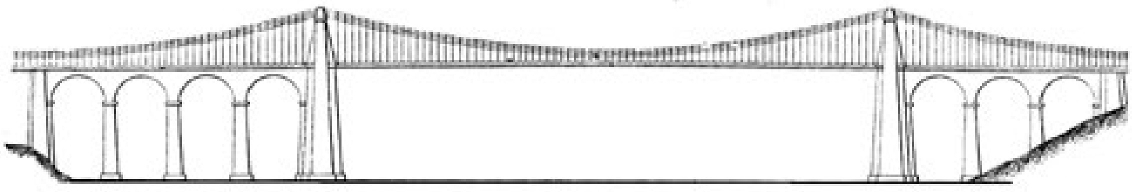
Lab 2 : Bridges – Suspension and Horizontal Cantilever

The lab will involve the study of two bridges. The Menai Straits Bridge illustrates the principles of a suspension bridge, while the Quebec Bridge is a classic horizontal cantilever bridge.

Lab 2a: Menai Straits Bridge

Transformation: Vertical Load into Horizontal Force

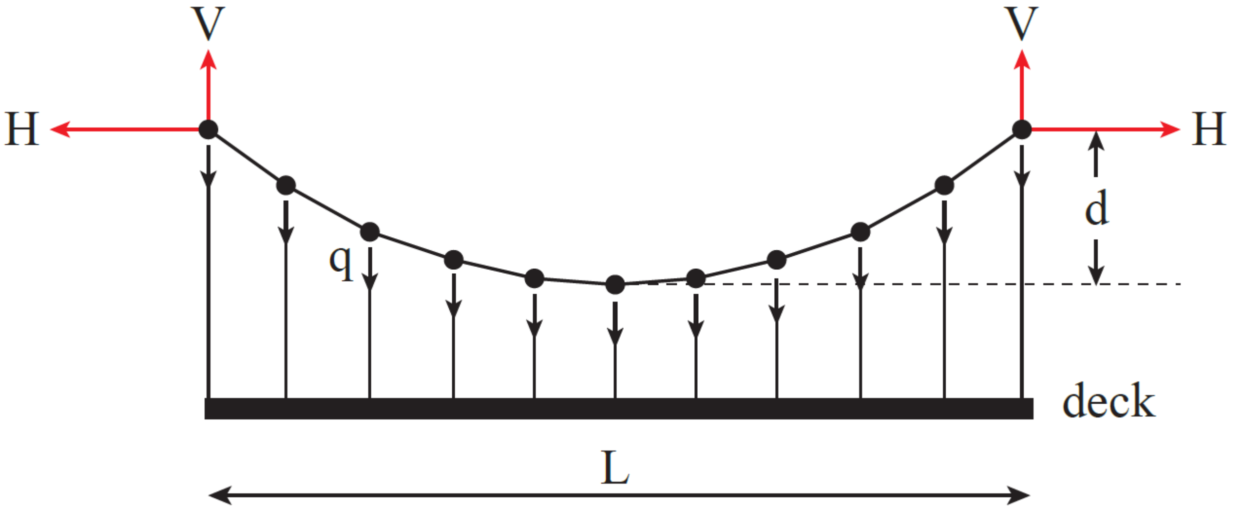
The purpose of this laboratory is to understand the forces in a cable suspension bridge. The laboratory is based on Thomas Telford's Menai Straits Bridge built in 1826. At the time it was erected, the Menai Straits Bridge was the longest spanning bridge in the world with a 579-foot middle span.



Telford's Menai Straits Bridge of 1826

It is the most important of Telford's flat form bridges and it served as a precursor to a large number of long-spanning bridges including Othmar Ammann's George Washington Bridge of 1931.

In a suspension bridge, deck loads are transferred by vertical suspenders through a sagging cable to the towers. Horizontal (H) and vertical (V) reaction forces occur at the towers as seen below.



