simple means to generate a strong magnetic field
- it may be used to convert electric currents into mechanical motion and force; for example, in a switch

A solenoid acts in some ways like a permanent magnet – but one which can be reversed, and turned on and off, at will.

Viewgraph 2

The Magnetic Field of a Solenoid

If we look deep within a solenoid, far from the ends, we can use Ampere's Law to calculate the field strength:



The magnetic field (deep) within the solenoid has a uniform value B, and outside the coils, has value <u>zero</u>. If there are n coils per meter, then

$$I_{enclosed} = I(nL)$$

and the integral

$$\oint \vec{B} \cdot d\vec{s} = BL + 0 + 0 + 0$$
$$= BL$$

Viewgraph 3

Ampere's Law allows us to calculate the strength of the magnetic field.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{encl.}$$

$$BL = \mu_0 I n L$$

$$B = \mu_0 n I$$

where

I is current through wire
n is number of coils per meter

Ex: in order to generate a magnetic field as strong as the earth's, $B = 5 \times 10^{-5}\ T$, with a current of only $I = 1\ A$, we need a solenoid with

$$n = \frac{B}{\mu_0 I} = \frac{5 \times 10^{-5}\ T}{\left(4\pi \times 10^{-7} \frac{N}{A^2}\right)(1A)}$$

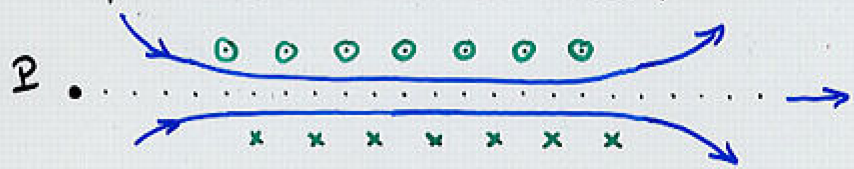
$$\cong 40 \text{ turns per meter}$$

$$\approx 1 \text{ turn per inch}$$

Viewgraph 4

Magnetic Field along axis of Solenoid

Suppose we need to know the magnetic field near the edge of a solenoid, or outside the coils. The magnetic field strength is no longer uniform, so we can't use Ampere's Law. What can we do?

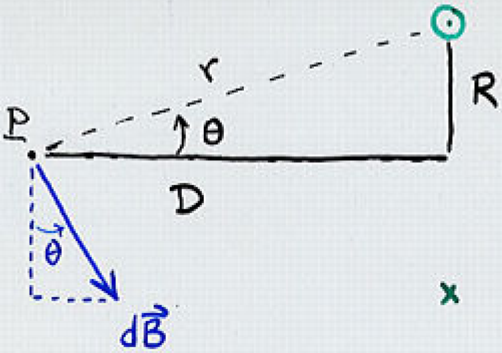


For locations along the axis of the coils, we can

- consider $d\vec{B}$ due to one ring of current
- integrate over all coils to find $\vec{B}$

In essence, we apply the Principle of Superposition.

Viewgraph 5



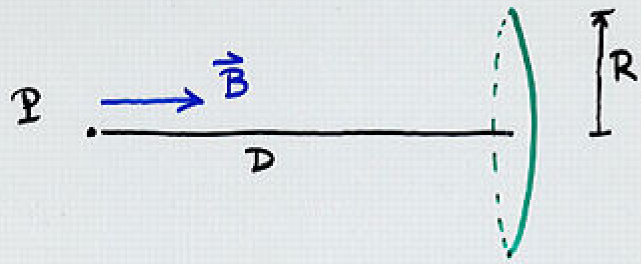
A coil of wire, radius R, carries a current I. It creates a magnetic field at point P. The magnetic field due to a tiny segment of the coil, up at top of loop, is

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$

which points <u>down</u> and to the right. A field of the same strength will be created by segment at bottom of loop — but it will point <u>up</u> and to the right. The vertical components cancel out, leaving

$$dB_{horiz} = \frac{\mu_0 I}{4\pi} \frac{ds}{D^2 + R^2} \cdot \frac{R}{\sqrt{D^2 + R^2}}$$

Viewgraph 6



If we integrate over every tiny segment in the wire, we find the total magnetic field

$$\vec{B} = \int dB_{horiz}$$

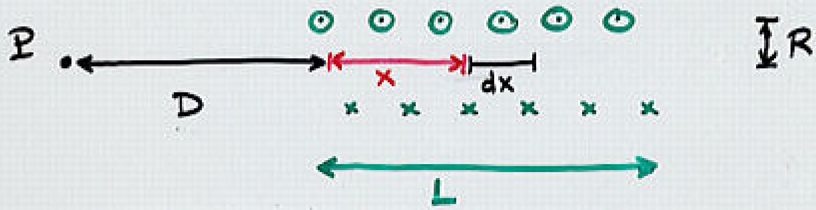
$$= \frac{\mu_0 I}{4\pi} \frac{R}{(D^2+R^2)^{3/2}} \int ds$$

$$= \frac{\mu_0 I}{4\pi} \frac{R}{(D^2+R^2)^{3/2}} \cdot 2\pi R$$

$$= \frac{\mu_0 I}{2} \frac{R^2}{(D^2+R^2)^{3/2}}$$ <u>to right</u>

Viewgraph 7

That gives us the magnetic field along the axis of a single loop. To calculate $\vec{B}$ along axis of a solenoid, we must integrate over all the loops:



Consider a section of the solenoid of length dx. The total current winding around the solenoid in that section is

$$I_{tot} = I(n \cdot dx)$$

This section is located a distance

$$\text{distance} = D + x$$

away from point P. So it contributes

$$B = \frac{\mu_0 I n\, dx}{2} \frac{R^2}{\left([D+x]^2 + R^2\right)^{3/2}}$$

Viewgraph 8

And so the total magnetic field at a point P which is D away from the edge of the solenoid is

$$B_{tot} = \int_{\text{near edge}}^{\text{far edge}} B$$

$$= \int_{x=0}^{x=L} \frac{\mu_0 I n \, dx}{2} \frac{R^2}{\left([D+x]^2 + R^2 \right)^{3/2}}$$

$$= \frac{\mu_0 I n R^2}{2} \int_{x=0}^{x=L} \frac{dx}{\left([D+x]^2 + R^2 \right)^{3/2}}$$

It simplifies a bit to substitute

$$z = D + x$$

$$B_{tot} = \frac{\mu_0 I n R^2}{2} \int_{z=D}^{z=D+L} \frac{dz}{\left(z^2 + R^2 \right)^{3/2}}$$

Viewgraph 9

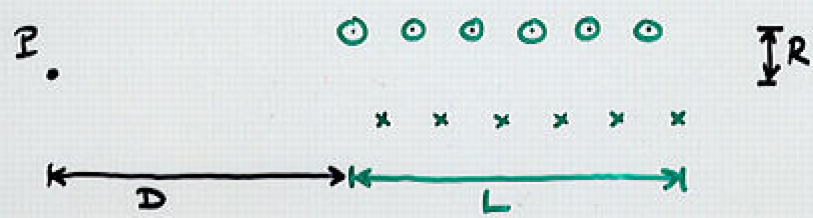
The integral can be done,

$$\int \frac{dz}{(z^2+R^2)^{3/2}} = \frac{z}{R^2(z^2+R^2)^{1/2}}$$

so

$$B_{tot} = \frac{\mu_0 I n R^2}{2} \cdot \frac{1}{R^2}\left(\frac{z}{(R^2+z^2)^{1/2}} \right|_{D}^{D+L}$$

$$= \frac{\mu_0 I n}{2}\left[\frac{D+L}{\sqrt{(D+L)^2+R^2}} - \frac{D}{\sqrt{D^2+R^2}} \right]$$



At $D=0$ (edge of solenoid)

$$B = \frac{\mu_0 I n}{2}\left[\frac{L}{\sqrt{L^2+R^2}} \right]$$

$$\approx \frac{\mu_0 I n}{2} \qquad \text{if } L \gg R$$

Viewgraph 10

How hard does a solenoid pull?

A solenoid can create a magnetic field — but how much force can it exert on a switch?

A simple but approximate answer is given by energy conservation. When a solenoid is empty, the energy of the magnetic field within it is

$$U(\text{empty}) = (\text{volume})(\text{mag. energy density})$$
$$= (\text{volume})\,\frac{B^2}{2\mu_0}$$

But if one fills the space within with some substance, one must use the magnetic permeability of that substance to calculate the energy of the magnetic field

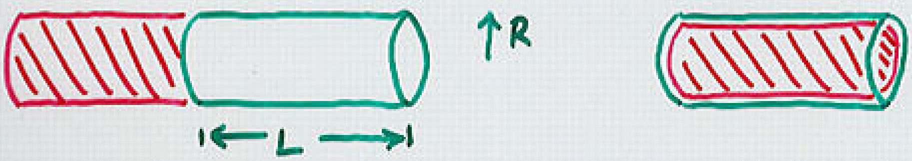
$$U(\text{filled}) = (\text{volume})\,\frac{B^2}{2\mu_0}\left(\frac{\mu_m}{\mu_0}\right)$$

Viewgraph 11

Iron (and cobalt and nickel) have very large magnetic permeabilities; for some types of iron,

$$\mu_m \gtrsim 10^4 \mu_0$$
$$= 4\pi \times 10^{-3}\ T/A^2$$

The force on an iron rod as it is pulled into a solenoid can be approximated as



$$U_{empty} = (L\pi R^2)\frac{B^2}{2\mu_0} \qquad U_f = (L\pi R^2)\frac{B^2}{2\mu_0}\left(\frac{\mu_m}{\mu_0}\right)$$

$$\Delta U = \frac{B^2 L\pi R^2}{2\mu_0}\left(\frac{\mu_m}{\mu_0} - 1\right)$$

and force is change in energy divided by distance moved by the rod.

Viewgraph 12

$$F = \frac{\text{change in energy}}{\text{distance}}$$

$$= \frac{\Delta U}{L} = \frac{\pi R^2 B^2}{2\mu_0}\left(\frac{\mu_m}{\mu_0} - 1\right)$$

For a typical household solenoid,

$$R = 5\,\text{mm}$$
$$B = 10^{-3}\ T$$
$$\mu_m = 4\pi \times 10^{-3}\ T/A^2$$

so the force pulling the iron rod when current is turned on may be

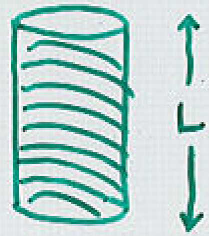
$$F \sim 0.3\ N$$

Viewgraph 13

Challenge Question

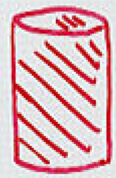
You are given a solenoid with

$L = 20$ cm
$R = 2$ cm
$N = 200$ turns



and an iron bar which just fits inside the solenoid, with

$\mu_m = 2 \times 10^{-2}$ T/A^2



How much current must you run through the solenoid to keep the iron bar from falling out the bottom?

A. 1 A
B. 10 A
C. 100 A
D. 1,000 A

Viewgraph 14