FORCES AND DIMENSIONS IN A CABLE BRIDGE

The horizontal and vertical reaction forces are governed by the following equations.

$$H = \left(\frac{1}{8}\right)(qL)\left(\frac{L}{d}\right) = \frac{qL^2}{8d}$$

$$V = \left(\frac{1}{2}\right)(qL)$$

where q is the deck load per unit length, L is the span, and d is the cable "sag" (see Figure above). The factor (qL) is the total weight of the deck, and the factor (L/d) is a non-dimensional ratio that expresses the major visual feature of the bridge, its flatness. We see from the second equation above that the vertical reaction at each tower is half of the total deck weight and that this quantity is independent of the sag. This is a reasonable given that structures are in static equilibrium. Static equilibrium means that all forces are balanced. The downward gravitational force on the deck therefore is exactly balanced by the sum of upward tower reaction forces, V. We see from the first equation above that since H depends upon L/d and qL, that for a constant L, the smaller the sag, the larger the H. Examining the equation for H, we see how a cable transforms a vertical load (q) into a horizontal reaction.

The larger the H, the larger the cable area, A, since:

$$A = \frac{T}{f} \approx \frac{H}{f}$$

Here A is the cable cross-sectional area, T is the cable tension (which is related to H), and f is the allowable stress. For small d, T may be replaced by H without much error since V is so much smaller than H[1]. Considerations such as cable thickness and cable length affect the cost of the bridge, as well as the amount of materials needed to construct it. The heavier cable is more expensive and uses more material than a light cable, however, the overall cable length and the length of the deck hangers is less with a "flat form" bridge (i.e. a bridge with small d). A flat form also means that the towers can be relatively 'short' and therefore they are less expensive and easier to build. Design choices by Telford, a structural artist, concerning the Menai Straits Bridge were made based on the combined factors of economy (cost), efficiency (materials), and elegance.

In the laboratory you will make measurements on a plastic scale-model of the Menai Straits Bridge. You will simulate the deck load with lead weights and measure vertical and horizontal reaction forces with force sensors, and compare your measurements with those calculated from the formulas above. You will also calculate the minimum cross-sectional area of wrought iron cables ($f \approx 30{,}000$ psi) needed to support the deck load in the small-scale Menai Bridge. Finally you will estimate the safety factor of a model railroad bridge based on the ideas explored in this laboratory experiment.

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[1] The *real* force in the cable is the square root of (H^2+V^2). At midspan V = 0 so the force equals H. At the towers, V is the greatest so the force in the cables will be the greatest; but for the purposes of this class, we can assume the cable force equals H throughout.

Important Equations

Variables:

L = span (in)
d = sag (in)
Q = Total load (lbs)
q = distributed load (lbs/in) = Q/L
    q_L = applied live distributed load
    q_D = distributed dead load of structure
A_s = cross sectional area of a cable (in^2)
f = stress in cable at midspan (lbs/in^2)
S.F. = safety factor

Equations

For a load, q, distributed uniformly along the <u>entire</u> span:

$$H = \frac{qL^2}{8d} = \text{horizontal reaction (lbs), constant along bridge}$$

$$V = \frac{qL}{2} = \text{vertical reaction (lbs)}$$

$$Q = qL$$

For a load q, distributed uniformly along one-half of the span with location 1 adjacent to the load and location 2 at the opposite end:

$Q = qL/2$
$H_1 = H_2 = qL^2 / 16d$
$V_1 = 3qL / 8$
$V_2 = qL / 8$
$A_s = (\pi * (diameter)^2) / 4$
$f = H / A_s$
S.F. = *(breaking stress)* / (*applied stress)*

Useful Conversions

16 oz = 1 lb

Notes

1. For most parts of the course, we use historical units, e.g. lbs and in
2. Use consistent units when adding and multiplying.
3. Percent error = *100% * (experimental - theoretical) /(theoretical)*

Apparatus

The experiment uses a model of the Menai Straits Bridge made out of plastic K'nex pieces, as seen in Figure 1 (with the hanging weights, as you will use in the experiment). The main cables between the two towers are the focus of the lab. The vertical towers rest on scales, and a chain with a turnbuckle connected to a horizontal force sensor supports the cables at each end. The scales and the force sensors read the vertical and horizontal components respectively, of the cable tension.

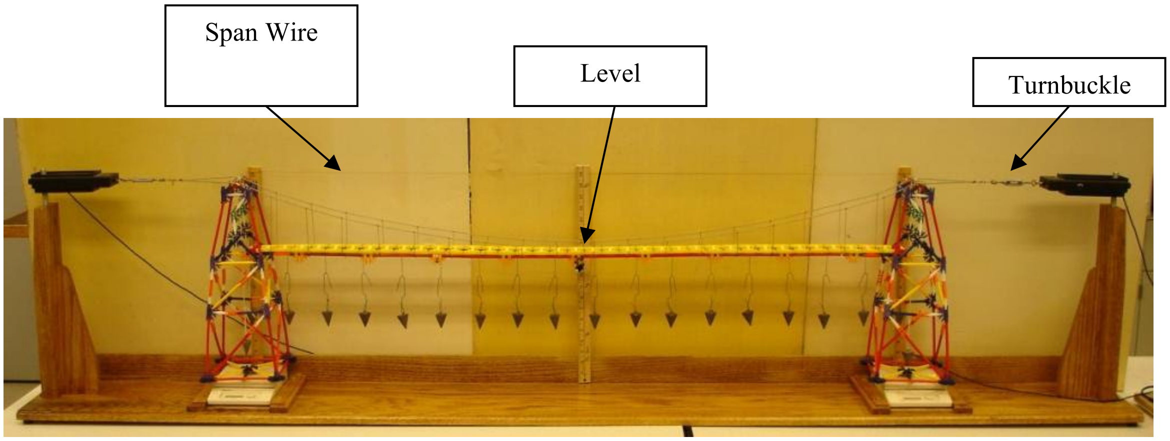

Figure 1: Menai Straits Experimental Model

Experiment 1:

Read the introduction to this lab before attempting to perform it.

Note: The two scales beneath the towers will turn off if left untouched for too long, so it is important to tare these scales immediately before you add weight to the bridge, and work quickly to get a reading before the scales power off.

Preliminary Set-Up:

Check the support conditions for the towers. At each tower, there should be a weight hanging freely that is directly over the center K'nex piece. The towers should be free to rotate at the intersection of the two knife-edges. Be sure that no part of the bridge is touching the wood base or the vertical rulers, as this would affect the results.

Ensure that the deck is level by looking from the sides of the towers with your eye at the height of the deck. Use the level located next to the deck to verify. Make sure the deck is not bowed or twisted. Adjust the deck using the turnbuckles if necessary, and try to adjust evenly between the two turnbuckles. Use the movable spring wire on the ruler to record the depth of the cable at mid-span. Make sure that the span wire is taut.

Measure and record the span (L) and the sag (d) of the cable in inches using a ruler (note: measure the height change of the suspension cable, not the height of the supports above the deck). Use the level to make sure you get an accurate reading.

Open Pasco Capstone worksheet "Menai.cap" from the "cee102_262" folder on the desktop. Select "Fast Monitor Mode" from the pull down menu near the bottom left of the screen, and click the "Monitor" button to the immediate left. Press the tare button on each force sensor, and turn on the scales beneath the bridge towers. When you have finished this step, the two scales under the towers and the displays on the computer will read zero. **If left too long, the scales under the towers will turn off and the displays on the monitor will drift. It is important, therefore, to take readings immediately after loading the bridge.**

Procedure

Note: The force sensors and the scales should be "tared" before each trial, so the output will reflect the applied live loads only and not the dead loads of the bridge itself.

1. Case A: You will place 18 small weights, each weighing a little over an ounce and marked with a "1," at even intervals along the bridge (this will be the loading q). First, place all 18 weights on the separate scale, convert the reading to pounds, record the value of Q, and calculate the uniformly distributed load, q.

   Q ___________________ q ___________________

2. Tare the force sensors and the scales beneath the towers. Distribute the weights on each of the hanging hooks along the deck. Observe how the deck deforms under these loads and sketch the deflected shape in Table 2 (below). **Be sure to measure the sag and label on your deflected shape!**
3. Adjust the two horizontal turnbuckles at the force sensors to return the deck to its original sag (d) as accurately as possible. Try to adjust the turnbuckles the same amount on the two sides.
4. Record the horizontal and vertical forces from the scales and force sensors **in pounds** in Table 1 below. Be sure not to wait too long to read the scales at the base of the towers because they will shut off and you will have to start over. Note that although only one horizontal and vertical reaction is recorded at each tower, two cables carry the loads.
5. Case B: Repeat the entire procedure and calculations for a second loading of two-ounce weights (each marked with a "2") at each suspender. In your lab report, you will consider how the actual factor of safety of Case A compares to that of Case B. Think about this.

   Q ___________________ q ___________________

Calculations (To be done while in lab for cases A and B).

1. Fill in Table 1
2. Fill in Table 2.
3. The experimental values that you recorded are for two cables. How much experimental horizontal tension force is carried by just one cable?

Table 1: Support Reactions for <u>TWO</u> cables

Load Case	*Theoretical*			*Experimental*			*% Error*		
	H	V_L	V_R	H	V_L	V_R	H	V_L	V_R
A									
B									

Experimental:

Case A Horizontal Tension Force of only one cable ____________________

Case B Horizontal Tension Force of only one cable ____________________

Table 2: Loading and Deflected Shape		
Load Case	*Sketch of Loading*	*Sketch of Deflected Shape*
A		
B		

Lab 2b: Quebec Bridge

Balanced Cantilevers and Continuous Beams

The purpose of this laboratory is to understand forces in a cantilever-constructed bridge. The laboratory is based on the Quebec Bridge completed in 1917. At the time it was erected, the Quebec Bridge was the longest cantilever bridge in the world with cantilever arms extending 580 feet beyond the main piers and supporting a 640-foot suspended span, giving the bridge the record-breaking 1800-foot clear span.

Background[2]

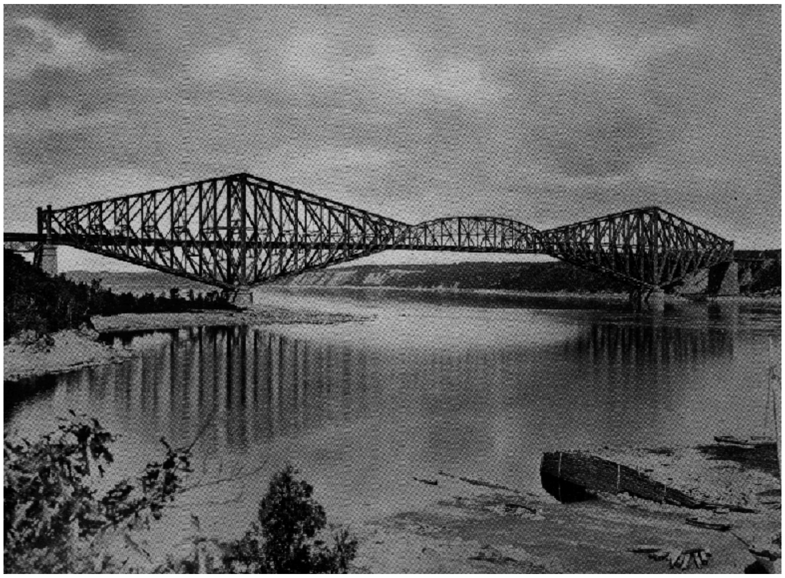

FIGURE 1: QUEBEC BRIDGE[3]

In 1890, the Phoenix Bridge Company of Pennsylvania was contracted to build a cantilever bridge with a main span of 1800 feet that would have eclipsed Scotland's Firth of Forth Bridge as the longest cantilever span in the world. By summer 1907, the structure was well advanced, with the cantilever arms projected out from both shores of the river, when a disastrous failure of the south arm plunged 76 workmen to their deaths. A Royal Commission attributed the failure to defective design and errors in judgment by the engineers.

A year later, the Canadian government appointed a board of engineers to try again, chief among them the noted American bridge designer Ralph Modjeski. The new bridge would have anchor arms of 515 feet, with cantilever arms extending 580 feet beyond the main piers and supporting a 640-foot suspended span, giving the bridge the same record-breaking 1800 foot clear span planned for the previous design. But this time the engineers proceeded with extreme caution. The bridge was designed to support much heavier loads than the earlier one had been, extensive tests of materials were made, and design calculations were checked and rechecked. Even though the designers used newly developed nickel alloy steel that could safely support stresses 40 percent greater than could the carbon steel used in the earlier design, the critical lower chord members of the cantilevers arms were made several times larger and heavier than those of the failed bridge.

Work began on the new bridge in 1909 and was nearing completion seven years later when disaster struck again. The 5000-ton center span was being lifted into place when a bearing failed, allowing

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[2] Story of the Quebec Bridge courtesy of http://geocities.com/Colosseum/Bench/7918/cssq/ponta.html

[3] Photo courtesy of http://www.civeng.carleton.ca/Exhibits/Quebec_Bridge/intro.html

the span to fall into the river. This time, 13 workers were killed. A year later, a new span was successfully lifted into place, and in October 1917, the first train crossed the great bridge. The bridge has stood firmly astride the St. Lawrence ever since, helping to link the Maritime Provinces and eastern Quebec with the rest of Canada. Now the property of Canadian National Railway, the bridge carries some freight as well as VIA Rail Canada's Quebec-Montreal passenger trains. The record its builders set in 1917 still stands, for the Quebec Bridge remains the longest railroad cantilever span ever built.

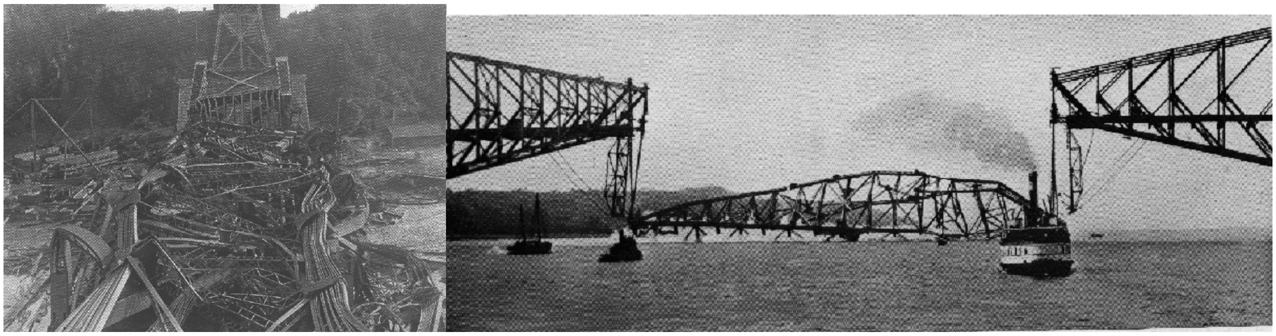
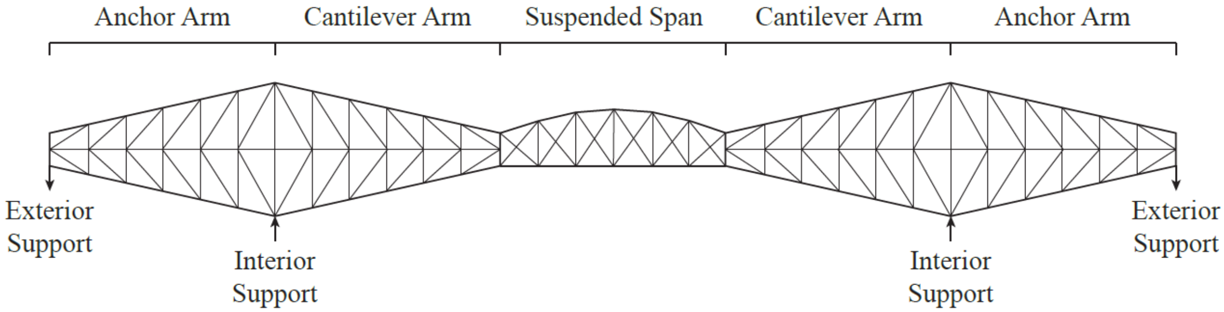


FIGURE 2: QUEBEC BRIDGE COLLAPSES OF 1907 AND 1916[4]

Experimental Procedure

Part I: Cantilever Construction

During construction, the bridge behaves as a large cantilever. If weight is added to the end of the cantilever arm, it will begin to overturn. This will create an upward reaction on the interior support and a downward reaction on the exterior support (see lecture slides on the Firth of Forth Bridge). We can predict the values of these reactions using the equations of static equilibrium. **Have your TA detach the center (suspended) span of the model.**



In order to simulate the load due to constructing the suspended span, you will use the lead weights to apply a concentrated load of approximately 7-ounces to the free end of the model. Conduct this on the cantilever section on the right. When this load is applied, will the top chord of the truss be in tension or compression? This question will be considered in your report guide.

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[4] Photos courtesy of http://www.civeng.carleton.ca/Exhibits/Quebec_Bridge/intro.html

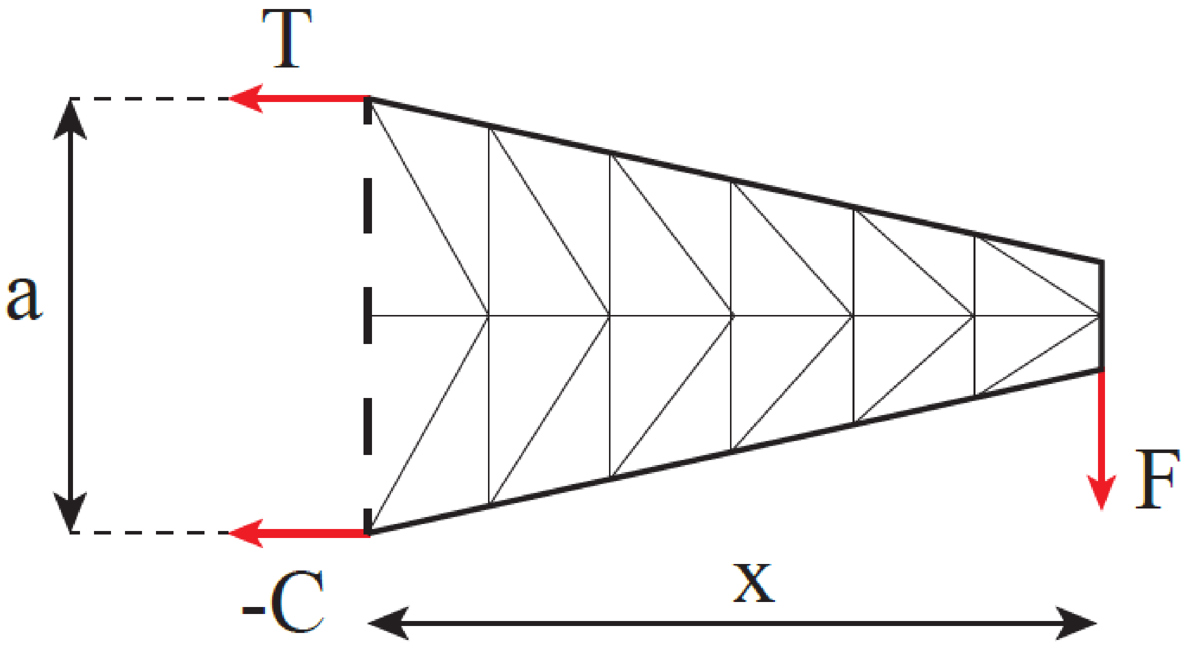
1. Measure the distance between the interior and exterior supports, as well as the length of the cantilever segment, and the length of the suspended span. Next, measure the vertical depth of the model at the interior support and at a point halfway between the interior support and the free end of the model.

2. Use the digital scale to weigh three 2-ounce weights and one 1-ounce weight. Record the exact weight (convert to lbs).

3. Determine the theoretical vertical reactions.

4. Use the identity M = F*x to predict the internal bending moment at the interior support of the bridge. Repeat this calculation for a point located halfway between the interior support and the free end of the model.

5. Use the identity T = -C = M/a (where 'a' is a vertical depth) to find the tension and compression forces in the upper and lower chords of the truss at the interior support. (Recall that the Quebec Bridge collapsed in 1907 because these forces were too large.)



6. Open Capstone worksheet "Quebec.cap" from the "cee102_262" folder on the desktop. Select "Fast Monitor Mode" from the pull down menu near the bottom left of the screen. Then click the "Monitor" button to the immediate left.

7. Tare (zero) the scale at the interior support, and tare the force sensor at the exterior support by pushing the button on the side of the sensor.

8. Hang the weights from the free-end of the model and record the support reactions. Note the direction of each reaction. Compare these experimental results to your predicted values.

Repeat the above procedure and calculations for a load of **seven 2-ounce weights**.

Part II: Suspended Span

Once the cantilever arm was constructed, the suspended span was prefabricated on the ground and then lifted into place using a large crane. Once this segment is fastened to the cantilever arms, it places a large point-load on each cantilever arm. **Have your TA re-attach the suspended span of the model.**

For this section, you will hang seven 2-ounce weights along the suspended span, thus imparting a total load of 14-ounces on the bridge. This distributed load will behave in the same way as if we applied a 7-ounce concentrated load at the free end of each cantilever arm.

1. Weigh seven 2-ounce weights on the scale and record the total weight.

2. Determine the theoretical vertical reactions.

3. Tare the scales at the interior supports, and tare the force sensors at the exterior supports.

4. Hang the seven 2-ounce weights along the suspended span. The weights should be evenly spaced. For convenience, hang them from the k'nex marked with green tape.

5. Record the support reactions and note the direction of each reaction. Compare these experimental results to your predicted values.

Repeat the above procedure and calculations for **fourteen 2-ounce weights.**

Compare your results from Part II with the results from Part I. Do you find similar results?

Part III: Continuous Beam

Once the bridge is made continuous, it will undergo loading due to heavy rail-traffic. In this section, you will load the bridge with three distributed load cases and record the support reactions.

1. Use the scale to weigh 25 2-ounce weights and record the value.

2. Tare the scales and force sensors.

3. Hang these weights along the entire bridge length, one on each green piece of tape. Record all four support reactions and note the direction of each reaction. Remove the weights.

Distance Measurements (to include in your sketch below)

Distance between interior and exterior supports ______________
Length of cantilever segment _____________
Length of suspended span ____________
Vertical depth of model at interior support _______________
Vertical depth at point halfway between interior support and free end ______________________

Table 1: Part 1 Results

	Theoretical							Experimental	
Load Case	Exact Weight (lbs)	Ext. Reaction	Int. Reaction	M at Int. Support	M at Halfway	T in Chord	C in Chord	Ext. Reaction	Int. Reaction
≈ 7 oz									
≈ 14 oz									

Table 2: Part 2 & Part 3 Results

		Theoretical Reactions				Experimental Reactions			
		Exterior		Interior		Exterior		Interior	
Load Case	Exact Weight (lbs)	Left	Right	Left	Right	Left	Right	Left	Right
≈ 14 oz total weight									
≈ 28 oz total weight									
Entire Bridge Loaded		N/A	N/A	N/A	N/A